A STUDY OF GENERALIZED SECOND LAW OF THERMODYNAMICS IN MAGNETIC UNIVERSE IN THE LIGHT OF NON-LINEAR ELECTRODYNAMICS

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Outline

1. The magnetic universe in non-linear electrodynamics is considered
2. The interaction between matter and magnetic field has been studied
3. The validity of the generalized second law of thermodynamics (GSLT) of the magnetic universe bounded by Hubble, apparent, particle and event horizons is discussed using Gibbs' law and the first law of thermodynamics

Introduction

Studying the equations of the non-linear electrodynamics (NLED) is an attractive subject of research in general relativity. Exact solutions of the Einstein field equations coupled to NLED may hint at the relevance of the non-linear effects in strong gravitational and magnetic fields. An exact regular black hole solution has recently been obtained proposing Einstein-dual non-linear electrodynamics. Also the General Relativity coupled with NLED effects can explain the primordial inflation.

Brief overview of non-linear electrodynamics

The Lagrangian density in the Maxwell electrodynamics $L=-\frac{1}{4\mu_0}F^{\mu\nu}F_{\mu\nu}=-\frac{1}{4\mu_0}F$ where $F^{\mu\nu}$ is electromagnetic field strength, $\mu_0\rangle 0$ is magnetic permeability and $F=2(B^2-E^2)$

The energy momentum tensor $T_{\mu\nu}=\frac{1}{\mu_0}\left(F_{\mu\alpha}F_{\nu}^{\alpha}+\frac{1}{4}Fg_{\mu\nu}\right)$

Let us take the homogeneous, isotropic FRW model of the universe $ds^2=dt^2-a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+Sin^2\theta\, d\phi^2)\right]$

The energy density and the pressure of the NLED field should be evaluated by averaging over volume. We define the volumetric spatial average of a quantity X at the time t by $\langle X\rangle\equiv\lim_{V\to V_0}\frac{1}{V}\int X\sqrt{-g}\,d^3x$ and impose the following restrictions for the electric field E_i and the magnetic field B_i for the electromagnetic field to act as a source of the FRW model

$$\langle E_i\rangle=0,\ \langle B_i\rangle=0,\ \langle E_iE_j\rangle=-\frac{1}{3}E^2g_{ij},\ \langle B_iB_j\rangle=-\frac{1}{3}B^2g_{ij},\ \langle E_iB_j\rangle=0$$

Additionally, it is assumed that the electric and magnetic fields, being random fields, have coherent lengths that are much shorter than the cosmological horizon scales. Using all these and comparing

$\langle F_{\mu\alpha}F_{\nu}^{\alpha}\rangle=\frac{2}{3}\left(\varepsilon_0E^2+\frac{B^2}{\mu_0}\right)u_\mu u_\nu+\frac{1}{3}\left(\varepsilon_0E^2-\frac{B^2}{\mu_0}\right)g_{\mu\nu}$ with the average value of the energy momentum tensor $\langle T_{\mu\nu}\rangle$ we have

$\rho=\frac{1}{2}\left(\varepsilon_0E^2+\frac{B^2}{\mu_0}\right)$ and $p=\frac{1}{3}\rho\rightarrow$ This shows that Maxwell electrodynamics generates the radiation type fluid in FRW universe

Now let us consider the generalization of the Maxwell electromagnetic Lagrangian up to the second order terms

$$L=-\frac{1}{4\mu_o}F+\omega F^2+\eta F^{*^2},\ \omega\rangle 0 \text{ and } \eta \text{ are arbitrary constants}, F^*\rightarrow \text{dual of } F$$

Corresponding energy momentum tensor $T_{\mu\nu}=-4\frac{\partial L}{\partial F}F_{\mu}^{\alpha}F_{\alpha\nu}+\left(\frac{\partial L}{\partial F^*}F^*-L\right)g_{\mu\nu}$

Now we consider that the electric field *E* in plasma rapidly decays $\rightarrow B^2$ dominates over E^2. Thus $F=2B^2$

In this case the energy density and the pressure for magnetic field become

$\rho_B=\frac{B^2}{2\mu_0}\left(1-8\mu_0\omega B^2\right)$ and $p_B=\frac{B^2}{6\mu_0}\left(1-40\mu_0\omega B^2\right)=\frac{1}{3}\rho_B-\frac{16}{3}\omega B^4$ where B must satisfy the inequality $B\langle\frac{1}{2\sqrt{2\mu_0\omega}}$

Interaction Between Matter and Magnetic Field

Considering Einstein field equations $H^2+\frac{k}{a^2}=\frac{8\pi G}{3}\rho_{total}$

$$\dot{H}-\frac{k}{a^2}=-4\pi G(\rho_{total}+p_{total})$$

The energy conservation equation becomes $\dot{\rho}_{total}=-3H(\rho_{total}+p_{total})$

where the energy density and pressure for matter obeys the equation of state $p_m=\omega_m\rho_m$

Now let us consider an interaction Q between matter and magnetic field as $Q=3\delta H\frac{B}{\mu_0}(1-16\mu_0\omega B^2),\ \delta\rangle 0$

Then the conservation equation can be solved to obtain

$$B=-\frac{3}{2}\delta+\frac{B_0}{a^2}\ \text{ and }\ \rho_m=\frac{3\delta}{2\mu_0}\left[-\frac{32\mu_0\omega B_0^{\ 2}}{3(\omega_m-1)a^6}+\frac{144\delta\mu_0\omega B_0^{\ 2}}{(3\omega_m-1)a^4}-\frac{\delta(1-36\mu_0\omega\delta^2)}{\omega_m+1}-\frac{2B_0(108\mu_0\omega\delta^2-1)}{(3\omega_m+1)a^2}\right]+\rho_0a^{-3(1+\omega_m)}$$

$$\rho_0, B_0 \text{ int const}$$

Generalized Second Law of Thermodynamics

The sum of entropy of total matter enclosed by the horizon and the entropy of the horizon does not decrease with time.

Let us consider Hubble, apparent, particle and event horizons with radii $R_H=\frac{1}{H},\ R_A=\frac{1}{\sqrt{H^2+\frac{k}{a^2}}},\ R_P=a\int_0^a\frac{da}{Ha^2},\ R_E=a\int_a^\infty\frac{da}{Ha^2}$

Considering the net amount of energy crossing through the horizons in time dt as $-dE=4\pi R_{horizon}^3H(\rho_{total}+p_{total})dt$, assuming the first law of thermodynamics $-dE=TdS_{horizon}$ on the horizons and from the Gibbs' equation $TdS_I=dE_I+p_{total}dV$, we get the rate of change of total entropy. Here $E_I=\frac{4}{3}\pi R_{horizon}^3\rho_{total}\rightarrow$ internal energy and $S_I\rightarrow$ entropy inside the horizon

Validity of GSLT of The Universe Bounded by Different Horizons

Hubble Horizon

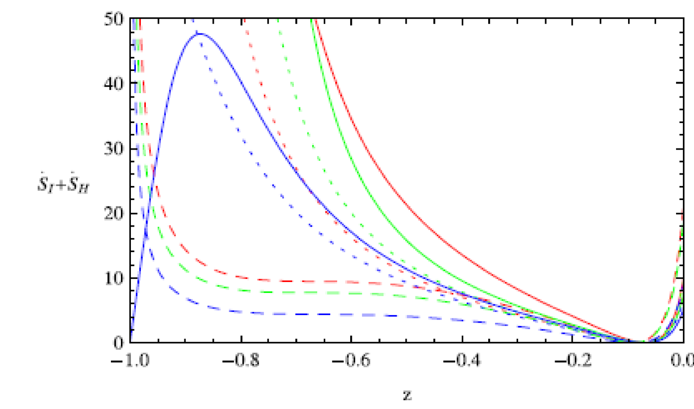

Fig. 1. Figure represents rate of change of total entropy of Hubble horizon, i.e., $\dot{S}_H+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ without interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

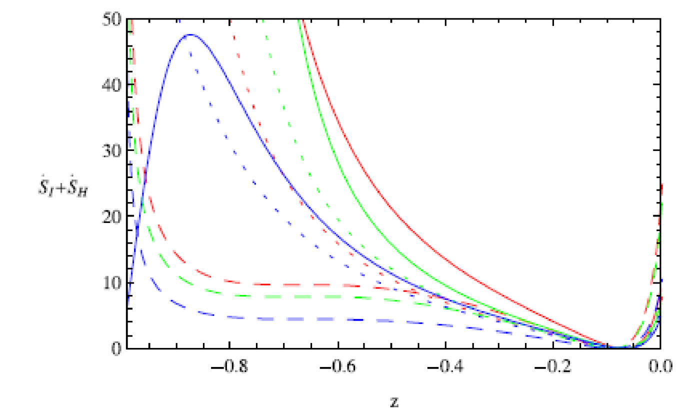

Fig. 2. Figure represents rate of change of total entropy of Hubble horizon, i.e., $\dot{S}_H+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ with interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Apparent Horizon

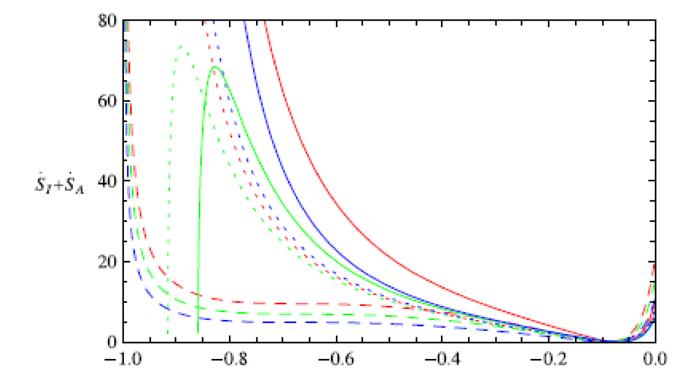

Fig. 3. Figure represents rate of change of total entropy of apparent horizon, i.e., $\dot{S}_A+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ without interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

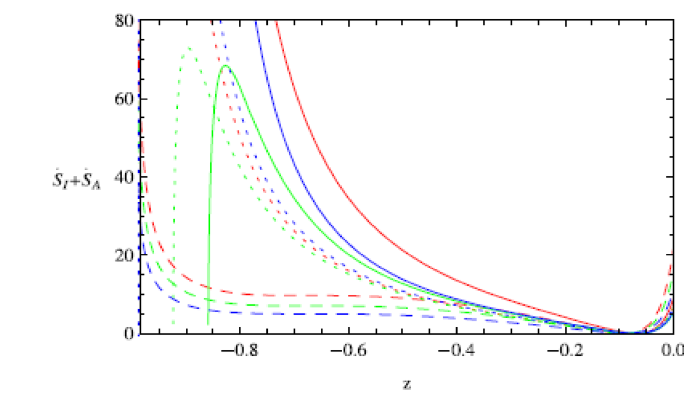

Fig. 4. Figure represents rate of change of total entropy of apparent horizon, i.e., $\dot{S}_A+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ with interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Particle Horizon

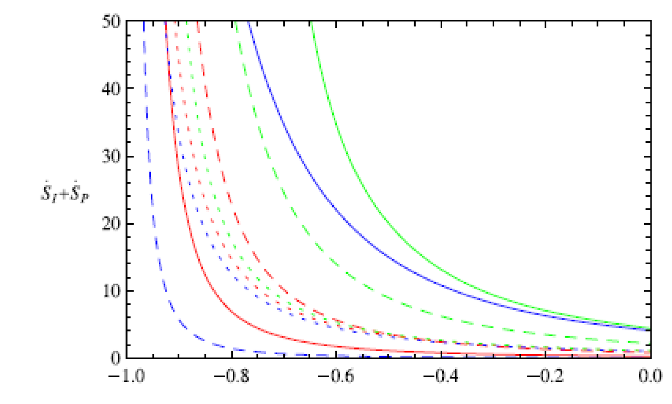

Fig. 5. Figure represents rate of change of total entropy of particle horizon, i.e., $\dot{S}_P+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ without interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

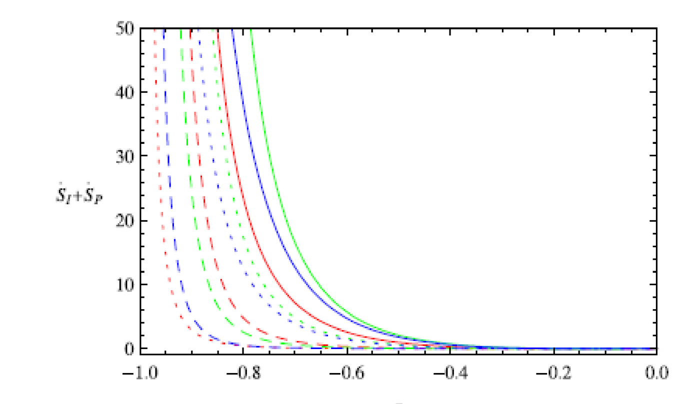

Fig. 6. Figure represents rate of change of total entropy of particle horizon, i.e., $\dot{S}_P+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ with interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Event Horizon

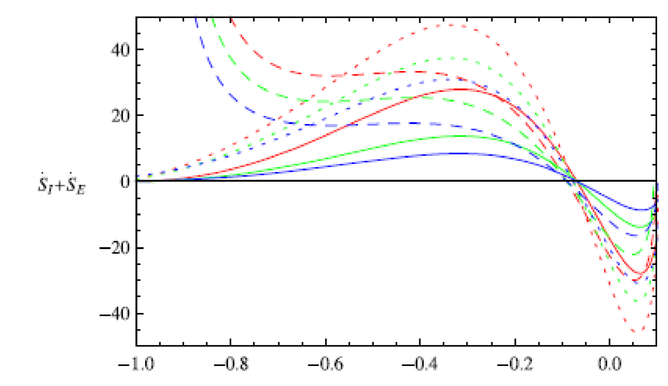

Fig. 7. Figure represents rate of change of total entropy of event horizon, i.e., $\dot{S}_E+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ without interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

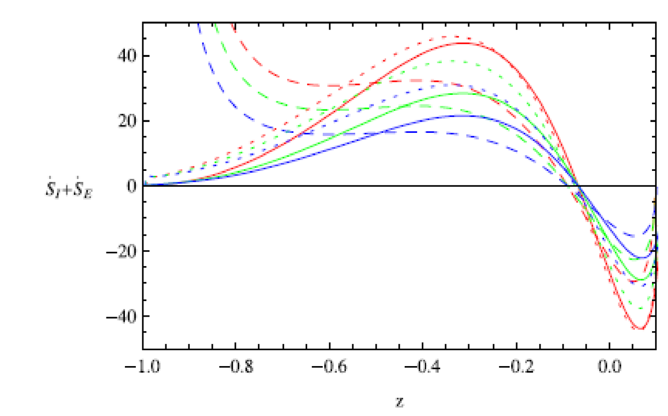

Fig. 8. Figure represents rate of change of total entropy of event horizon, i.e., $\dot{S}_E+\dot{S}_I$ against redshift $z=(\frac{1}{a}-1)$ with interaction for $w_m=1/3$ (solid line), $w_m=0$ (dotted line) and $w_m=-0.5$ (dashed line) and $k=0$ (red line), $k=+1$ (green line) and $k=-1$ (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Conclusions

1. From figs. 1–6, we see that the rate of change of total entropy is always positive level for interacting and non-interacting scenarios of the magnetic universe bounded by Hubble, apparent and particle horizons and therefore GSLT is always satisfied for them in magnetic universe
2. Figs. 7–8 show that the rate of change of total entropy is negative up to a certain stage (about $z>-0.1$) and positive after that for non-interacting and interacting scenarios of the magnetic universe bounded by event horizon. Thus in this case, GSLT cannot be satisfied initially but in the late time, it is satisfied.

Key References

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