QNM and Thermodynamical Aspects of the 3D Lifshitz Black Hole

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Framework

- ♣ Lifshitz spacetimes

$$ds^2 = -\frac{r^{2z}}{l^{2z}}dt^2 + \frac{l^2}{r^2}dr^2 + \frac{r^2}{l^2}d\vec{x}^2\,, \tag{1}$$

where z is the dynamical exponent.

- ♣ They are possible gravity duals for Lifshitz fixed points.
- ♣ BH solutions in the gravity side $\longrightarrow$ Finite temperature in the gauge side.

Lifshitz Black Holes in NMG

The action of NMG is given by [E.Bergshoeff, O.Hohm, and P.Townsend, PRL 102, 201301 (2009); E.Ayón-Beato, A.Garbarz, G.Giribet, and M.Hassaine, PRD 80, 104029 (2009).]

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\,\left[R - 2\lambda - \frac{1}{m^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\right)\right]. \qquad (2)$$

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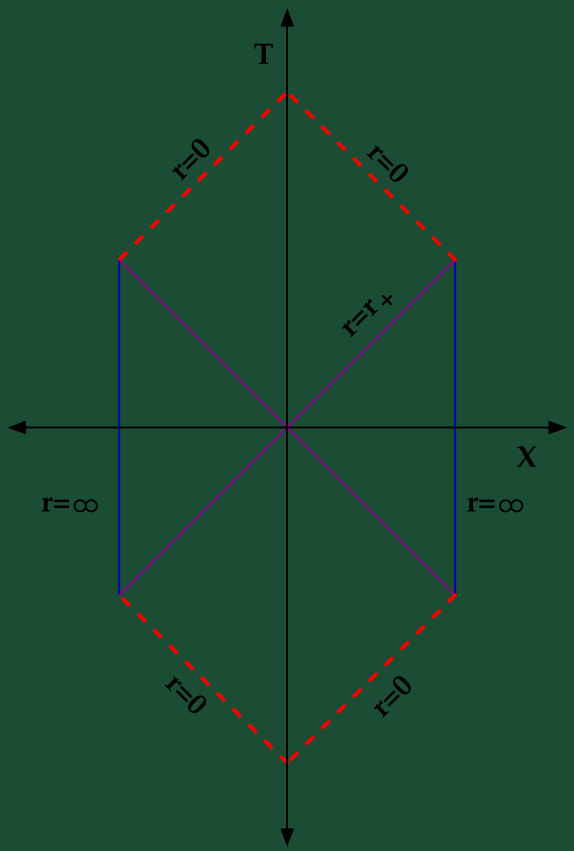
Black hole solutions:

- At $m^2l^2 = 1/2$, $z = 1 \rightarrow$ BTZ.
- At $m^2l^2 = -1/2$, $z = 3$,

$$ds^2 = -a(r)\frac{\Delta}{r^2}dt^2 + \frac{r^2}{\Delta}dr^2 + r^2d\phi^2 , \tag{3}$$

with $a(r) = \frac{r^4}{l^4}$, $\Delta = -Mr^2 + \frac{r^4}{l^2}$, and $\lambda = -\frac{13}{2l^2}$.

Penrose-Carter Diagram



(4)

Figure 1: Diagram for the Lifshitz BH with horizon $r_+ = l\sqrt{M}$.

Scalar Perturbation

We consider a scalar field $\Phi(t,r,\phi)=\Psi(t,r)e^{i\kappa\phi}$ obeying

$$\Box\Phi=\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}\partial_N\right)\Phi=m^2\Phi\,. \tag{5}$$

Thus,

$$-\partial_t^2\Psi+\frac{r^4}{l^6}\left(1-\frac{Ml^2}{r^2}\right)\left(\frac{5r^3}{l^2}-3Mr\right)\partial_r\Psi+\frac{r^8}{l^8}\left(1-\frac{Ml^2}{r^2}\right)^2\partial_r^2\Psi- \\ -\frac{r^4}{l^6}\left(m^2r^2+\kappa^2\right)\left(1-\frac{Ml^2}{r^2}\right)\Psi = 0\,. \tag{6}$$

Analytical Solution

Setting $\Psi(t,r) = R(r)e^{-i\omega t}$ and $r = l\sqrt{M}/y$,

$$R(y) = C_1 y^{2+\alpha}(1-y^2)^{\beta/2}\,\mathsf{HeunC}\left(0,\alpha,\beta,-\frac{\beta^2}{4},\frac{\alpha^2}{4}+\frac{\kappa^2}{4M},y^2\right) + \\ +C_2 y^{2-\alpha}(1-y^2)^{\beta/2}\,\mathsf{HeunC}\left(0,-\alpha,\beta,-\frac{\beta^2}{4},\frac{\alpha^2}{4}+\frac{\kappa^2}{4M},y^2\right)\,, \quad (7)$$

where $\alpha = \sqrt{4+m^2l^2}$ and $\beta = -i\,l\omega/M^{3/2}$.

Imposing the Dirichlet condition at infinity and ingoing waves at the horizon the QNF are

$$\omega = 2i\frac{M^{3/2}}{l}\left[1+2N+\sqrt{4+m^2l^2}- -\sqrt{7+\frac{3}{2}m^2l^2+\frac{\kappa^2}{2M}+(3+6N)\sqrt{4+m^2l^2}+6N(N+1)}\right]. \quad (8)$$

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The asymptotic frequency ($N\to\infty$) is given by

$$\omega_\infty = -2(\sqrt{6}-2)\,i\,\frac{M^{3/2}}{l}N<0\,. \quad (9)$$

Numerical Solution

We rewrite the KG equation in terms of the tortoise coordinate

$$r_* = l^4 \left[-\frac{1}{M^{3/2} l^3} \operatorname{arccoth}\left(\frac{r}{l\sqrt{M}} \right) + \frac{1}{M l^2 r} \right] , \tag{10}$$

as

$$-\partial_t^2 X + \partial_{r_*}^2 X = V(r) X , \tag{11}$$

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with $X(t, r_*) = \sqrt{r}\,\Psi$, and the potential

$$\begin{aligned} V(r) &= \left(\frac{7}{4l^8} + \frac{m^2}{l^6} \right) r^6 - \left(\frac{5M}{2l^6} + \frac{M m^2}{l^4} - \frac{\kappa^2}{l^6} \right) r^4 + \\ &\quad + \left(\frac{3M^2}{4l^4} - \frac{M\kappa^2}{l^4} \right) r^2 \,. \end{aligned} \tag{12}$$

(13)

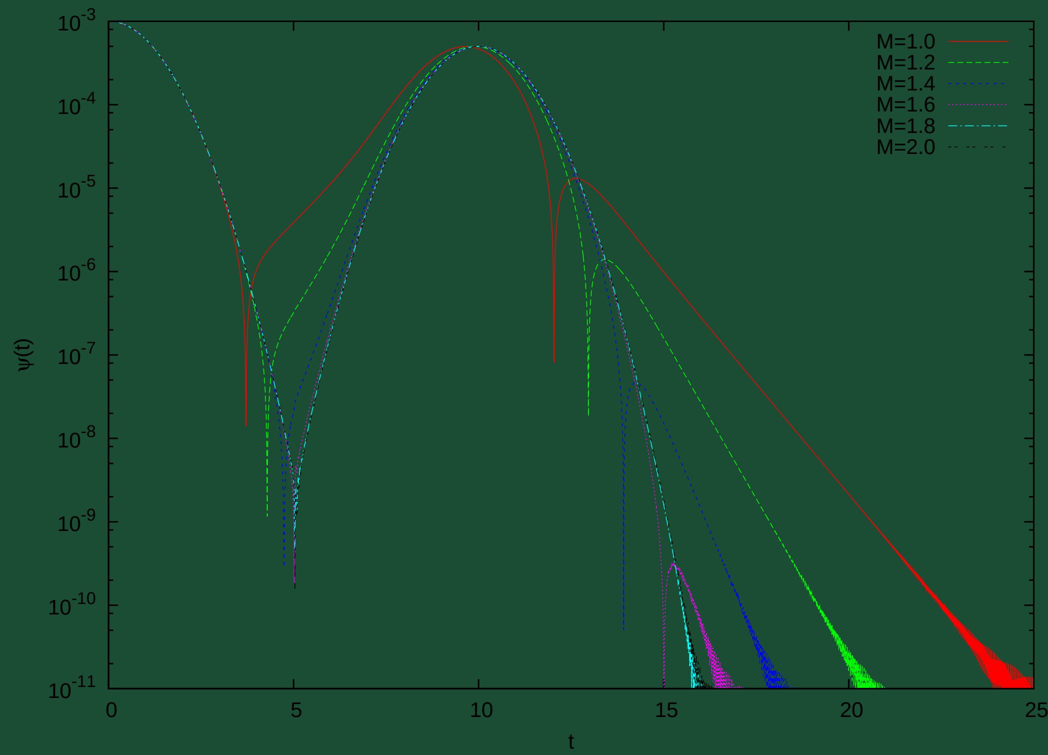

Figure 2: Decay of scalar field with $m = 1$, $l = 1$.

(14)

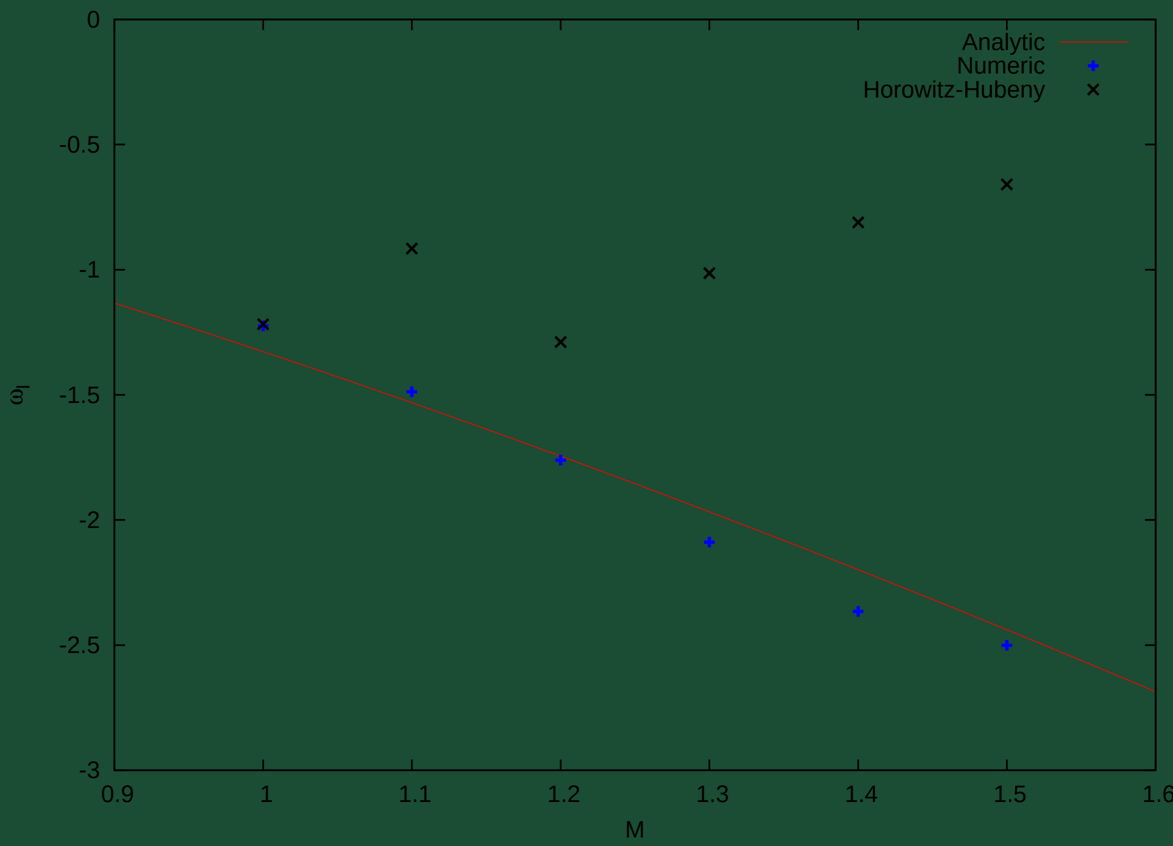

Figure 3: Imaginary part of the QNF using analytical, numerical, and HH methods.

Spinorial Perturbation

We consider a two component spinor Ψ obeying the Dirac equation,

$$i\gamma^{(a)}{e_{(a)}}^{\mu}\nabla_{\mu}\Psi - \mu_s\Psi = 0\,. \tag{15}$$

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Analogously to the scalar case, after using the tortoise coordinate and redefining Ψ, the components of this Eq. can be reduced to

$$\left(\partial^2_{\hat{r}_*} + \omega^2\right) X_{\pm} = V_{\pm} X_{\pm}\,. \tag{16}$$

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The superpartner potential for a massless spinor reduces to

$$V_{\pm} = \left(-\frac{m^2M}{l^4} \mp \frac{mM}{l^5}\sqrt{r^2 - Ml^2}\right) r^2 + \left(\frac{m^2}{l^6} \pm \frac{2m}{l^7}\sqrt{r^2 - Ml^2}\right) r^4\,. \tag{17}$$

(18)

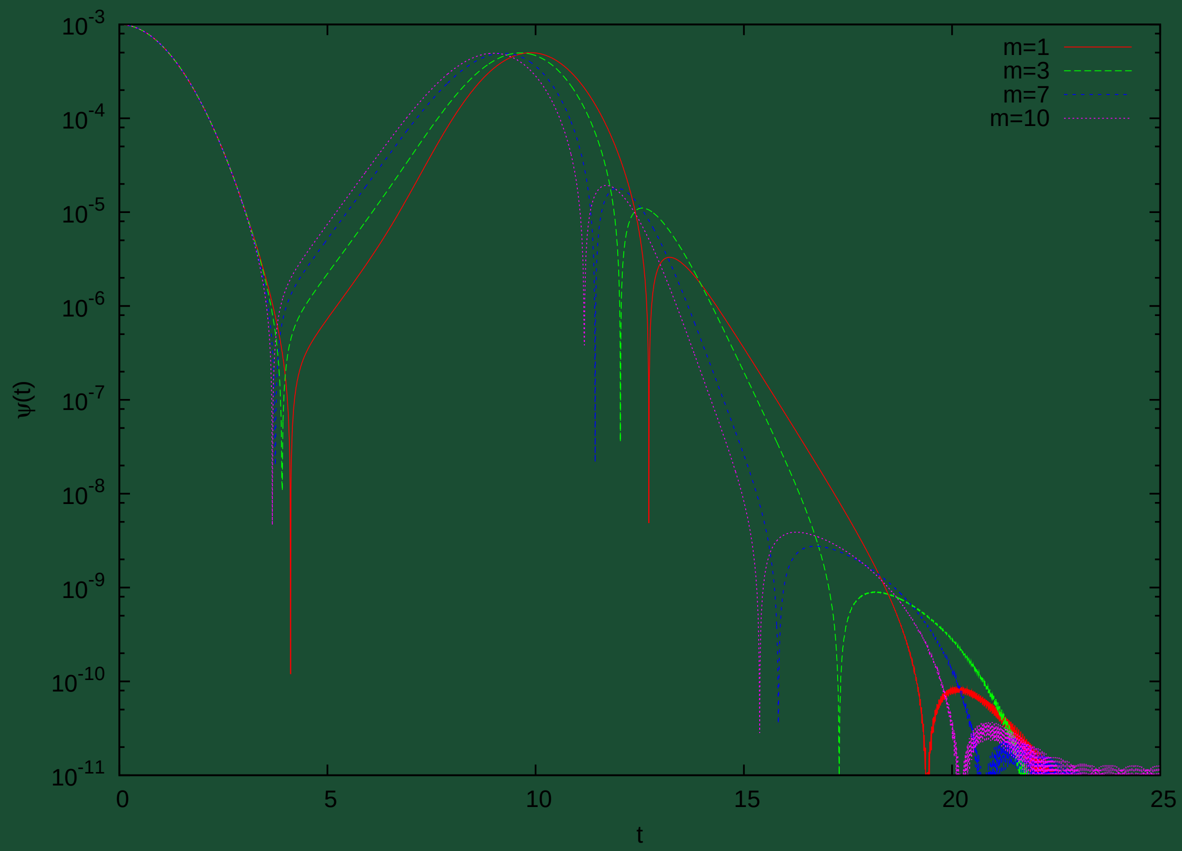

Figure 4: Decay of massless field with $l = 1$ and $M = 1.5$.

Area Spectrum

The proper physical frequency of the damped harmonic oscillator equivalent to QNM is given by [M.Maggiore, PRL 100, 141301 (2008).]

$$\omega_p = \sqrt{\omega_R^2 + \omega_I^2} = \omega_\infty = 2(\sqrt{6} - 2)\frac{M^{3/2}}{l}N\,. \qquad (19)$$

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This can be inserted in the adiabatic invariant

$$I = \int \frac{d\mathcal{M}}{\Delta\omega} = \int \frac{M}{\Delta\omega} dM\,, \tag{20}$$

which is Bohr-Sommerfeld quantized in the semiclassical limit, to finally obtain

$$A = 2\pi(\sqrt{6} - 2) n\hbar\,. \tag{21}$$

As $S \propto A$ for this BH, $\Rightarrow S$ is also quantized with an evenly spaced spectrum.

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♣ We show that this solution is stable under scalar and spinorial perturbations.

♣ In the scalar case we found that analytical and finite differences method are in perfect agreement, while HH method can not be applied.

♣ The area and entropy of this BH are quantized and have an equally spaced spectrum.