Astrophysical tests of general relativity and black hole bombs

1) **Quasinormal modes: tests within GR**
   No-hair theorem, area theorem
   Formation history of supermassive black holes

2) **Massive scalar-tensor theories: tests of GR**
   Weak field constraints
   [Gravitational-wave observations]

3) **Superradiance of massive bosonic fields in Kerr**
   Scalar fields: floating orbits
   Vector fields: best bounds on photon mass

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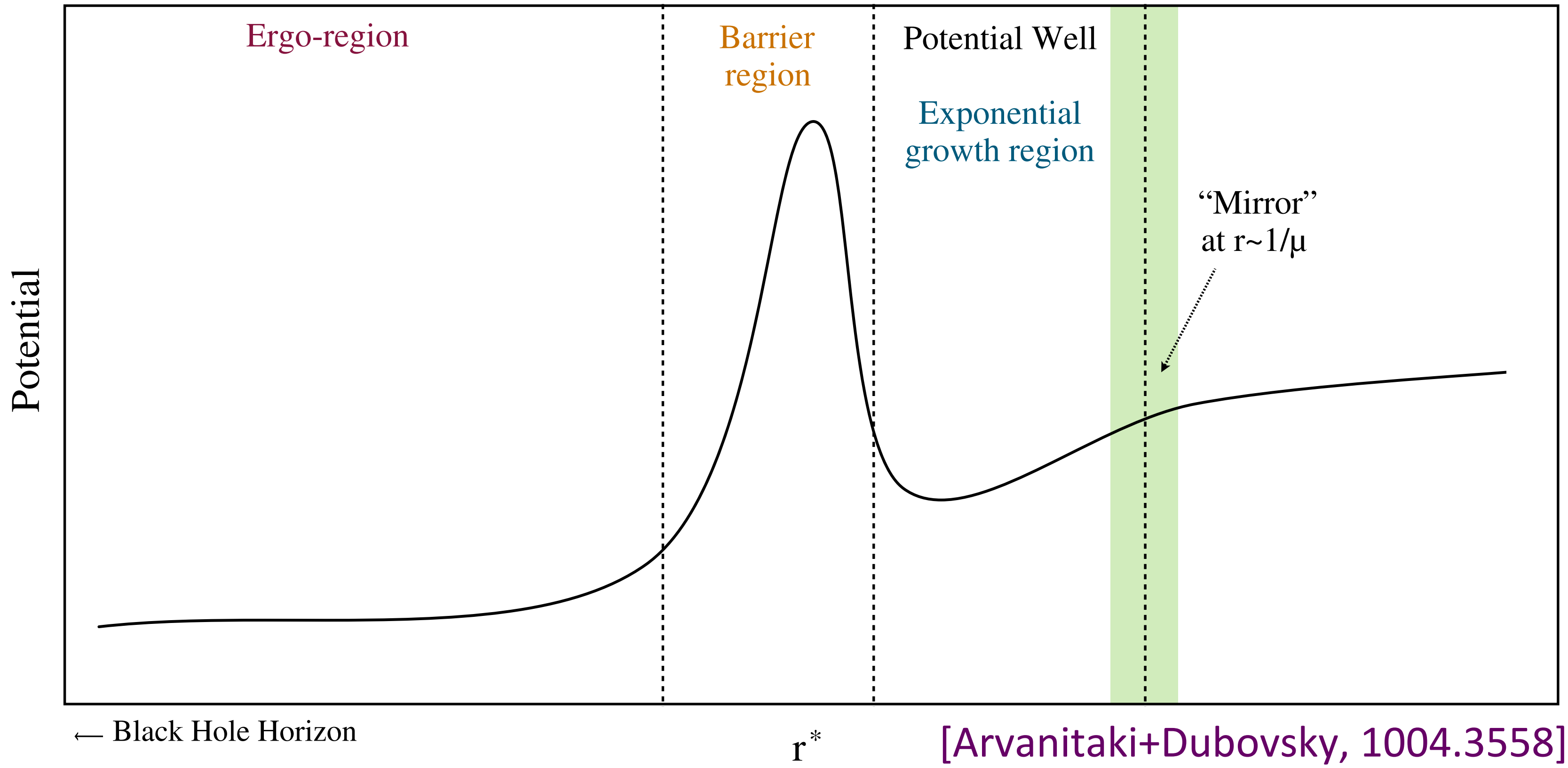
NEB15, Chania, Crete, June 20 2012

Wave scattering in rotating black holes



[Arvanitaki+Dubovsky, 1004.3558]

Quasinormal modes:

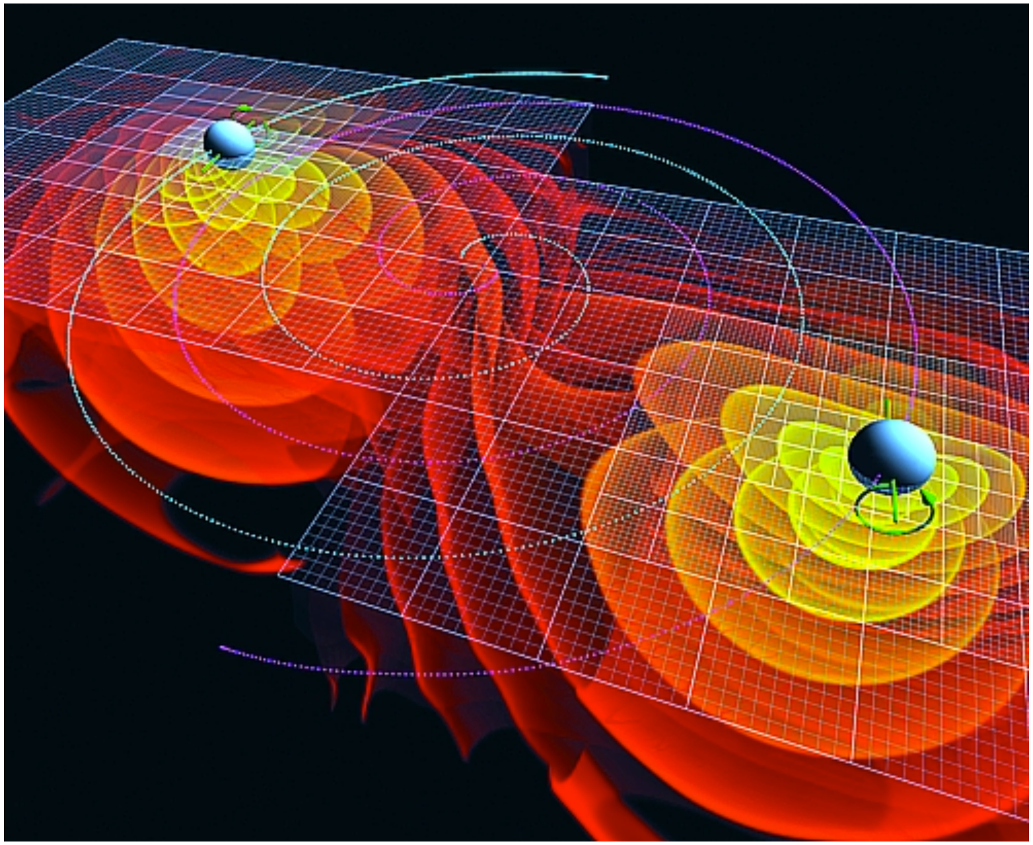
- Ingoing waves at the horizon, outgoing waves at infinity
- Discrete spectrum of damped exponentials ("ringdown") [EB++, 0905.2975]

Massive scalar field:

- Superradiance: black hole bomb when $0 < \omega < m\Omega_H$
- Hydrogen-like, unstable bound states [Detweiler, Zouros+Eardley…]

Quasinormal modes

[Visualization: NASA Goddard]



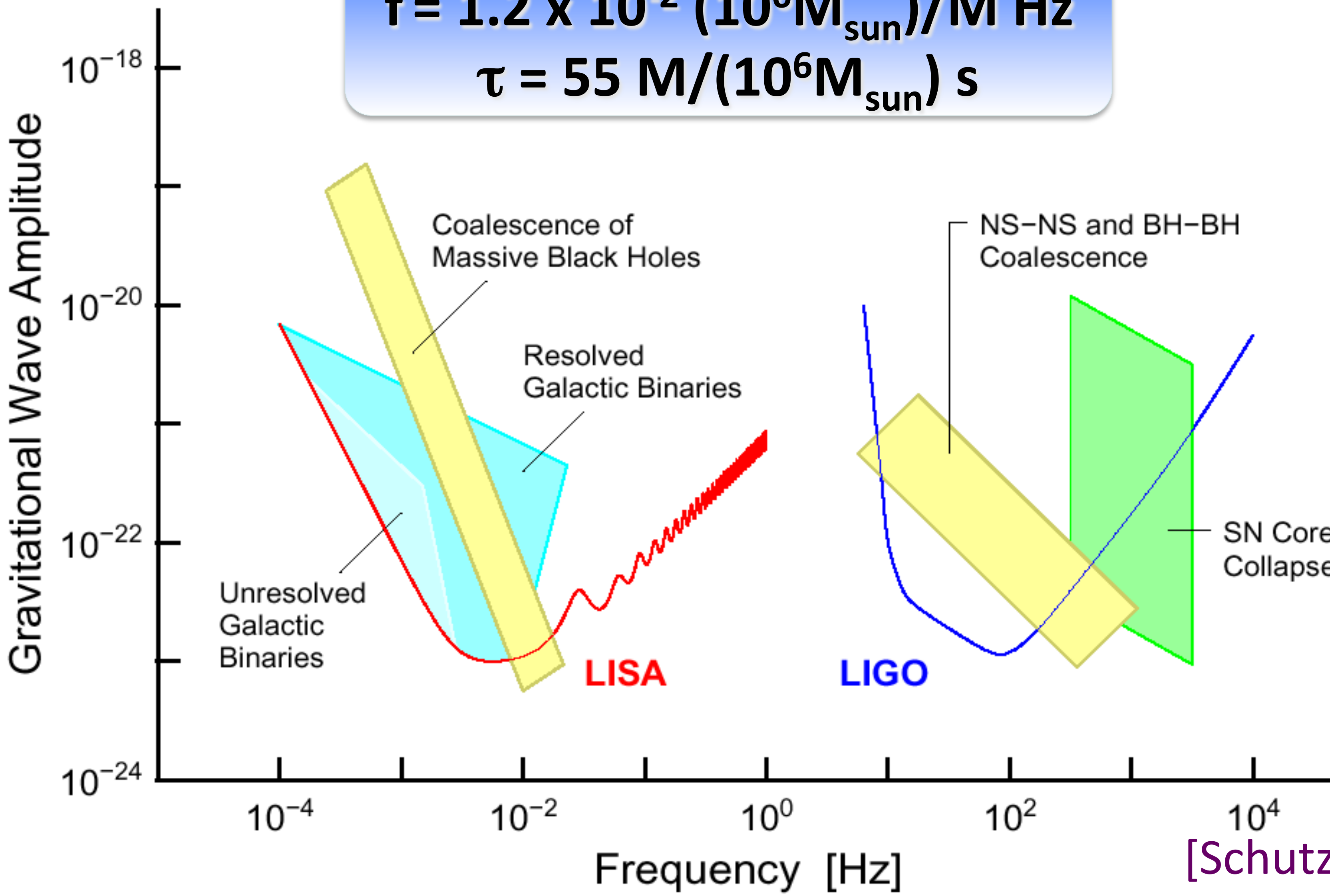
- In GR, each mode determined uniquely by mass and spin
- One mode: **(M,a)**
  Any other mode frequency:
  No-hair theorem test
- Relative mode amplitudes: pre-merger parameters [Kamaretsos++,Gossan++]
- Feasibility depends on SNR:
  Need SNR>30 [EB++, 2005/07]
  1) Noise $S(f_{QNM})$
  2) Signal $h \sim E^{1/2}$, $E = \varepsilon_{rd} M$
  $\varepsilon_{rd} \sim 0.01(4\eta)^2$ for comparable-mass mergers, $\eta = m_1 m_2/(m_1+m_2)^2$

$$f = 1.2 \times 10^{-2} \, (10^6 M_{sun})/M \text{ Hz}$$
$$\tau = 55 \, M/(10^6 M_{sun}) \text{ s}$$

(e)LISA vs. LIGO

$f = 1.2 \times 10^{-2} (10^6 M_{sun})/M$ Hz

$\tau = 55\ M/(10^6 M_{sun})$ s

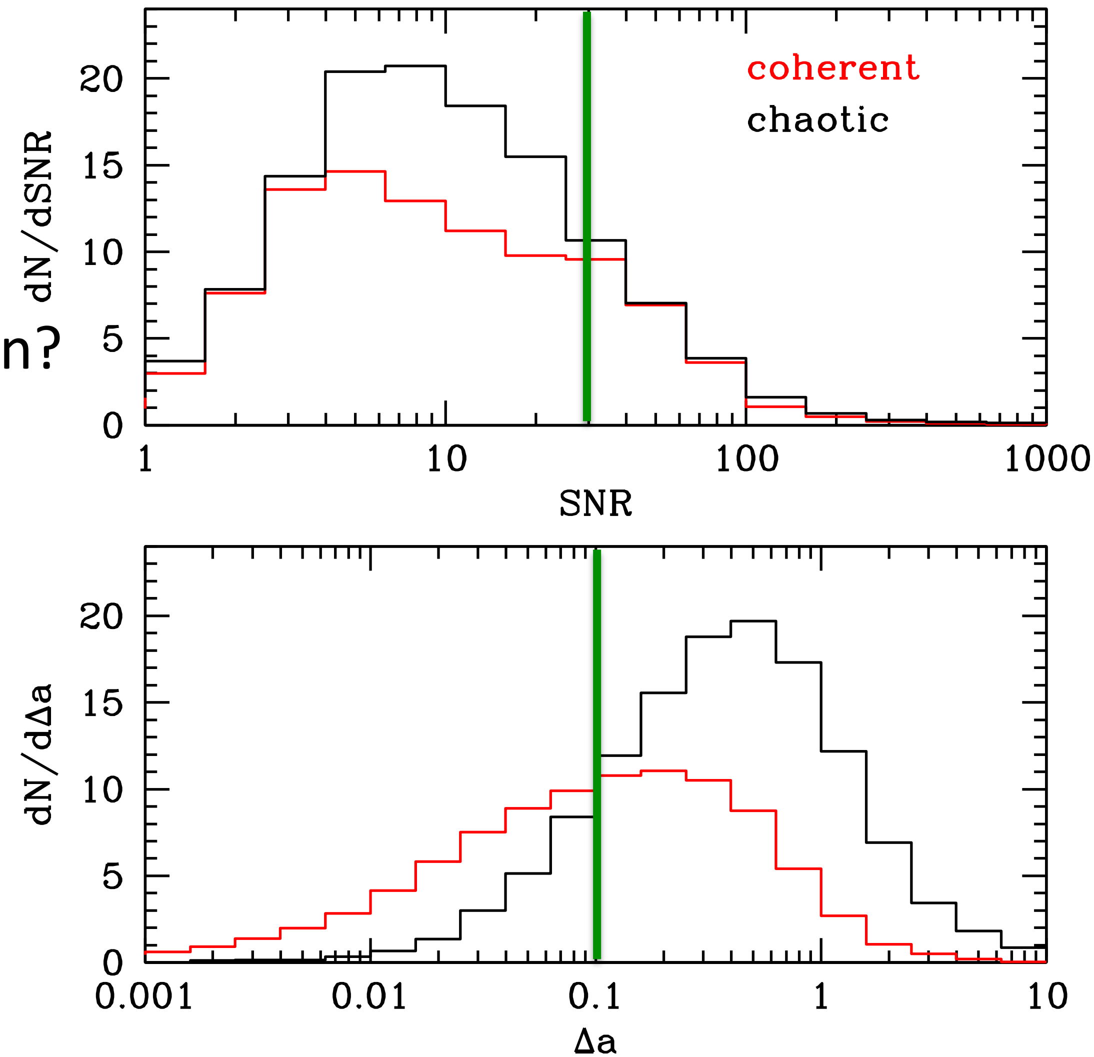


[Schutz]

SNR=h/S: S comparable, $h \sim \eta M^{1/2}$

Ringdown as a probe of SMBH formation

- LISA/eLISA studies: merger-tree models of SMBH formation
- Light or heavy seeds? Coherent or chaotic accretion? [Arun++, 0811.1011]
- eLISA can easily tell whether seeds are light or heavy [Sesana++, 1011.5893]
- Mergers: a~0.7
  Chaotic accretion: a~0
  Coherent accretion: a~1
  [EB+Volonteri, 0802.0025]



[Sesana++, 2012]

- **>10 binaries can be used for no-hair tests**
- **Spin observations constrain SMBH formation**

Part 2: extending GR. Why massive scalar fields?

1) Phenomenology

- ❑ Modern equivalent of planets [Bertschinger]
- ❑ Well-posed, flexible (Damour & Esposito-Farése "spontaneous scalarization")
- ❑ f(R) and other theories equivalent to scalar-tensor theories

2) High-energy physics

- ❑ Standard Model extensions predict massive scalar fields (dilaton, axions, moduli…)
- ❑ Not seen yet: dynamics must be frozen
  - ✓ **small coupling** ξ - or equivalently **large** $\omega_{BD}\sim 1/\xi$
  - ✓ **large mass** $m>1/R$ ($1\text{AU}\sim 10^{-18}$eV!)

3) Cosmology

- ❑ "String axiverse": light axions, **$10^{-33}\text{eV} < m_s < 10^{-18}\text{eV}$** [Arvanitaki++, 0905.4720]
  Striking astrophysical implications: bosenovas, floating orbits

4) Open problems in scalar-tensor theory:

- ❑ Are black hole binaries **indistinguishable** in GR and scalar-tensor theories?
  [Horbatsch+Burgess, 1111:4009; Healy++, 1112.3928]

Post-Newtonian effects in massive scalar-tensor theories

$$S = \frac{1}{16\pi}\int\left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + M(\phi)\right](-g)^{1/2}d^4x + \int \mathcal{L}_{\rm M}(g^{\mu\nu}, \Psi)d^4x,$$

[Alsing++, 1112.4903]

- ✓ Shapiro time delay **(Cassini)**
- ✓ Nordtvedt effect **(Lunar Laser Ranging)**
- ✓ Orbital period derivative **(binary pulsars)**

$$\frac{\dot P}{P} = -\frac{8}{5}\frac{\mu m^2}{r^4}\kappa_1 - \frac{\mu m}{r^3}\kappa_D \mathcal{S}^2$$

$$\kappa_1 = \mathcal{G}^2\left[12 - 6\xi + \xi\Gamma^2\left(\frac{4\omega^2 - m_s^2}{4\omega^2}\right)^2\Theta(2\omega - m_s)\right],$$

$$\kappa_D = 2\mathcal{G}\xi\frac{\omega^2 - m_s^2}{\omega^2}\Theta(\omega - m_s),$$

$$\xi = \frac{1}{2 + \omega_{\rm BD}},$$

$$\mathcal{G} = 1 - \xi(s_1 + s_2 - 2s_1 s_2),$$

$$\Gamma = 1 - 2\frac{s_1 m_2 + m_1 s_2}{m}.$$

1) No dipole if **$\mathcal{S}=s_1-s_2=0$** **(need NS-BH!)**

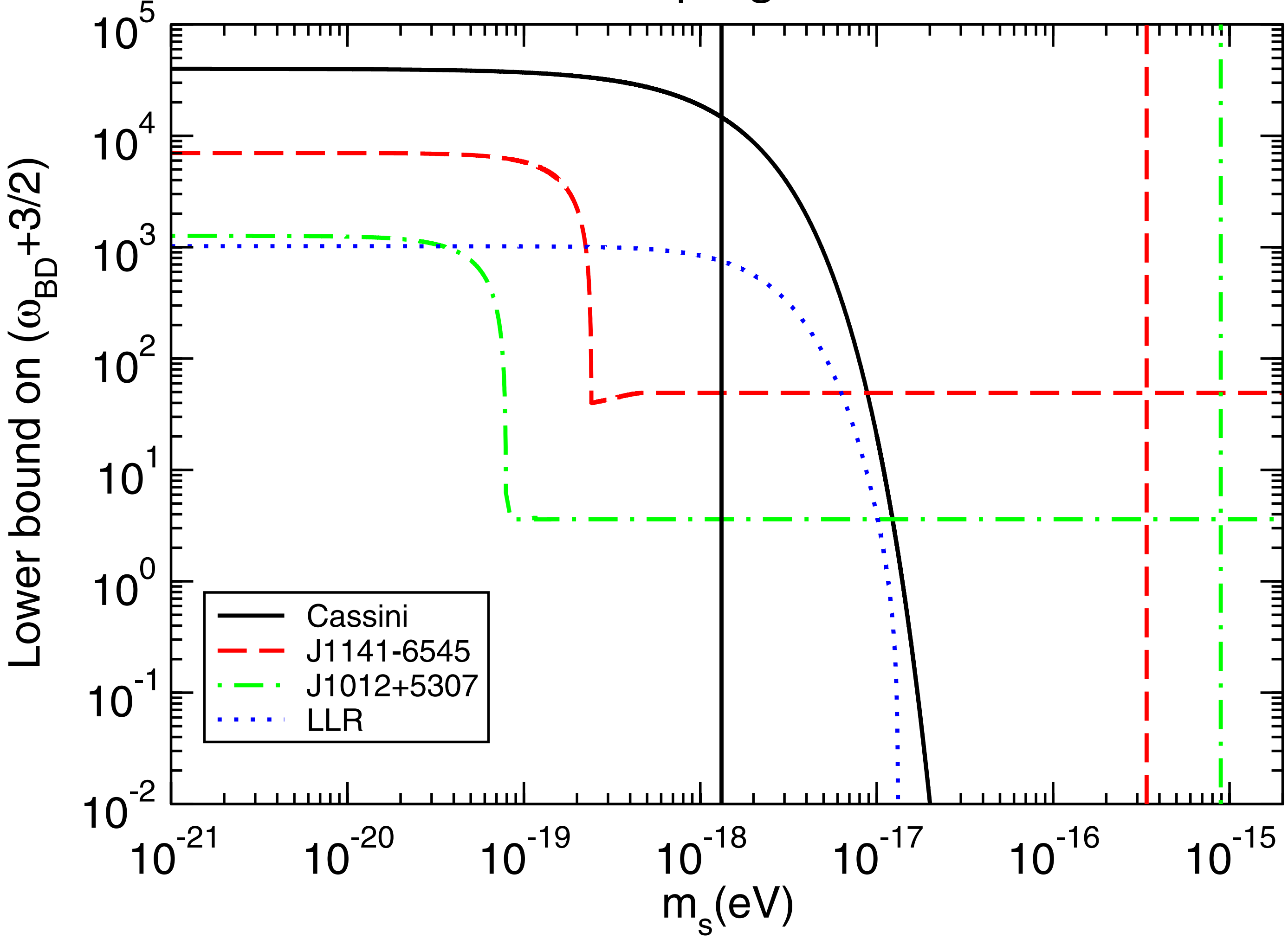
2) For binary black holes **$\Gamma=0$**: indistinguishable from GR?

Small coupling or small mass?

Bounds from:

- ✓ Shapiro time delay [Perivolaropoulos]
- ✓ Lunar Laser Ranging
- ✓ Binary pulsars - new binary pulsar: **ω_{BD}>25,000** [Freire++, 1205.1450]

For light scalars we can assume small coupling



Bounds with Earth- and space-based GW detectors

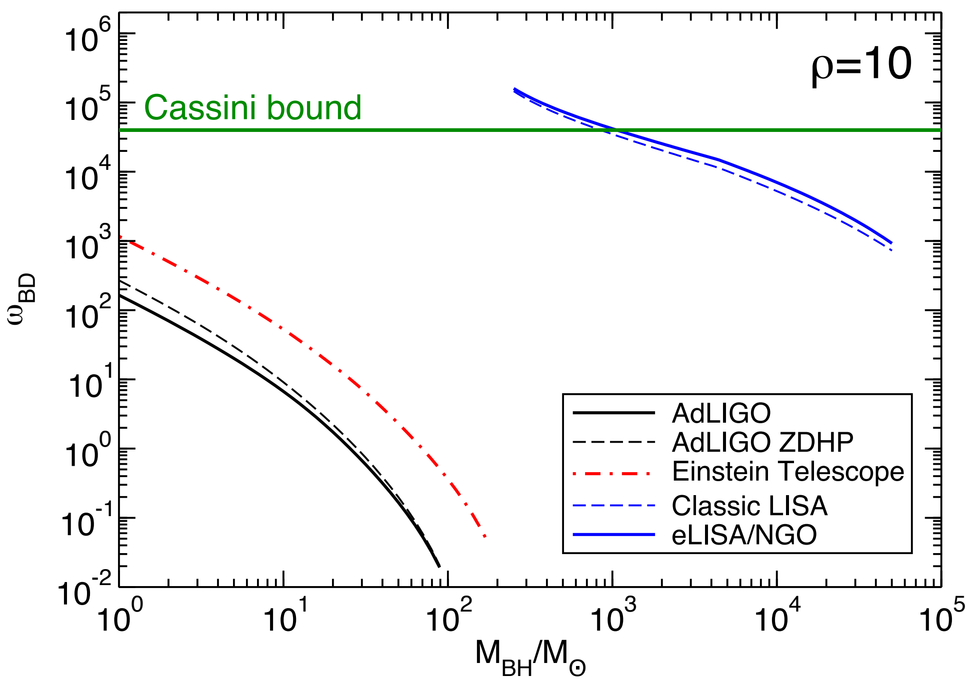
$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128(\pi \mathcal{M} f)^{5/3}} \times$$
$$\times \left\{1 + \xi \left[\frac{2}{3}(s_1 + s_2 - 2 s_1 s_2) + \frac{1}{2} - \frac{\Gamma^2}{12}\Theta(2\pi f - m_s)\right]\right.$$
$$+ \frac{20}{9} A v^2 - 16\pi v^3 + \ldots$$
$$+ \xi \nu \Gamma^2 \left[\frac{5}{462} v^{-6} - \frac{\nu}{1632} v^{-12}\right] \Theta(2\pi f - m_s)$$
$$\left. + \xi \mathcal{S}^2 \left[\frac{25\nu}{1248} v^{-8} - \frac{5}{84} v^{-2}\right] \Theta(\pi f - m_s)\right\}$$

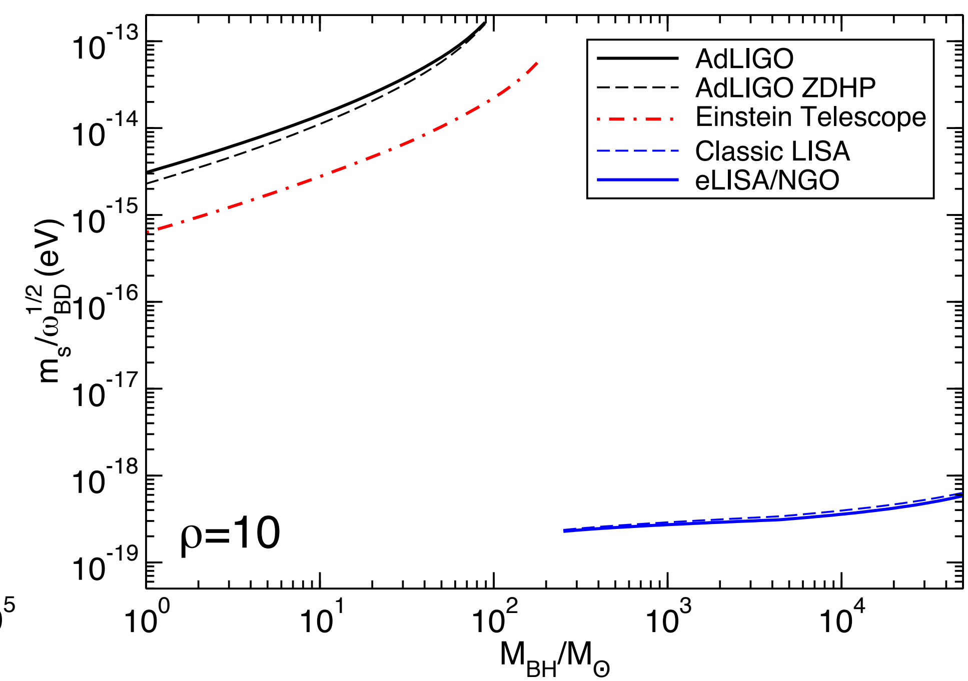
$$v = (\pi m f)^{1/3} = (\pi \mathcal{M} f)^{1/3} \eta^{-1/5}$$

$m_s(\mathrm{eV}) = 6.6 \times 10^{-16} f(\mathrm{Hz})$

Mass-dependent terms always scale with

$\nu\xi \sim m_s^2/\omega_{BD}$ $(\nu = m_s^2/m^2)$





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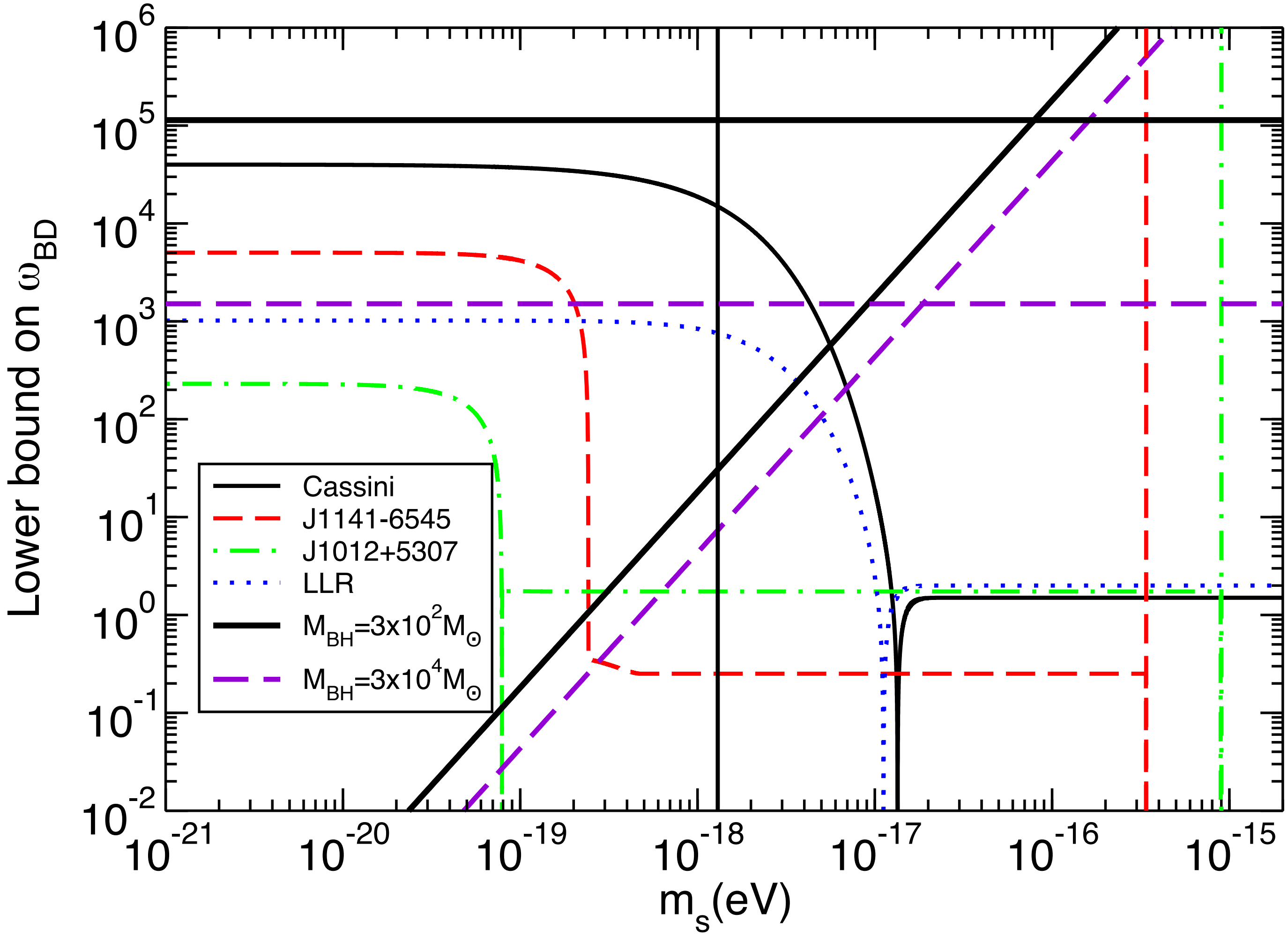
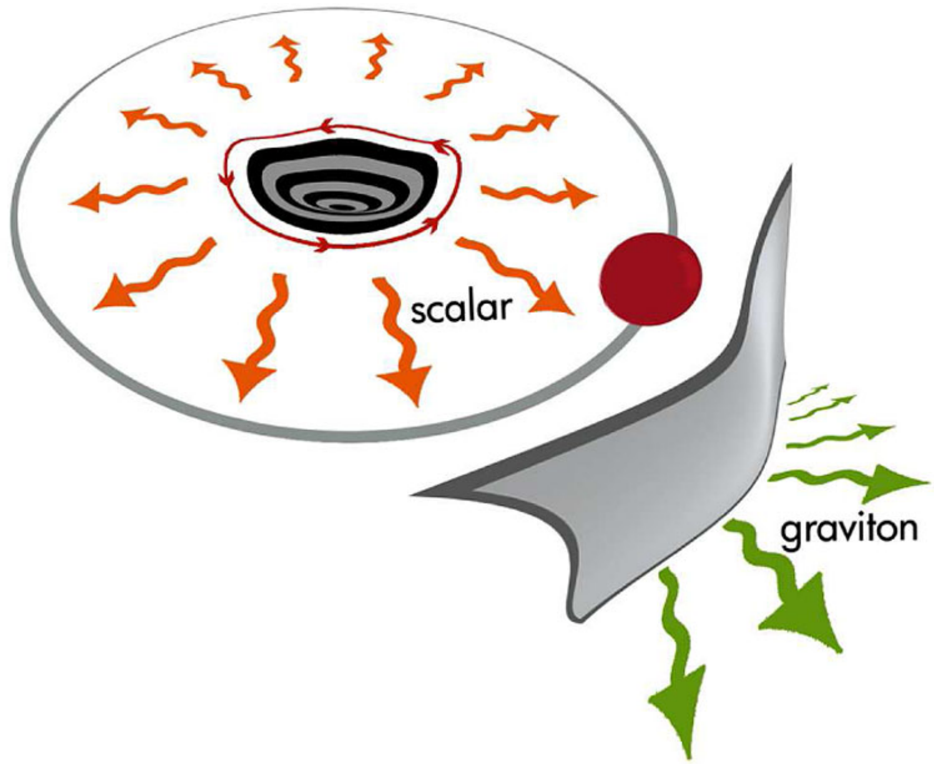


Part 3: massive bosonic fields and superradiant instabilities

Superradiance when **$\omega < m\Omega_H$**



Any light scalar can trigger a black hole bomb ("bosenova") [Yoshino+Kodama, 1203.5070]

Strongest instability: **$\mu_s M \sim 1$** [Dolan, 0705.2880]

For **μ_s=1eV, M=M_{sun} : $\mu_s M \sim 10^{10}$**
Need light scalars (or primordial black holes!)

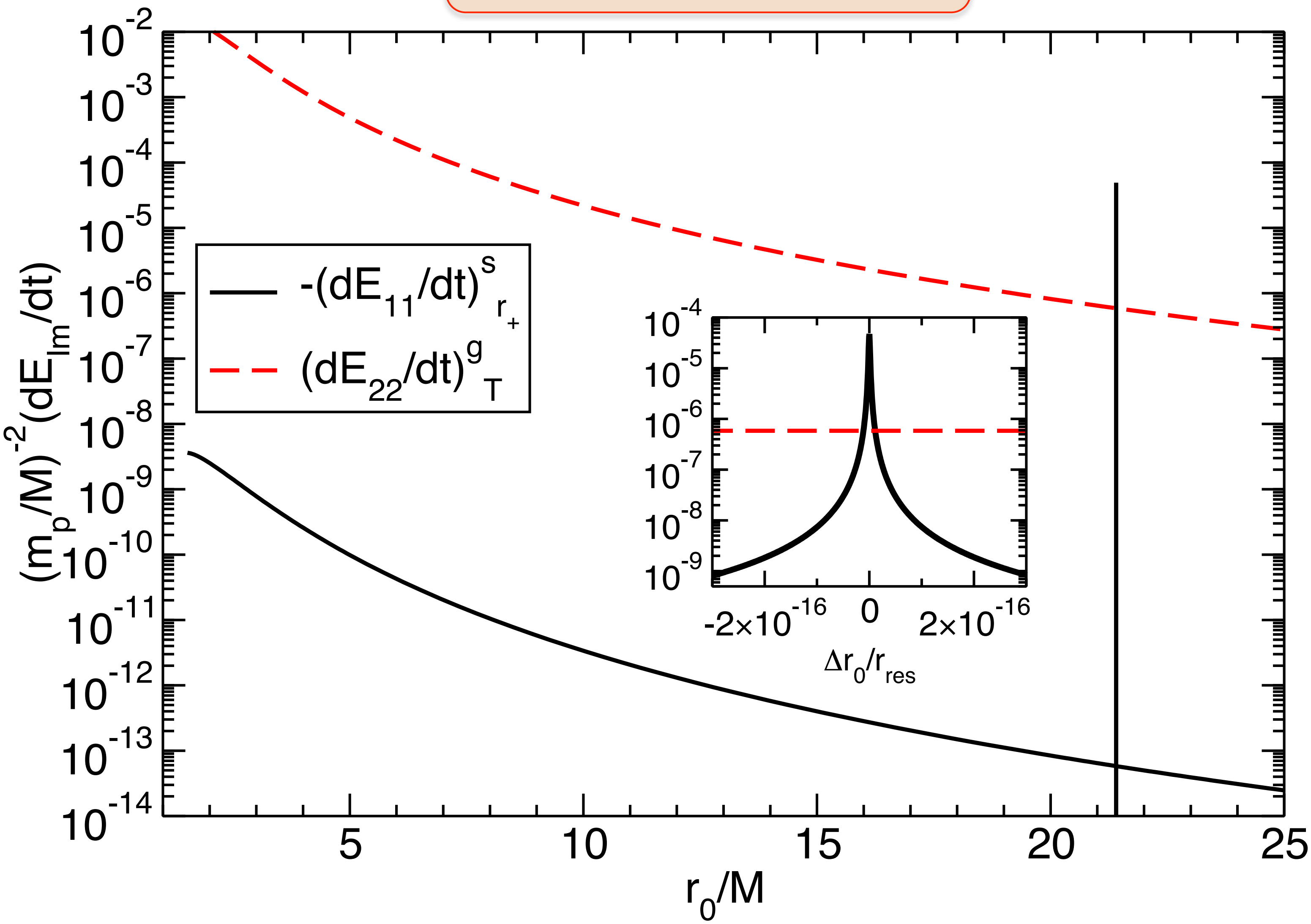
Negative scalar flux at the horizon close to superradiant resonances at

$$\omega_{\rm res}^2 = \mu_s^2 - \mu_s^2 \left(\frac{\mu_s M}{l+1+n}\right)^2, \quad n = 0, 1, \ldots$$

[Detweiler 1980]

Light scalars: floating orbits (Press & Teukolsky 1972)

$$\dot{E}_p + \dot{E}^g + \dot{E}^s = 0$$



[Cardoso++ 1109.6021; Yunes++, 1112.3351]

Photon mass bound from rotating black holes

$$\nabla_\sigma F^{\sigma\rho} - \mu^2 A^\rho = 0$$

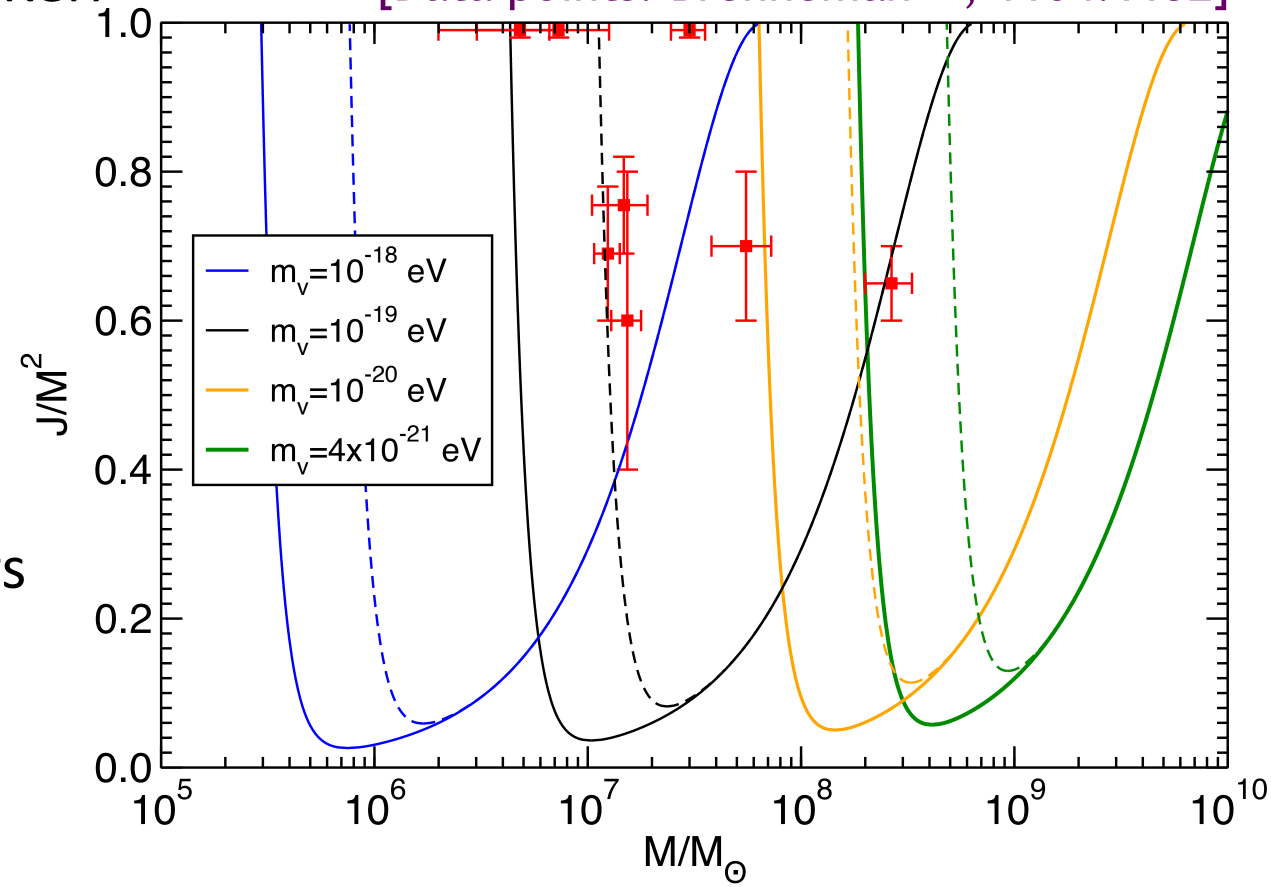
Proca perturbations in Kerr do **not** decouple

Use Kojima's slow-rotation approximation

Stronger instability than for massive scalars

Maximum (again) for $\mu_s M \sim 1$

[Data points: Brenneman++, 1104.1132]



$m_\gamma < 10^{-20}$ (or 4×10^{-21}) eV
PDG: $m_\gamma < 10^{-18}$ eV

$$M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)\left(M\mu\right)^{4\ell+5+2S}$$

[Pani++, submitted]

Summary

Quasinormal modes

1) **(e)LISA: Tens of events could allow us to test the no-hair theorem**
   Advanced LIGO/ET can also test no-hair theorem - if IMBHs exist!
2) **Spin measurements constrain SMBH merger/accretion history**

Massive scalar fields

1) **Weak-field: Solar System, binary pulsars**
   Cassini: **$\omega_{BD}>40{,}000$ for $m_s<2.5\text{x}10^{-20}$ eV** [Alsing++, 1112.4903]
   Binary pulsars will do better in a few years
2) **Future gravitational-wave observations:**
   $m_s/(\omega_{BD})^{1/2}<10^{-19}$eV, $\omega_{BD}>10^5$
   for (e)LISA observations of NS-BH with $M_{BH}=300M_{sun}$ [EB++, 1204.4340]

Superradiant instabilities in the Kerr background

1) **Massive scalars: floating orbits**
   [Cardoso++, 1109.6021; Yunes++, 1112.3351]
2) **Massive vectors and SMBH spins: best bounds on photon mass**
   $m_\gamma<10^{-20}$ ($4\text{x}10^{-21}$eV) (Particle Data Group: $m_\gamma<10^{-18}$eV)
   [Pani++, submitted]