Scalar vacuum polarization by cosmic string in constant curvature spacetime

Eugênio R. Bezerra de Mello

Phys. Dept. (UFPB), Brazil

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Main references

1. E. R. Bezerra de Mello, V. B. Bezerra, A. A. Saharian, A. S. Tarloyan, *Phys. Rev. D* **74**, 025017 (2006).
2. E. R. Bezerra de Mello, A. A. Saharian, *Vacuum polarization by a cosmic string in de Sitter spacetime*, J. High Energy Phys. **04**, 046 (2009).
3. E. R. Bezerra de Mello, A. A. Saharian, *Vacuum polarization induced by a cosmic string in anti-de Sitter spacetime*, J. Phys. A: Math. Theor. **45**, 115402 (2012)

Plan of the talk

- Gravitational field of a cosmic string in Minkowski spacetime.
- Maximally symmetric spacetime: dS and AdS spacetimes.
- Gravitational field of a cosmic string in dS and AdS spacetimes.
- The Klein-Gordon equation
- Wigthman functions
- Vacuum polarizations induced by cosmic strings

Cosmic strings

- Cosmic strings generically arise within the framework of grand unified theories and could be produced in the early universe as a result of symmetry breaking phase transitions

- Observational data on the cosmic microwave background radiation have ruled out cosmic strings as the primary source for primordial density perturbations

- Cosmic strings are still candidates for the generation of a number of interesting physical effects:
  - *Gravitational lensing*
  - *Generation of gravitational waves*
  - *High energy cosmic rays*
  - *Gamma ray bursts*

String gravity: Simplified model

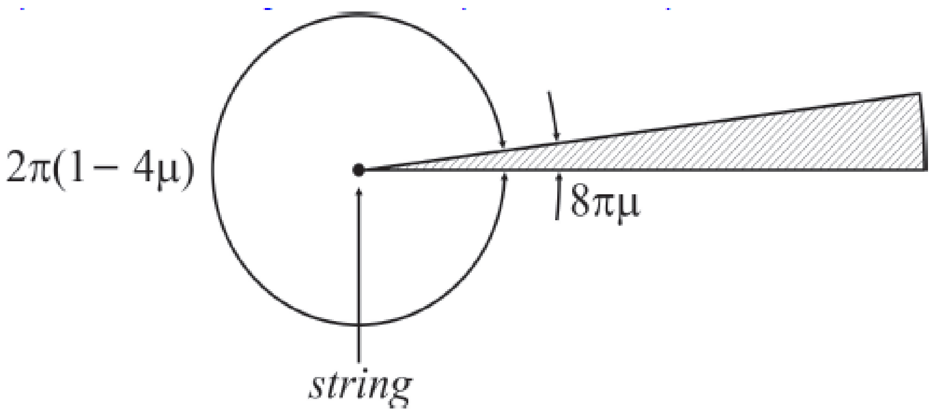
- Cosmic string is approximated as a line of zero width with a distributional δ-function energy-momentum tensor

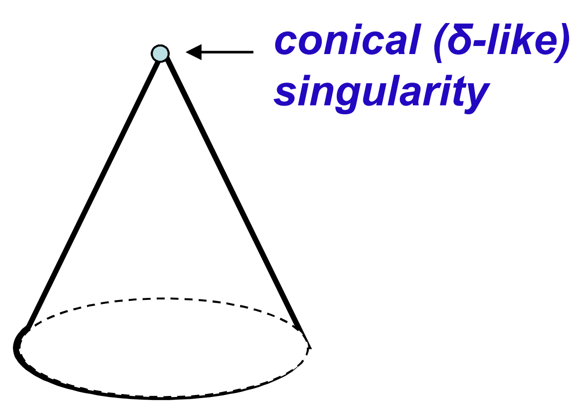
$$T^{\mu}_{\nu} = \mu \ \mathrm{diag}(1,\ 0,\ 0,\ 1)\delta^2(\vec{\rho})$$

- Line element $\quad ds^2 = dt^2 - dr^2 - r^2 d\varphi^2 - dz^2$

- Definition: $\varphi \in [0,\ 2\pi/q]$, $q = 1/\beta$ with $\beta = 1 - 4\mu < 1$
  μ: Linear mass density.

- This space in locally flat and presents a planar deficit angle.





Maximally symmetric spacetimes

- Many physical problem can be exactly solvable

  - De Sitter (dS): Inflationary model $\Leftrightarrow$ Expanding Universe with a positive cosmological constant. (Birrel and Davies, 99)

$$\begin{aligned} ds^2 &= dt^2 - e^{2t/\alpha}(dx^2 + dy^2 + dz^2) \ , \\ \Lambda &= \frac{3}{\alpha^2} \ , \ R = \frac{12}{\alpha^2} \ . \end{aligned}$$

Conformal time: $\tau = -\alpha e^{-t/\alpha}$, $-\infty < \tau < 0$.

$$ds^2 = \frac{\alpha^2}{\tau^2}\left[d\tau^2 - (dx^2 + dy^2 + dz^2)\right] \ .$$

α/τ: Conformal factor.

(continuation)

- Anti-de Sitter (AdS): Randall-Sundrum scenario (Randall and Sundrum, 99). AdS_4/CFT_2 (Maldacena, 00)

$$\begin{aligned} ds^2 &= e^{-2z/a}\left[dt^2 - dx^2 - dy^2\right] - dz^2 \ , \\ \Lambda &= -\ \frac{3}{a^2} \ , \ R = -\frac{12}{a^2} \ . \end{aligned}$$

Poincaré coordinate: $\omega = ae^{z/a}$, $\omega \in [0,\ \infty)$.

$$ds^2 = \frac{a^2}{\omega^2}\left[dt^2 - dx^2 - dy^2 - d\omega^2\right] \ .$$

a/ω: Conformal factor

$\omega = 0$: AdS boundary

$\omega = \infty$: Horizon.

Cosmic string in dS and AdS

- **dS:**

$$
\begin{aligned}
ds^2 &= dt^2 - e^{2t/\alpha}\left(dr^2 + r^2 d\varphi^2 + dz^2\right) \ , \\
\varphi &\in [0,\ 2\pi/q] \ , \ q = 1/\beta \ .
\end{aligned}
$$

By using conformal time:

$$
\begin{aligned}
ds^2 &= \frac{\alpha^2}{\tau^2}\left(d\tau^2 - dr^2 - r^2 d\varphi^2 - dz^2\right) \ , \\
R &= \frac{12}{\alpha^2} \ .
\end{aligned}
$$

The metric tensor is conformally related with the cosmic string one.

(Continuation)

- **AdS:**

$$ds^2 = e^{-2z/a}\left[dt^2 - dr^2 - r^2 d\phi^2\right] - dz^2 \ .$$
$$\varphi \in [0, \ 2\pi/q] \ , \ q = 1/\beta \ .$$

By using Poincaré coordinate:

$$ds^2 = \frac{a^2}{\omega^2}\left[dt^2 - dr^2 - r^2 d\phi^2 - d\omega^2\right] \ ,$$
$$R = -\frac{12}{a^2}.$$

The metric tensor is conformally related with the cosmic string one.

Vacuum polarization by a cosmic string in a constant curvature spacetimes (dS and AdS)

- The non-trivial topology of the cosmic string spacetime results in the distortion of the zero-point vacuum fluctuations of quantized fields
- The non-vanishing curvature of dS/AdS produces also modification of the zero-point vacuum fluctuations of quantized fields
- Non-zero vacuum expectation values (VEVs) for physical observables (field squared, energy-momentum tensor,...)

The Klein-Gordon equation

- Equation of motion obeyed by a scalar field in a general background spacetime with an arbitrary curvature coupling:

$$\left(g^{\mu\nu}\nabla_\mu\nabla_\mu + m^2 + \xi R\right)\phi(x) = 0 \ .$$

m: Mass associated with the field.

ξ: Curvature coupling constant.

$\xi = 0$: minimum coupling.

$\xi = 1/6$: conformal coupling.

R: Scalar curvature.

Expansion of field operator

- Commutation relation for annihilation and creation operators.

$$
\begin{aligned}
[a_\sigma,\ a_{\sigma'}] &= 0 \ , \\
[a^\dagger_\sigma,\ a^\dagger_{\sigma'}] &= 0 \ , \\
[a_\sigma,\ a^\dagger_{\sigma'}] &= \delta_{\sigma,\sigma'} \ .
\end{aligned}
$$

- Defenition of vacuum state |0>: $a_\sigma|0>=0$

- Expansion of field operator:

$$\hat{\Phi} = \sum_\sigma [a_\sigma \phi_\sigma + a^\dagger_\sigma \phi^*_\sigma] \ ,$$

$\{\phi_\sigma,\ \phi^*_\sigma\}$: Complete set of orthonormal solution of the classical equation.

σ: Complete set of quantum number.

- Orthogonalization condition:

$$-i \int d^3x \sqrt{|g|} g^{00} [\phi_\sigma(x) \partial_\tau \phi^*_{\sigma'}(x) - \phi^*_{\sigma'}(x) \partial_\tau \phi^*_\sigma(x)] = \delta_{\sigma,\sigma'} \ .$$

Vacuum polarization
Wigthmann function (WF)

- Among the most important local characteristics of the vacuum state are the vacuum expectation values (VEVs) of the:
  - Field squared $\langle \Phi^2(x) \rangle$
  - Energy-momentum tensor, $\langle T_{\mu\nu}(x) \rangle$
- These VEVs are obtained from the positive frequency Wightman function in the coincidence limit of the arguments
- WF is presented as the mode sum

$$G(x, x') =< 0|\hat{\Phi}(x)\hat{\Phi}(x)|0 >= \sum_{\sigma} \phi_\sigma(x)\phi^*_\sigma(x') \ .$$

- In addition, WF determines the response of a particle detector in an arbitrary state of motion

(Continuation)

- Field squared:

$$\langle \Phi^2(x) \rangle = \lim_{x' \to x} G(x, x') \ .$$

- Energy-momentum tensor:

$$\begin{aligned} \langle T_{\mu\nu}(x) \rangle \ &= \ \lim_{x' \to x} \partial_{\mu'} \partial_\nu G(x, x') + [(\xi - 1/4)\, g_{\mu\nu} \Box \\ &- \ \xi \nabla_\mu \nabla_\nu - \xi R_{\mu\nu}] \langle \phi^2(x) \rangle \ . \end{aligned}$$

- These procedures provide divergent results

- Renormalizations are required

Renormalization procedure

- Analysis of the Wightman function near the coincidence limit.

- Identify the divergent terms.

- Subtraction scheme: From the expansion one subtracts the divergent terms.

- This procedure can be made in a manifest form:

$$G_{Ren}(x', x) = G(x', x) - G_H(x', x)$$

$G_H(x', x)$: Hadamard function.

$$G_H(x', x) = \frac{1}{8\pi^2}\left[\frac{1}{\sigma} + \frac{1}{2}\Big(m^2 + (\xi - 1/6)R\Big)\ln(m_{DS}^2\sigma/2)\right]$$

Vacuum polarization by a cosmic string in Minkowski spacetime

- For a massless scalar field the renormalized VEV of the field squared has the form (units $\hbar = c = 1$)

$$\langle \varphi^2 \rangle = \frac{q^2 - 1}{48\pi^2 r^2},\ q = 2\pi/\phi_0,\ 0 \leqslant \phi \leqslant \phi_0 \quad \phi_0 = 2\pi\beta$$

- Energy-momentum tensor

Curvature coupling parameter

$$\langle T_0^0 \rangle = \langle T_3^3 \rangle = -\frac{q^2 - 1}{96\pi^2 r^4}\left(8\xi + \frac{q^2 - 19}{15}\right),$$

$$\langle T_1^1 \rangle = -\langle T_2^2 \rangle /3 = -\frac{q^2 - 1}{96\pi^2 r^4}\left(\frac{q^2 + 11}{15} - 4\xi\right)$$

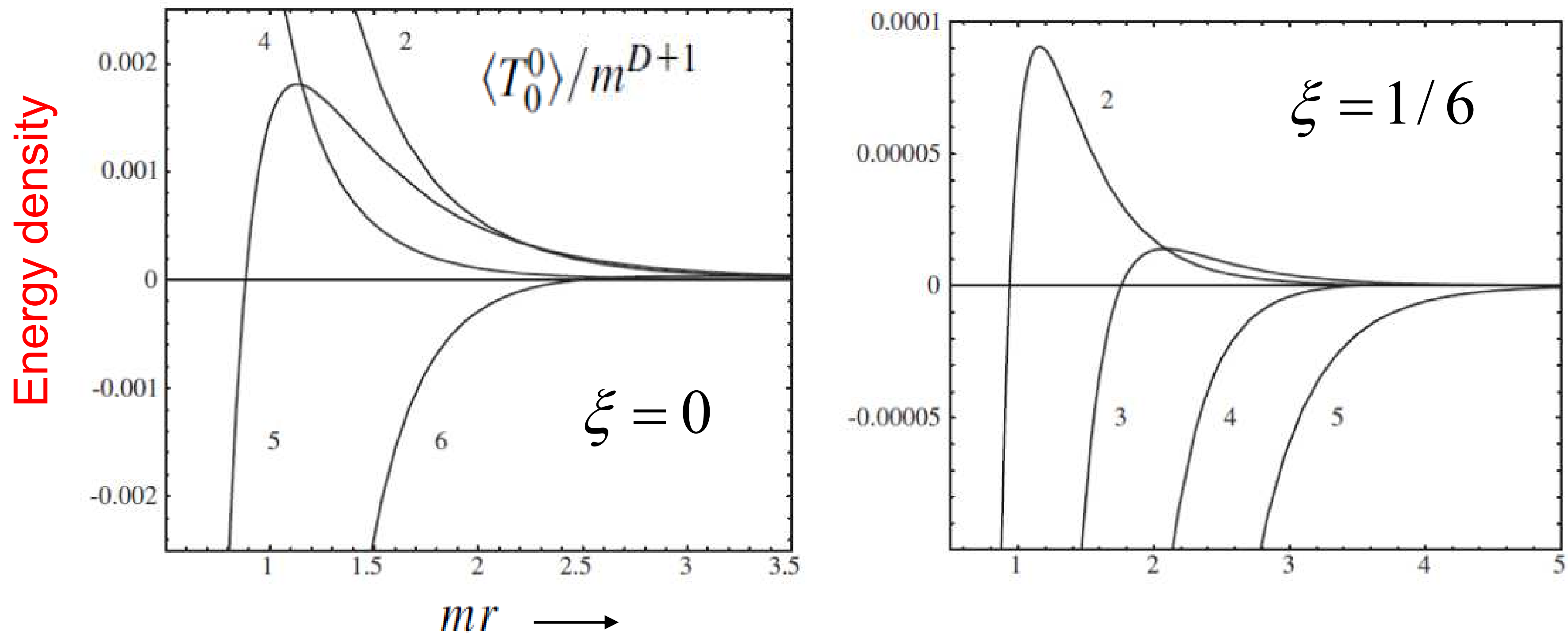
Massive scalar field

- For integer values of the parameter $q = 2\pi/\phi_0$ (D=3)

$$\langle \varphi^2 \rangle = \frac{m^{D-1}}{(2\pi)^{\frac{D+1}{2}}} \sum_{l=1}^{q-1} \frac{K_{(D-1)/2}(2mry_l)}{(2mry_l)^{(D-1)/2}}, \quad y_l = \sin(\pi l/q)$$

- At large distances $mr \gg 1$:

$$\langle \varphi^2 \rangle \approx \frac{m^{D/2-1}}{(4\pi r)^{D/2}} \frac{e^{-2mr\sin(\pi/q)}}{\sin^{D/2}(\pi/q)}$$



Wightman functions in dS/AdS with cosmic string

- The Wightman functions in cosmic string dS/AdS spacetimes.
  These functions are composed by two terms:

$$G(x,x') = G_{dS,AdS}(x,x') + G_c(x,x') \ .$$

where $G_{dS,AdS}$ corresponds to the functions in dS/AdS spacetime, i.e., in the absence of cosmic string ($q = 1$).
$G_c(x',x)$: The contribution to the Wightman function due to the presence of the defect.

Although q is an parameter greater than unity, here we shall consider only a very special case where it is an integer number.

Special case: q being an interger number

- **dS:** For dS case the Wightman function reads (ERBM and A. Saharian)

$$G(x', x) = \sum_{k=0}^{q-1} G_k(x', x) \ ,$$

with

$$G_k(x', x) = \frac{\Gamma\left(\frac{3}{2}+\nu\right)\Gamma\left(\frac{3}{2}-\nu\right)}{8\alpha^2\pi^2(1-u_k^2)^{1/2}} P^{-1}_{\nu-1/2}(u_k) \ ,$$

$$u_k = -1 + \frac{\Delta z + r^2 + r'^2 - 2rr' \cos(\Delta\phi + 2\pi k/q) - (\Delta\eta)^2}{2\eta\eta'} \ ,$$

$$\nu = \sqrt{9/4 - 12\xi - m^2\alpha^2} \ .$$

(Continuation)

- **AdS:** For AdS case the Wightman function reads (ERBM and A. Saharian)

$$G(x',x) = \sum_{k=0}^{q-1} G_k(x',x) \ ,$$

with

$$G_k(x',x) = -\frac{1}{4\pi^2 a^2} \frac{Q^1_{\nu-1/2}(u_k)}{(u_k^2-1)^{(1/2)}} \ ,$$

$$\begin{aligned} u_k &= \frac{r^2 + r'^2 - 2rr' \cos(\Delta\phi - 2\pi k/q) + \omega^2 + \omega'^2 - \Delta t^2}{2\omega\omega'} \ , \\ \nu &= \sqrt{9/4 - 12\xi + m^2 a^2} \ . \end{aligned}$$

(Continuation)

The $k = 0$ components for both cases correspond to the dS/AdS Wightman functions respectively.
The parts in the corresponding Wightman functions induced by the cosmic string, $G_C(x, x')$, are given by:

$$G_c(x, x') = \sum_{k=1}^{q-1} G_k(x, x') \ ,$$

which is finite at the coincidence limit.

These functions are composed by two terms:

$$G(x, x') = G_{dS,AdS}(x, x') + G_c(x, x') \ .$$

where $G_{dS,AdS}$ corresponds to the functions in dS/AdS space-time, i.e., in the absence of cosmic string ($q = 1$).

(Continuation)

As consequence of this decomposition:

$$\begin{aligned} \langle \Phi^2(x) \rangle &= \langle \Phi^2(x) \rangle_{dS,AdS} + \langle \Phi^2(x) \rangle_c \ , \\ \langle T_{\mu\nu}(x) \rangle &= \langle T_{\mu\nu}(x) \rangle_{dS,AdS} + \langle T_{\mu\nu}(x) \rangle_c \ . \end{aligned}$$

Due to the maximal symmetry of dS/AdS spacetimes, the analisis of the polarization effects provides point independent contributions.

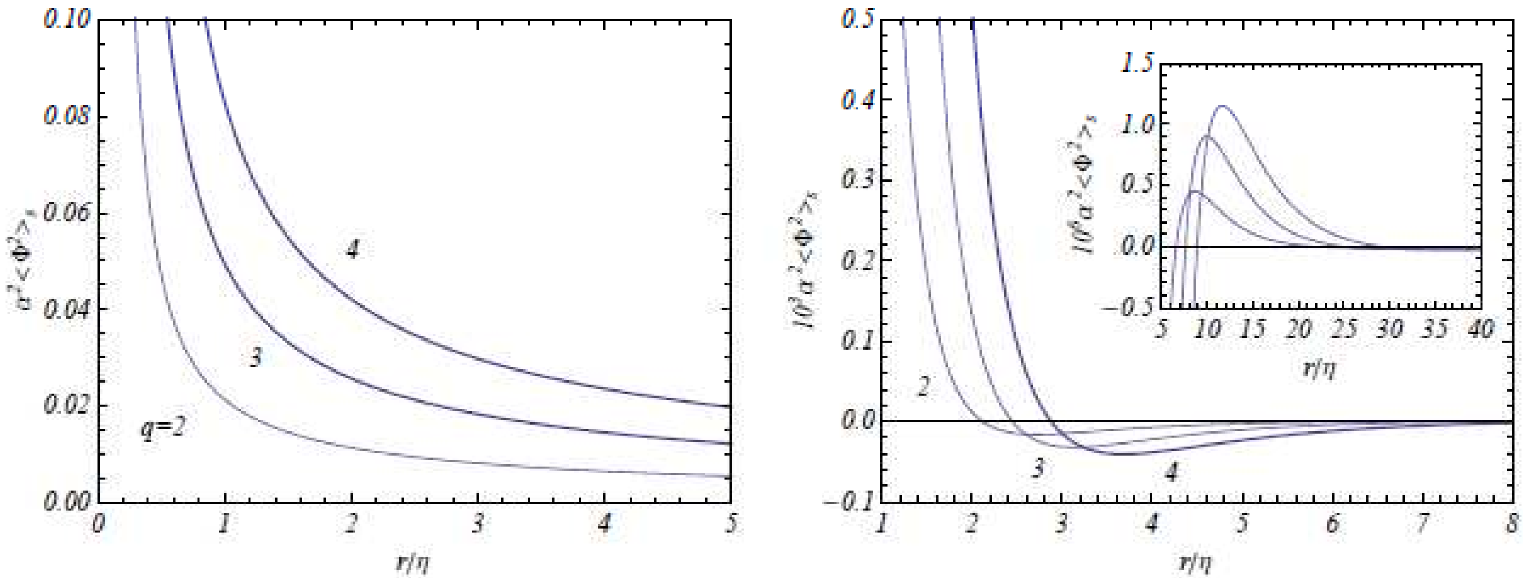
Here we are mainly interested in quantum effects induced by the presence of the cosmic string.

(Continuation)

- **dS:**

$$\langle \Phi^2(x) \rangle_c = \frac{\Gamma\left(\frac{3}{2}+\nu\right)\Gamma\left(\frac{3}{2}-\nu\right)}{8\alpha^2\pi^2} \sum_{k=1}^{q-1} \frac{P^{-1}_{\nu-1/2}(u^0_k)}{[1-(u^0_k)^2]^{1/2}} ,$$

$$u^0_k = -1 + 2(r/\eta)^2 \sin^2(\pi k/q) .$$

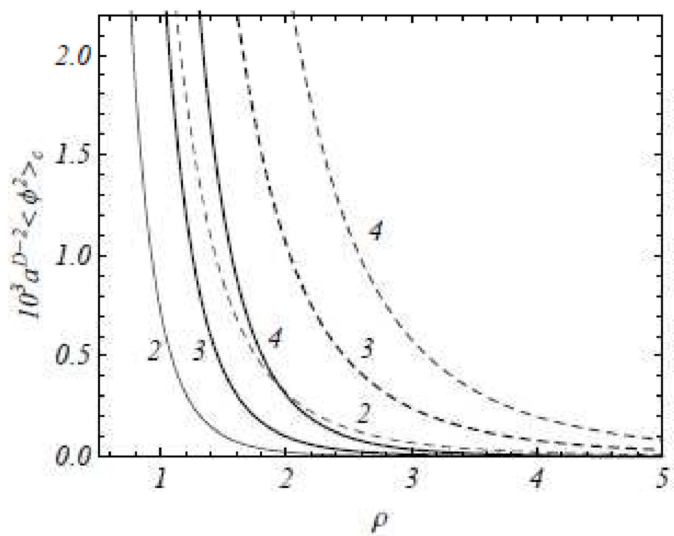


VEV of field squared as a function of the ratio r/η for various values of the parameter q for a minimally coupled scalar field with $m\alpha = 1$ (left panel) and $m\alpha = 2$ (right panel).

(Continuation)

- **AdS:**

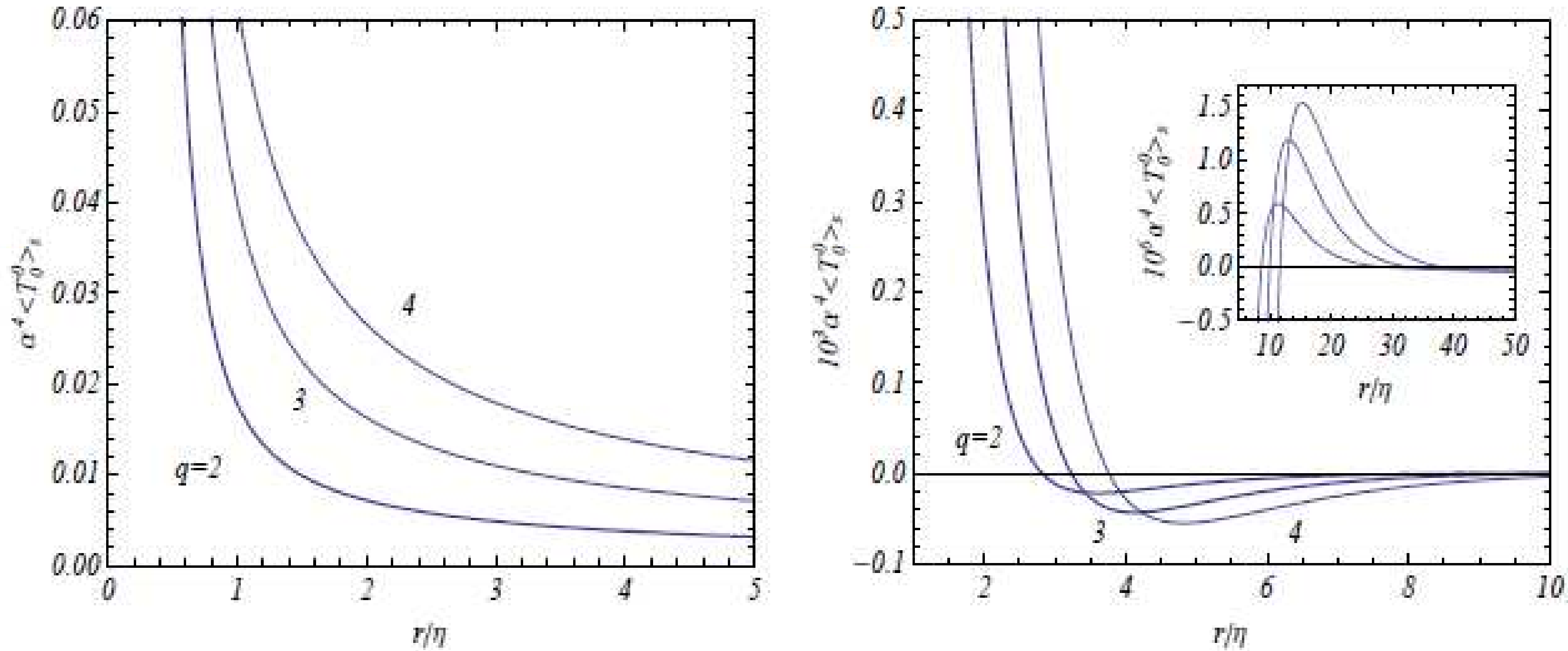
$$\langle \Phi^2(x) \rangle_c = -\frac{1}{4\pi^2 a^2} \sum_{k=1}^{q-1} \frac{Q^1_{\nu-1/2}(u_k^0)}{((u_k^0)^2-1)^{(1/2)}} ,$$
$$u_k^0 = 1 + 2(r/\omega)^2 \sin^2(\pi k/q) .$$



VEV of field squared as a function of the ratio $\rho = r/\omega$ for various values of the parameter q, for a minimal (full curves) and conformal (dashed curves) massless scalar fields.

Computation of energy density $\langle T_0^0(x) \rangle$

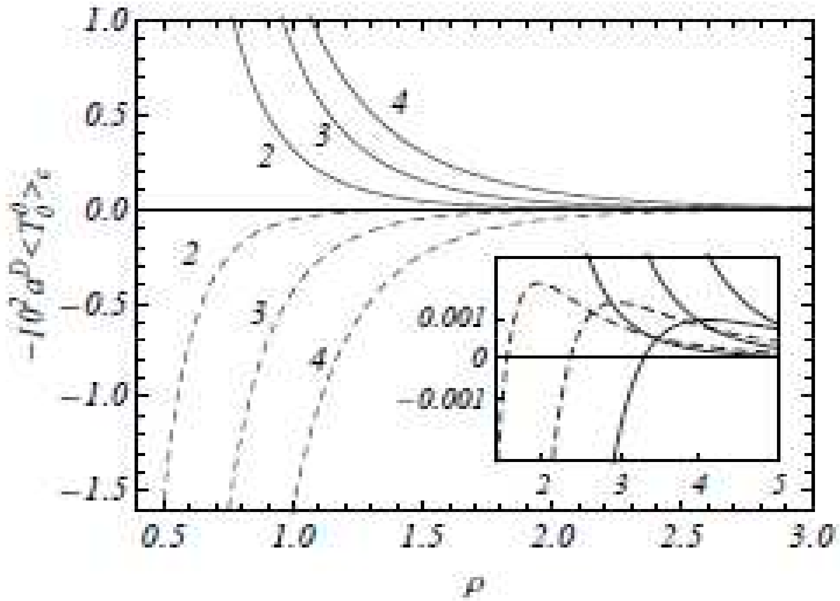
- dS:



The vacuum energy density induced by the string in 4-dimensional dS spacetime versus r/η for a minimally coupled scalar field with $m\alpha = 1$ (left panel) and $m\alpha = 2$ (right panel).

(Continuation)

- **AdS:**



The string-induced part in the VEV of the energy density as a function of the proper distance from the string (measured in units of the AdS curvature radius) for $D = 4$ minimally (full curves) and conformally (dashed curves) massless scalar fields. The numbers near the curves correspond to the values of the parameter q.

Final remarks

- We have investigated the one-loop quantum effects arising from vacuum fluctuations associated with massive scalar field on the background of dS/AdS spacetimes considering the presence of a cosmic string.

- These effects took into account the presence of the positive/negative curvature of the spacetime and the non-trivial topology associated with the two-dimensional conical subspace.

Final remarks (Continuation)

- The position-dependent contributions to the VEVs induced by the cosmic string dominate the pure dS/AdS parts for points near the string.

- Although cosmic strings produced in phase transitions before or during early stages of inflation would have been drastically diluted by the expansion, they could be continuously created during inflation by quantum-mechanical tunneling.

- The analysis of string evolution in a expanding universe, has been developed by many authors. They have founded by computer simulation that the final string density under the Hubble horizon is about 16 ± 4 at late time.

Thank you!

ΕΥΧΑΡΙΣΤΩ