INSTITUT D'ASTROPHYSIQUE DE PARIS

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NEB 15 – Recent Developments in Gravity

GRAVITATIONAL RADIATION FROM COMPACT BINARY STAR SYSTEMS

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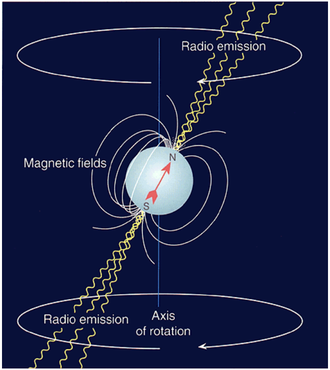
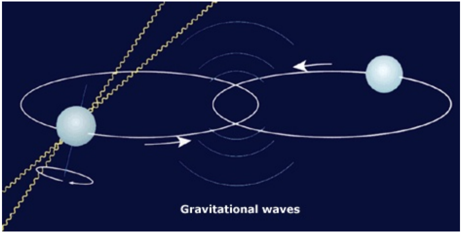
Outline of the talk

1. Gravitational waves from compact binaries
2. Post-Newtonian templates for binary inspiral
3. Post-Newtonian versus self-force predictions
4. First law of point mass binary systems

GRAVITATIONAL WAVES FROM COMPACT BINARIES

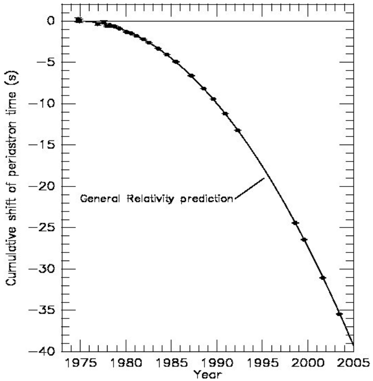
The binary pulsar PSR 1913+16





- The pulsar PSR 1913+16 is a rapidly rotating neutron star emitting radio waves like a lighthouse toward the Earth.
- This pulsar moves on a (quasi-)Keplerian close orbit around an unseen companion, probably another neutron star

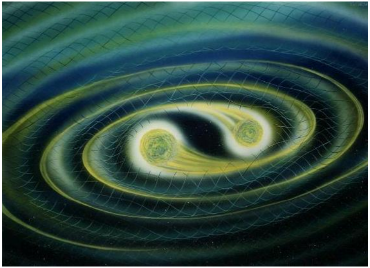
The orbital decay of the binary pulsar [Taylor & Weisberg 1989]



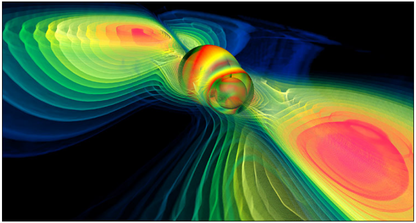
Prediction from general relativity theory

$$\dot{P} = -\frac{192\pi}{5c^5}\,\frac{\mu}{M}\left(\frac{2\pi G\,M}{P}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \approx -2.4\times 10^{-12}$$

The inspiral and merger of compact binaries



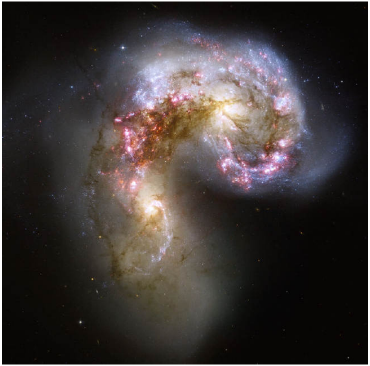
Neutron stars spiral and coalesce



Black holes spiral and coalesce

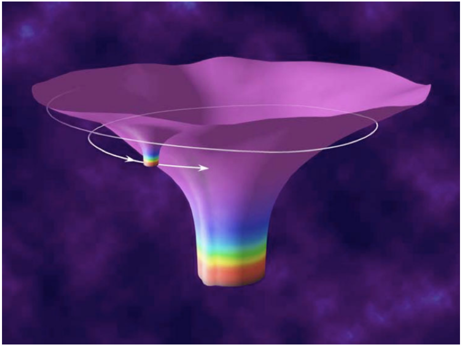
1. Neutron star ($M = 1.4\,M_\odot$) events will be detected by ground-based detectors LIGO/VIRGO/GEO
2. Stellar size black hole ($5\,M_\odot \lesssim M \lesssim 20\,M_\odot$) events will also be detected by ground-based detectors
3. Supermassive black hole ($10^5\,M_\odot \lesssim M \lesssim 10^8\,M_\odot$) events will be detected by the space-based detector LISA

Supermassive black-hole coalescences as detected by LISA



When two galaxies collide their central supermassive black holes may form a bound binary system which will spiral and coalesce. LISA will be able to detect the gravitational waves emitted by such enormous events anywhere in the Universe

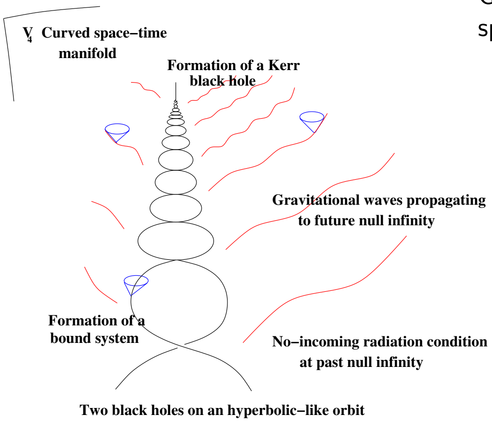
Extreme mass ratio inspirals (EMRI) for LISA



- A neutron star or a stellar black hole follows a highly relativistic orbit around a supermassive black hole. The gravitational waves generated by the orbital motion are computed using black hole perturbation theory
- Observations of EMRIs will permit to test the no-hair theorem for black holes, i.e. to verify that the central black hole is described by the Kerr geometry

POST-NEWTONIAN TEMPLATES FOR BINARY INSPIRAL

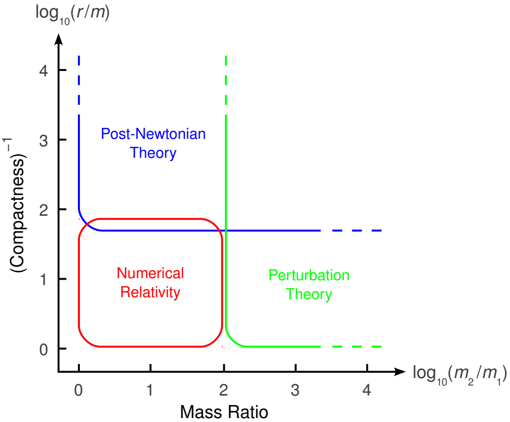
The two-body problem in General Relativity



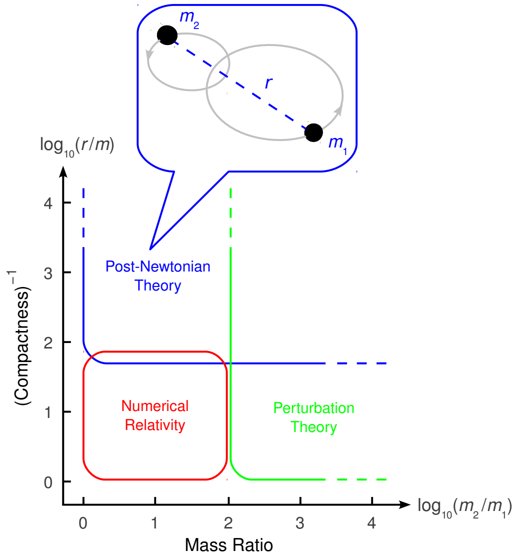
The solution of the two-body problem in General Relativity would consist of a space-time manifold describing

1. Two black holes on an initial hyperbolic-like (scattering) orbit
2. The formation of a bounded binary system by emission of gravitational radiation
3. The long inspiral phase where the black holes gradually come close to each other
4. The detailed process of merger of the two black hole horizons
5. The emission of quasi-normal mode radiation by the final object untill the formation of a stationary (Kerr) black hole

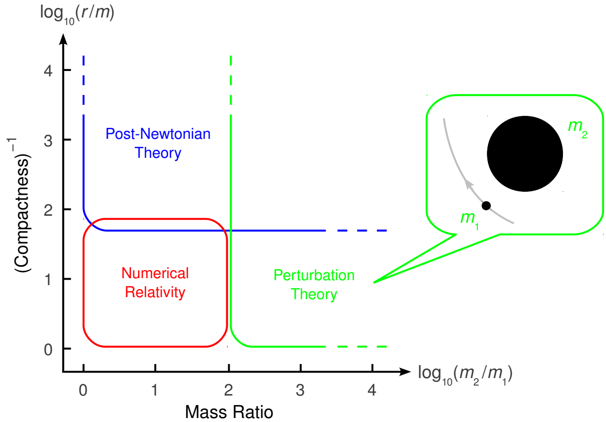
Methods to compute gravitational-wave templates



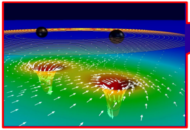
Methods to compute gravitational-wave templates

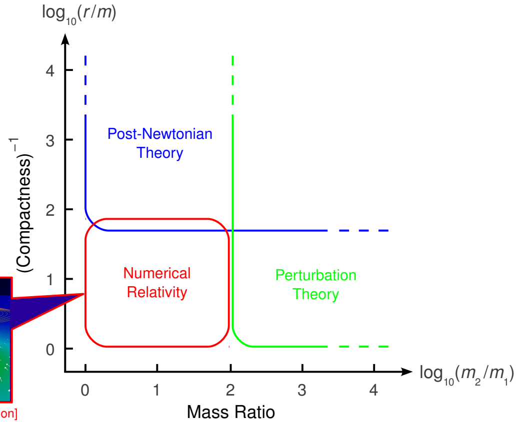


Methods to compute gravitational-wave templates

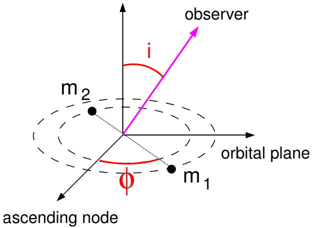


Methods to compute gravitational-wave templates





PN methods applied to Inspiralling Compact Binaries



The orbital phase $\phi(t)$ should be monitored in LIGO/VIRGO with precision

$$\delta\phi \sim \pi$$

$$\phi(t) = \phi_0 \underbrace{- \frac{1}{\nu}\left(\frac{GM\omega}{c^3}\right)^{-5/3}}_{\text{result of the quadrupole formalism (sufficient for the binary pulsar)}} \Big\{ 1 + \underbrace{\frac{1\text{PN}}{c^2} + \frac{1.5\text{PN}}{c^3} + \cdots + \frac{3\text{PN}}{c^6} + \cdots}_{\text{needs to be computed with 3PN precision}} \Big\}$$

Short history of the PN approximation

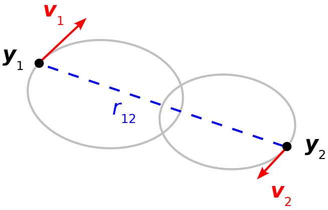
Equations of motion

- 1PN equations of motion [Lorentz & Droste 1917; Einstein, Infeld & Hoffmann 1938]
- Radiation-reaction controvercy [Ehlers *et al* 1979; Walker & Will 1982]
- 2.5PN equations of motion and GR prediction for the binary pulsar [Damour & Deruelle 1982, Damour 1983]
- The 3mn Caltech paper [Cutler, Flanagan, Poisson, Thorne 1993]
- 3.5PN equations of motion [Jaranowski & Schäfer 1999; LB & Faye 2001; Andrade, LB & Faye 2002; Itoh & Futamase 2003; Foffa & Sturani 2010]
- Ambiguity parameters resolved [Damour, Jaranowski & Schäfer 2001; LB, Damour & Esposito-Farèse 2003]

Radiation field

- 1918 Einstein quadrupole formula
- 1940 Landau-Lifchitz formula
- 1960 Peters-Mathews formula
- Epstein-Wagoner-Thorne moments [Thorne 1980]
- 1PN wave generation [Wagoner & Will 1976; LB & Schäfer 1989]
- Blanchet-Damour moments [LB & Damour 1989; LB 1995, 1998]
- 2PN wave generation [LB, Damour, Iyer, Will & Wiseman 1995]
- 3.5PN wave generation [LB, Iyer & Joguet 2002; LB, Faye, Iyer & Joguet 2003]
- Ambiguity parameters resolved [LB, Damour, Esposito-Farèse & Iyer 2004]

Post-Newtonian equations of motion



The equations of motion are written in Newtonian-like form (with $t = x^0/c$ playing the role of Newton's "absolute time")

$$\frac{\mathrm{d}\mathbf{v}_1}{\mathrm{d}t} = \mathbf{A}_1^{\mathrm{N}} + \frac{1}{c^2}\mathbf{A}_1^{1\mathrm{PN}} + \frac{1}{c^4}\mathbf{A}_1^{2\mathrm{PN}} + \underbrace{\frac{1}{c^5}\mathbf{A}_1^{2.5\mathrm{PN}}}_{\text{radiation reaction}} + \overbrace{\frac{1}{c^6}\mathbf{A}_1^{3\mathrm{PN}}}^{\text{very difficult term to compute}} + \underbrace{\frac{1}{c^7}\mathbf{A}_1^{3.5\mathrm{PN}}}_{\text{radiation reaction}} + \mathcal{O}\left(\frac{1}{c^8}\right)$$

1. The EOM reduce in the test-mass limit to the geodesic equations of Schwarzschild metric
2. They are derivable from a Lagrangian/Hamiltonian formalism (when the gravitational radiation reaction is neglected)
3. They are invariant under a global Lorentz-Poincaré transformation

Two equivalent PN wave generation formalisms

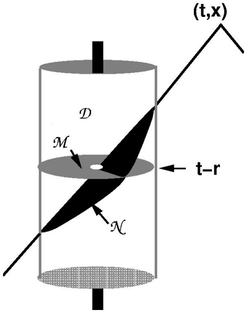
The field equations are integrated in the exterior of an extended PN source by means of a multipolar expansion

BD multipole moments [LB & Damour 1989; LB 1995, 1998]

$$M_L^{\mu\nu}(t) = \underset{B=0}{\text{Finite Part}} \int d^3x \, x_L \, \overline{\tau}^{\mu\nu}(\mathbf{x}, t)$$

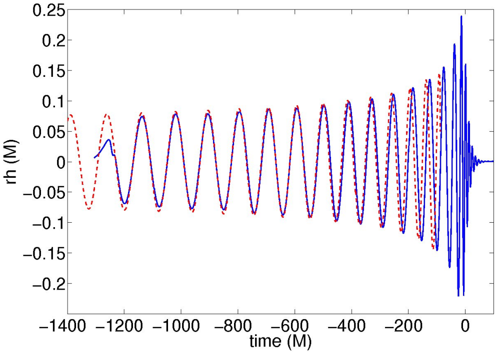
WW multipole moments [Will & Wiseman 1996]

$$W_L^{\mu\nu}(t) = \int_{\mathcal{M}} d^3x \, x_L \, \overline{\tau}^{\mu\nu}(\mathbf{x}, t)$$



These formalisms solved the long-standing problem of divergencies in the PN expansion for general extended sources

The gravitational chirp of compact binaries

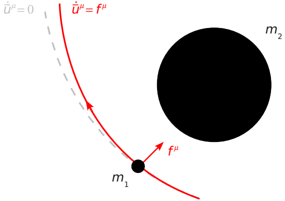


The waveform is obtained by matching a high-order post-Newtonian waveform describing the long inspiralling phase and a highly accurate numerical waveform describing the final merger and ringdown phases

POST-NEWTONIAN VERSUS SELF-FORCE PREDICTIONS

General problem of the self-force

- A particle is moving on a background space-time
- Its own stress-energy tensor modifies the background gravitational field
- Because of the "back-reaction" the motion of the particle deviates from a background geodesic hence the appearance of a self force



The self acceleration of the particle is proportional to its mass

$$\frac{\mathrm{D}\bar{u}^\mu}{\mathrm{d}\tau} = f^\mu = \mathcal{O}\left(\frac{m_1}{m_2}\right)$$

The gravitational self force includes both dissipative (radiation reaction) and conservative effects.

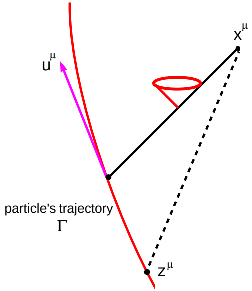
Self-force in perturbation theory

The space-time metric $g_{\mu\nu}$ is decomposed as a background metric $\bar{g}_{\mu\nu}$ plus

$$h_{\mu\nu} = \text{linearized parturbation of the background space-time}$$

The field equation in an harmonic gauge reads

$$\Box\, h^{\mu\nu} + 2R^{\,\mu\,\nu}_{\rho\,\sigma}\, h^{\rho\sigma} = -16\pi\, T^{\mu\nu}$$



The retarded solution is

$$h^{\mu\nu}(x) = 4m_1 \int_\Gamma G^{\mu\nu}_{\text{ret}\ \rho\sigma}(x,z)\,\bar{u}^\rho\,\bar{u}^\sigma + \mathcal{O}(m_1^2)$$

Green function responsible for the self-force [Detweiler & Whiting 2003]

The symmetric Green function is defined by the prescription

$$\underset{\mathsf{S}}{G} = \frac{1}{2}\Big[\underset{\mathsf{ret}}{G} + \underset{\mathsf{adv}}{G} - H\Big]$$

where H is homogeneous solution of the wave equation

- G_S is symmetric under a time reversal hence corresponds to stationary waves at infinity and does not produce a reaction force on the particle
- It has the same divergent behavior as G_{ret} on the particle's worldline
- It is non zero only when x and z are related by a space-like interval

The radiative Green function responsible for the self force is

$$\underset{\mathsf{R}}{G}(x,z) = \underset{\mathsf{ret}}{G}(x,z) - \underset{\mathsf{S}}{G}(x,z) = \frac{1}{2}\Big[\underset{\mathsf{ret}}{G} - \underset{\mathsf{adv}}{G} + H\Big]$$

Computation of the self-force [Mino, Sasaki & Tanaka 1997; Quinn & Wald 1997]

1. The metric perturbation is decomposed as

$$h_{\mu\nu} = \underset{\mathsf{S}}{h}_{\mu\nu} + \underset{\mathsf{R}}{h}_{\mu\nu}$$

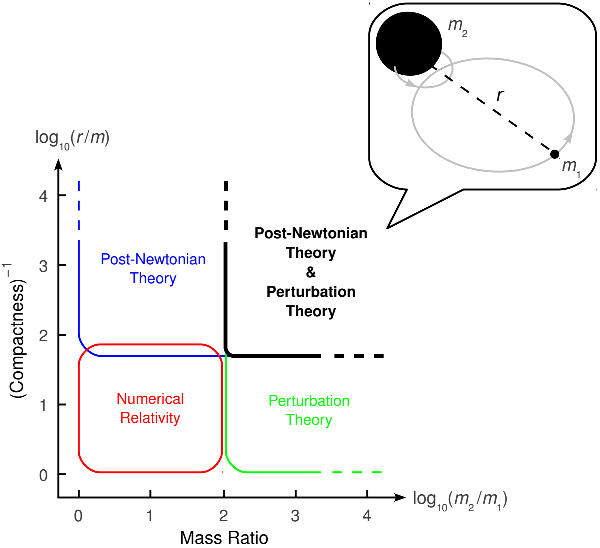
   where the particular solution $h_{\mathsf{S}}^{\mu\nu}$ (symmetric in a time reversal) diverges on the particle's location, but where the homogeneous solution $h_{\mathsf{R}}^{\mu\nu}$ is regular

2. The self-force f^{μ} is computed from the geodesic motion with respect to

$$g^{\mathsf{SF}}_{\mu\nu} = \bar{g}_{\mu\nu} + \underset{\mathsf{R}}{h}_{\mu\nu}$$

3. The divergence on the particle's trajectory due to G_S can be renormalized in a redefinition of the particle's mass
4. The result agrees with the MiSaTaQuWa expression of the self-force

Common regime of validity of SF and PN



Why and how comparing PN and SF predictions?

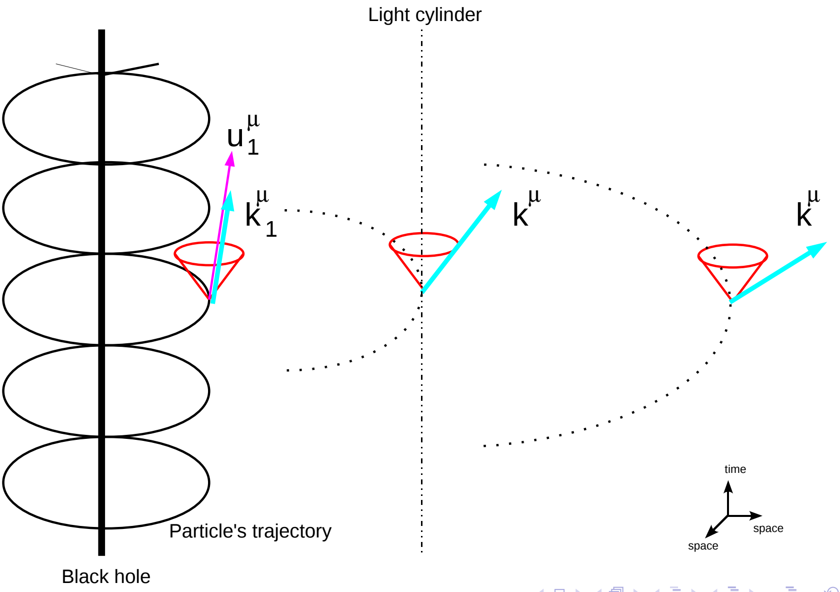
Both the PN and SF approaches use a self-field regularization for point particles followed by a renormalization. However, the prescription are very different

1. SF theory is based on a prescription for the Green function G_R that is at once regular and causal [Detweiler & Whiting 2003]
2. PN theory uses dimensional regularization and it was shown that subtle issues appear at the 3PN order due to the appearance of poles $\propto (d-3)^{-1}$

How can we make a meaningful comparison?

1. To restrict attention to the conservative part of the dynamics
2. To find a gauge-invariant observable computable in both formalisms

Circular orbits admit a helical Killing vector



Choice of a gauge-invariant observable [Detweiler 2008]

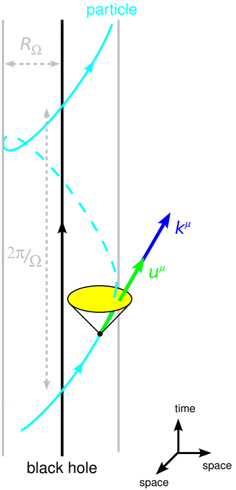
1. For exactly circular orbits the geometry admits a helical Killing vector with

$$k^\mu \partial_\mu = \partial_t + \Omega\, \partial_\varphi \quad \text{(asymptotically)}$$

2. The four-velocity of the particle is necessarily tangent to the Killing vector hence

$$u_1^\mu = u_1^T\, k_1^\mu$$

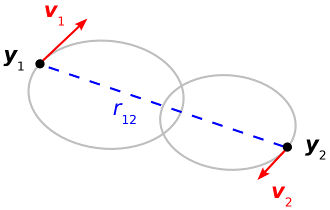
3. The relation $u_1^T(\Omega)$ is well-defined in both PN and SF approaches and is gauge-invariant



Post-Newtonian calculation

In a coordinate system such that $k^\mu \partial_\mu = \partial_t + \Omega\, \partial_\varphi$ everywhere this invariant quantity reduces to the zero component of the particle's four-velocity,

$$u_1^t = \Bigg(- \underbrace{\mathrm{Reg}_1 \left[g_{\mu\nu} \right]}_{\text{regularized metric}} \frac{v_1^\mu v_1^\nu}{c^2} \Bigg)^{-1/2}$$



One needs a self-field regularization

- Hadamard regularization will yield an ambiguity at 3PN order
- Dimensional regularization will be free of any ambiguity at 3PN order

High-order post-Newtonian result [LB, Detweiler, Le Tiec & Whiting 2010]

- The result is expressed in terms of $x = \left(\frac{GM\Omega}{c^3}\right)^{3/2}$ as

$$\begin{aligned} u^T &= 1 + A_0\,x + A_1\,x^2 + A_2\,x^3 + \overbrace{A_3\,x^4}^{\text{3PN}} \\ &+ \underbrace{\left[A_4 + B_4\ln x\right]x^5}_{\text{4PN}} + \underbrace{\left[A_5 + B_5\ln x\right]x^6}_{\text{5PN}} + o(x^6) \end{aligned}$$

- The coefficients depend on mass ratios $\nu = m_1 m_2/M^2$, $\Delta = (m_1 - m_2)/M$

$$\begin{aligned} A_3 &= \frac{2835}{256} + \frac{2835}{256}\Delta - \left[\frac{2183}{48} - \frac{41}{64}\pi^2\right]\nu + \text{other terms} \\ B_4 &= -\frac{32}{5}\nu(1+\Delta) + \frac{64}{15}\nu^2 \\ B_5 &= \frac{478}{105}\nu(1+\Delta) + \text{other terms} \end{aligned}$$

High-order PN prediction for the self-force

- We re-expand in the small mass-ratio limit $q = m_1/m_2 \ll 1$ so that

$$u^T = u^T_{\text{Schw}} + \underbrace{q\, u^T_{\text{SF}}}_{\text{self-force}} + \underbrace{q^2\, u^T_{\text{PSF}}}_{\text{post-self-force}} + \mathcal{O}(q^3)$$

- Posing $y = \left(\frac{G m_2 \Omega}{c^3}\right)^{3/2}$ we find

$$\begin{aligned} u^T_{\text{SF}} &= -y - 2y^2 - 5y^3 + \overbrace{\left(-\frac{121}{3} + \frac{41}{32}\pi^2\right) y^4}^{\text{3PN}} \\ &+ \underbrace{\left(a_4 + \frac{64}{5}\ln y\right) y^5}_{\text{4PN}} + \underbrace{\left(a_5 - \frac{956}{105}\ln y\right) y^6}_{\text{5PN}} + o(y^6) \end{aligned}$$

High-order PN fit to the numerical self-force

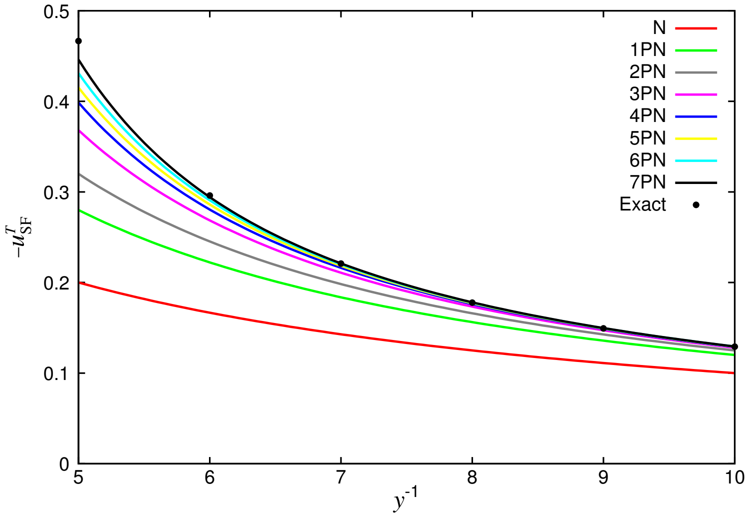
- Post-Newtonian coefficients are fitted up to 7PN order

PN coefficient	SF value
a_4	$-114.34747(5)$
a_5	$-245.53(1)$
a_6	$-695(2)$
b_6	$+339.3(5)$
a_7	$-5837(16)$

- The 3PN prediction agrees with the SF value with 7 significant digits

3PN value	SF fit
$a_3 = -\frac{121}{3} + \frac{41}{32}\pi^2 = -27.6879026\cdots$	$-27.6879034 \pm 0.0000004$

Comparison between PN and SF predictions



FIRST LAW OF POINT MASS BINARY SYSTEMS

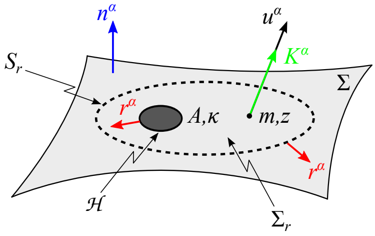
Komar like integral for helical symmetric space-times

1. Space-time with helical Killing vector

$$k^\alpha = t^\mu + \Omega \phi^\mu$$

2. The ADM mass and angular momentum are given by surface integrals at infinity

$$M = -\frac{1}{8\pi} \lim_{r\to\infty} \oint_{S_r} \nabla^\mu t^\nu \, \mathrm{d}S_{\mu\nu}$$
$$J = \frac{1}{16\pi} \lim_{r\to\infty} \oint_{S_r} \nabla^\mu \phi^\nu \, \mathrm{d}S_{\mu\nu}$$



Using the Einstein field equations for a smooth matter distribution we get

$$M - 2\Omega J = 2\int_\Sigma \left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right) n^\alpha k^\beta \sqrt{\gamma}\, \mathrm{d}^3 x$$

First law of perfect fluid mechanics [Friedman, Uryū & Shibata 2002]

Compare two nearby solutions of the Einstein field equations with Killing vector k^μ, corresponding to slightly different matter configurations:

$$\delta M - \Omega\delta J = -\int_\Sigma \Delta\big(\mathrm{d}\Sigma_\mu\, T^\mu{}_\nu\big)k^\nu + \frac{1}{2}\int_\Sigma \mathrm{d}\Sigma_\mu k^\mu\, T^{\rho\sigma}\Delta g_{\rho\sigma}$$

where Δ denotes the Lagrangian variation of the matter fluid.

Generalized law of perfect fluid and black hole mechanics [Friedman, Uryū & Shibata 2002]

$$\delta M - \Omega\delta J = \int_\Sigma \big[\bar{\mu}\,\Delta(\mathrm{d}m) + \bar{T}\,\Delta(\mathrm{d}S) + w^\mu\Delta(\mathrm{d}C_\mu)\big] + \sum_n \frac{\kappa_n}{8\pi}\,\delta A_n$$

where $\mathrm{d}m$ is the conserved baryonic mass element, and where $\overline{T} = zT$ and $\overline{\mu} = z(h - Ts)$ are the redshifted temperature and chemical potential.

PN derivation of the first law [Le Tiec, LB & Whiting 2012]

1. The ADM mass M and angular momentum J of the circular-orbit binary are computed through 3PN order augmented by 4PN and 5PN logarithmic contributions
2. We explicitly check through 3PN + 4PN/5PN$_{\text{log}}$ that they obey the relation

$$\frac{\partial M}{\partial \Omega} = \Omega \, \frac{\partial J}{\partial \Omega}$$

   used in computations of the binary evolution based on a sequence of quasi-equilibrium configurations [Gourgoulhon *et al* 2002]
3. However we find that they are also related to Detweiler's redshift observables $z_1 = 1/u_1^T$ and $z_2 = 1/u_2^T$ by

$$\begin{aligned} \frac{\partial M}{\partial m_1} - \Omega \frac{\partial J}{\partial m_1} &= z_1 \\ \frac{\partial M}{\partial m_2} - \Omega \frac{\partial J}{\partial m_2} &= z_2 \end{aligned}$$

First law of binary point particle mechanics [Le Tiec, LB & Whiting 2012]

- These relations can be summarized in the first law of binary binary point-particles (modelling binary black holes) mechanics

$$\boxed{\delta M - \Omega\,\delta J = z_1\,\delta m_1 + z_2\,\delta m_2}$$

- The first law tells how the ADM quantities change when the individual masses m_1 and m_2 of the particles vary (keeping the frequency Ω fixed)
- An interesting consequence of the first law is the remarkably simple relation

$$\boxed{M - 2\Omega J = m_1 z_1 + m_2 z_2}$$

- There is complete agreement with the generalized law of fluid and black hole mechanics [Friedman, Uryū & Shibata 2002] and the Komar integral of the first law

Higher PN terms in the binary's energy

The first law can be used to compute new PN coefficients in the binary's binding energy $E = M - m_1 - m_2$

$$
\begin{aligned}
E = & -\frac{1}{2} m \nu x \Bigg\{ 1 + \left(-\frac{3}{4} - \frac{\nu}{12} \right) x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) x^2 \\
& + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2 \right] \nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right) x^3 \\
& + \left(-\frac{3969}{128} + \nu\, e_4(\nu) + \frac{448}{15}\nu \ln x \right) x^4 \\
& + \left(-\frac{45927}{512} + \nu\, e_5(\nu) + \left[-\frac{4988}{35} - 6565\nu \right] \nu \ln x \right) x^5 \\
& + \left(-\frac{264627}{1024} + \nu\, e_6(\nu) + \nu\, e_6^{\mathrm{ln}}(\nu) \ln x \right) x^6 \Bigg\}
\end{aligned}
$$

Higher PN terms in the binary's energy

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$$
\begin{aligned}
E \quad = \quad & -\frac{1}{2} m\,\nu\,x \Bigg\{ 1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right) x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right) x^2 \\
& + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right) x^3 \\
& + \left(-\frac{3969}{128} + 153.8803\,\nu + \frac{448}{15}\nu \ln x\right) x^4 \\
& + \left(-\frac{45927}{512} - 55.13\,\nu + \left[-\frac{4988}{35} - 6565\nu\right]\nu \ln x\right) x^5 \\
& + \left(-\frac{264627}{1024} + 588.\,\nu - 1144.\,\nu\,\ln x\right) x^6 + \mathcal{O}(\nu^2) \Bigg\}
\end{aligned}
$$

Conclusions

1. Compact binary star systems are the most important source for gravitational wave detectors LIGO/VIRGO and LISA
2. Post-Newtonian theory has proved to be the appropriate tool for describing the inspiral phase of compact binaries up to the ISCO
3. The 3.5PN templates should be sufficient for detection and analysis of neutron star binary inspirals in LIGO/VIRGO
4. For massive BH binaries the PN templates should be matched to the results of numerical relativity for the merger and ringdown phases
5. The PN approximation is now tested against different approaches such as the SF and performs extremely well.