Boundedness and decay for linear waves on sub-extremal Kerr–Newman exterior spacetimes

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Kerr–Newman spacetimes

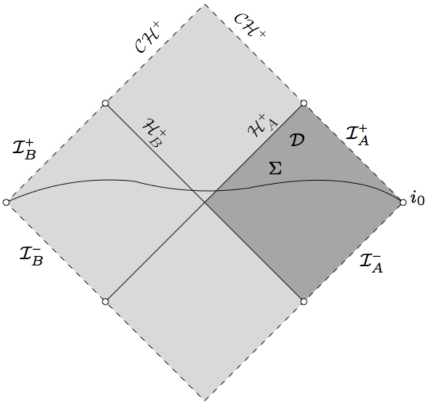

Figure: "Penrose diagram" of a subextremal Kerr–Newman spacetime

$g_{M,a,Q} = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\Big((r^2+a^2)d\phi - adt\Big)^2$

where $\Delta = r^2 - 2Mr + a^2 + Q^2$, $\rho^2 = r^2 + a^2\cos^2\theta$ and $0 \leq a^2 + Q^2 < M^2$.

The stability problem for Kerr–Newman

- **Uniqueness:**

  The Kerr/Kerr–Newman black holes are thought to be the unique family of stationary black hole solutions of the Einstein vacuum/electrovac equations.
  Hawking, Carter, Robinson, Chrusciel–Costa, Papapetrou, Alexakis–Ionescu–Klainerman, Yu, Wong

- **Stability problem:**

  Show that perturbations of a subextremal Kerr–Newman metric evolve via the Einstein equations into a member of the Kerr–Newman family in the black hole exterior region.

Linear stability of Kerr–Newman spacetime

- Following philosophy of Christodoulou–Klainerman '93, *"The global nonlinear stability of the Minkowski space".* Consider

$$\Box_g \psi = 0,$$

where $\psi : \mathcal{M} \to \mathbb{R}$ and $g = g_{M,a,Q}$ is a Kerr–Newman metric.

- Prescribe sufficiently regular initial data on a suitable Cauchy hypersurface Σ_0.

- Prove quantitative decay of energy of ψ in a robust manner.

Previous work

- Carter '68 – wave equation can be formally separated.
- Whiting '89 – *mode stability:* no modes with finite initial energy growing in time.
- Finster–Kamran–Smoller–Yau '03 & '06 – each mode decays under assumption of smoothness and support away from horizon.
- Without boundedness statement, one cannot make a statement about superposition of modes.
- Apparently no such results for Kerr–Newman.
- Non-quantitative results don't show clear dependence on data – this is needed for nonlinear stability problem.
- Dafermos–Rodnianski '09 & '10: Boundedness for $|a| \ll M$ and essential elements of decay for Kerr $|a| < M$.
- Aretakis '11: Extremal Kerr.
- Schlue '11: Schwarzschild–de Sitter case.

Linear stability theorem for Kerr–Newman [D.C.]

Consider $\Box_{g_{M,a,Q}}\psi = 0$ where ψ is supported only on modes of high azimuthal frequency $|m| > m_0$ or ψ is axisymmetric.

- Degenerate integrated local energy decay

$$\int_0^\tau \int_{\Sigma_t} \left((\partial_r \psi)^2 + \psi^2 + \chi(r) \left[(\partial_t \psi)^2 + |\nabla\!\!\!/ \psi|^2 \right] \right) \, dt \leq C \int_{\Sigma_0} |\partial\psi|^2$$

- Boundedness of nondegenerate energy flux

$$\int_{\Sigma_\tau} |\partial\psi|^2 \leq C \int_{\Sigma_0} |\partial\psi|^2$$

- Polynomial time decay of the energy flux

$$\int_{\Sigma_\tau} |\partial\psi|^2 \leq C\tau^{-2} \int_{\Sigma_0} \left[|\partial\psi|^2 + \left|\partial^2\psi\right|^2 \right]$$

Difficulties

- **Event horizon:** Usual energy (that related to timelike Killing field ∂_t) degenerates on horizon.
- **Ergoregion:** where Killing field ∂_t is spacelike. Conserved energy is not positive definite. For particles leads to Penrose process. For waves, superradiance – energy flux to null infinity may be larger than the initial energy.
- **Trapped geodesics:** There are null geodesics of constant r: they neither cross the event horizon nor asymptote to null infinity. Energy decay statement must degenerate on a complicated set.
- **Strong coupling** of superradiance and trapping in physical space.

Strategy

- DAFERMOS–RODNIANSKI '08 – Use redshift to get nondegenerate energy up to horizon.
- CARTER '68 – wave equation can be formally separated.
- Frequency localise – capture frequency specific phenomena.
- DAFERMOS–RODNIANSKI '09 – Miraculous decoupling of trapping and superradiance for Kerr family. D.C. – extend to Kerr–Newman.
- Prove frequency localised estimates.
- Return to physical space and control error terms
- Retrieve boundedness from integrated local energy decay.

Thank you for your attention