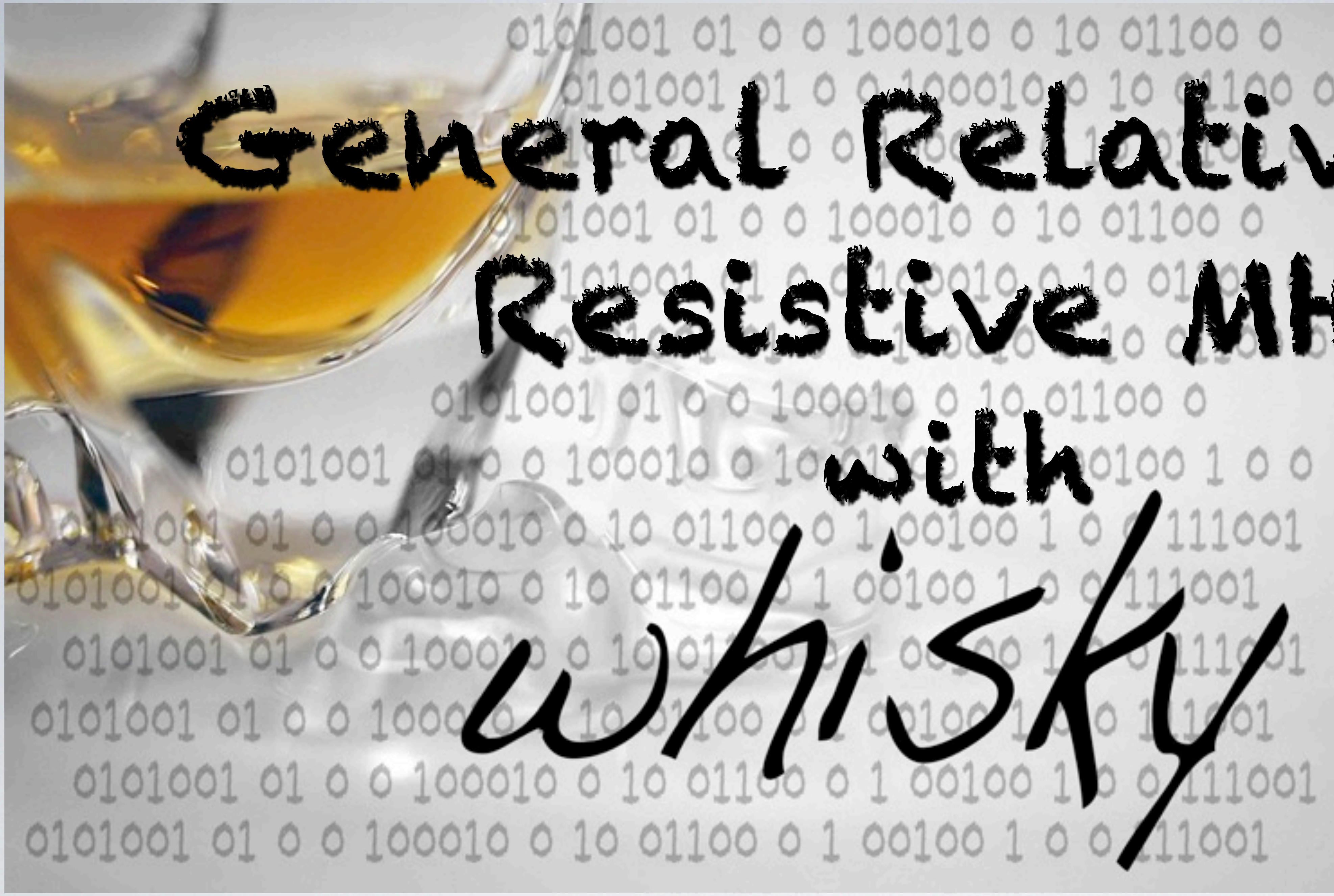

General Relativistic Resistive MHD with whisky

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Outline

Analytical and numerical setup:

- GR-Resistive MHD (with ideal MHD and Electrovacuum limits)
- Stiff terms and RKIMEX evolution schemes

Tests:

- Shock Tubes, Alfven Waves, Current Sheets (1D)

Numerical Evolution of Stars:

- Stable star with magnetic field extending outside the star
- Unstable star with magnetic field extending outside the star - gravitational collapse to a black hole

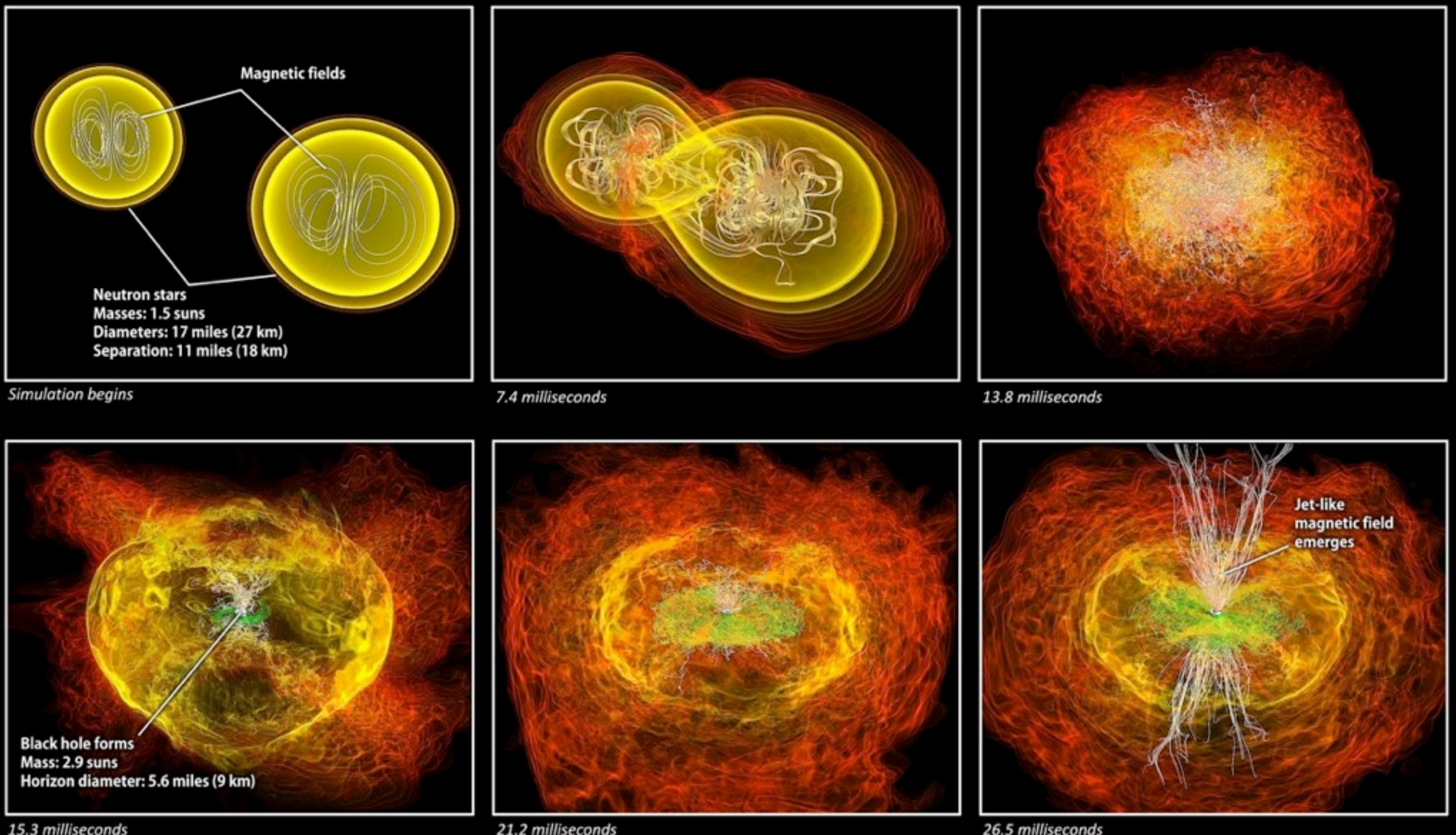
Motivation

GR-RMHD provides a single mathematical framework which can accurately describe the interior of a star, the magnetosphere and vacuum regions.

It allows resistive effects (like reconnection of the magnetic field lines) and offers some control over the amount of resistivity in the system.

The GR-RMHD system is hyperbolic with stiff relaxation terms, as the diffusive effects take place on different time-scales than the dynamical ones.

Crashing neutron stars can make gamma-ray burst jets



Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

$J/M^2 = 0.83$ $\quad$ $\mathrm{M_{tor}} = 0.063 M_\odot$ $\quad$ $\mathrm{t_{accr}} \simeq M_{\mathrm{tor}}/\dot{M} \simeq 0.3\ \mathrm{s}$

The Resistive MHD formalism

Maxwell equations: $B^i\,,E^i$

$$\mathcal{D}_t\,E^i - \epsilon^{ijk}\nabla_j(\,\alpha\,B_k\,) + \alpha\,\gamma^{ij}\nabla_j\,\Psi = \alpha\,K\,E^i - \alpha\,J^i$$
$$\mathcal{D}_t\,B^i + \epsilon^{ijk}\nabla_j(\,\alpha\,E_k\,) + \alpha\,\gamma^{ij}\nabla_j\,\Phi = \alpha\,K\,B^i$$

Divergence cleaning: $\phi\,,\psi$

$$\mathcal{D}_t\,\Psi + \alpha\,\nabla_i E^i = \alpha\,q - \alpha\kappa\,\Psi$$

$$\mathcal{D}_t\,\Phi + \alpha\,\nabla_i B^i = -\alpha\kappa\,\Phi$$

Charge Density: $q = \nabla_i E^i$

$$\mathcal{D}_t\,q + \nabla_i(\,\alpha J^i\,) = \alpha\,K\,q$$

Hydrodynamic equations: $D\,,S_i\,,\tau$

The Resistive MHD formalism

Maxwell equations: $B^i \, , E^i$

$$\mathcal{D}_t \, E^i - \epsilon^{ijk} \nabla_j (\alpha \, B_k) + \alpha \, \gamma^{ij} \nabla_j \, \Psi = \alpha \, K \, E^i - \alpha \, J^i$$
$$\mathcal{D}_t \, B^i + \epsilon^{ijk} \nabla_j (\alpha \, E_k) + \alpha \, \gamma^{ij} \nabla_j \, \Phi = \alpha \, K \, B^i$$

Divergence cleaning: $\phi \, , \psi$

$$\mathcal{D}_t \, \Psi + \alpha \, \nabla_i E^i = \alpha \, q - \alpha \kappa \, \Psi$$

$$\mathcal{D}_t \, \Phi + \alpha \, \nabla_i B^i = - \alpha \kappa \, \Phi$$

Charge Density: $q = \nabla_i E^i$

$$\mathcal{D}_t \, q + \nabla_i (\alpha J^i) = \alpha \, K \, q$$

Hydrodynamic equations: $D \, , S_i \, , \tau$

The Resistive MHD formalism

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i] + \ldots$$

Resistive regime: low σ

Require that the current is finite when σ → ∞

Ideal MHD limit: $E^i = -\epsilon^{ijk}v_j B_k$

Electrovacuum limit: $q \to 0, \ \sigma \to 0$

The conductivity σ can lead to a stiff term in the evolution equation of electric field.

RK IMEX Methods

(Pareschi & Russo 2005)

$$\partial_t \mathbf{W} = F_W(\mathbf{V},\mathbf{W})$$
$$\partial_t \mathbf{V} = F_V(\mathbf{V},\mathbf{W}) + \frac{1}{\epsilon(\mathbf{W})} R_V(\mathbf{V},\mathbf{W})$$

$$W \to \{D, S, \tau, q, B, \psi, \phi\}$$
$$V \to E\,,\ \frac{1}{\epsilon} \to \sigma$$

1) Explicit step

$$\mathbf{W}^* = \mathbf{W}^n + \Delta t \sum_{j=1}^{i-1} \tilde{a}_{ij} F_W(\mathbf{U}^{(j)})$$
$$\mathbf{V}^* = \mathbf{V}^n + \Delta t \sum_{j=1}^{i-1} \tilde{a}_{ij} F_V(\mathbf{U}^{(j)})$$
$$+ \Delta t \sum_{j=1}^{i-1} a_{ij} \frac{1}{\epsilon^{(j)}} R_V(\mathbf{U}^{(j)})$$

2) Implicit step

$$\mathbf{V}^{(i)} = \mathbf{V}^* + a_{ii} \frac{\Delta t}{\epsilon^{(i)}} R_V(\mathbf{V}^{(i)}, \mathbf{W}^{(i)})$$
$$\mathbf{W}^{(i)} = \mathbf{W}^*$$

a) Assuming linear dependence

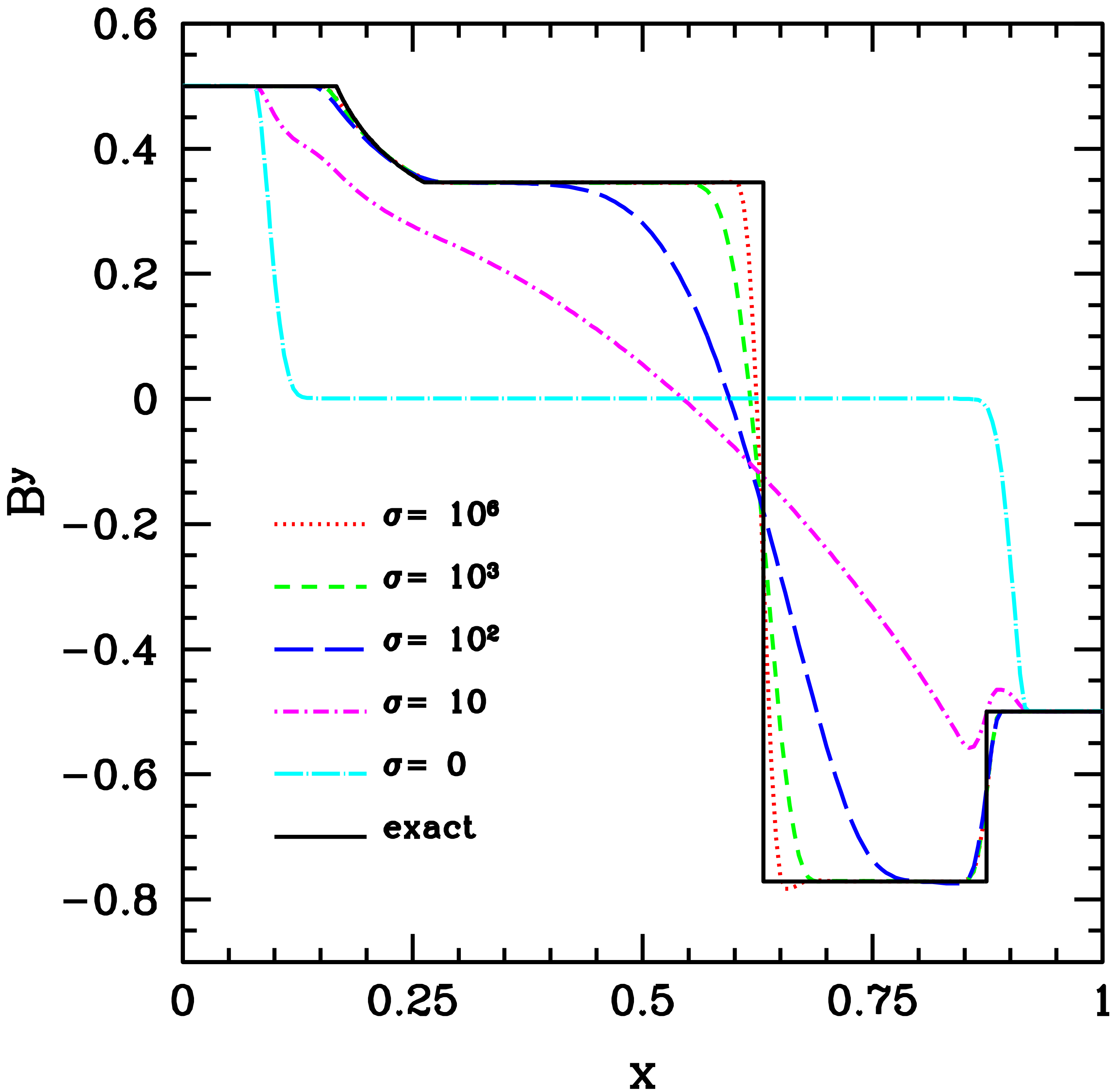
$$R_V(\mathbf{V},\mathbf{W}) = A(\mathbf{W})\mathbf{V} + S(\mathbf{W})$$

b) We simply invert

$$\mathbf{V}^{(i)} = [I - a_{ii} \frac{\Delta t}{\epsilon^{(i)}} A(\mathbf{W}^*)]^{-1} \left(\mathbf{V}^* + a_{ii} \frac{\Delta t}{\epsilon^{(i)}} S(\mathbf{W}^*)\right)$$
$$\mathbf{W}^{(i)} = \mathbf{W}^*$$

Results

Test 1D: Shock-tube

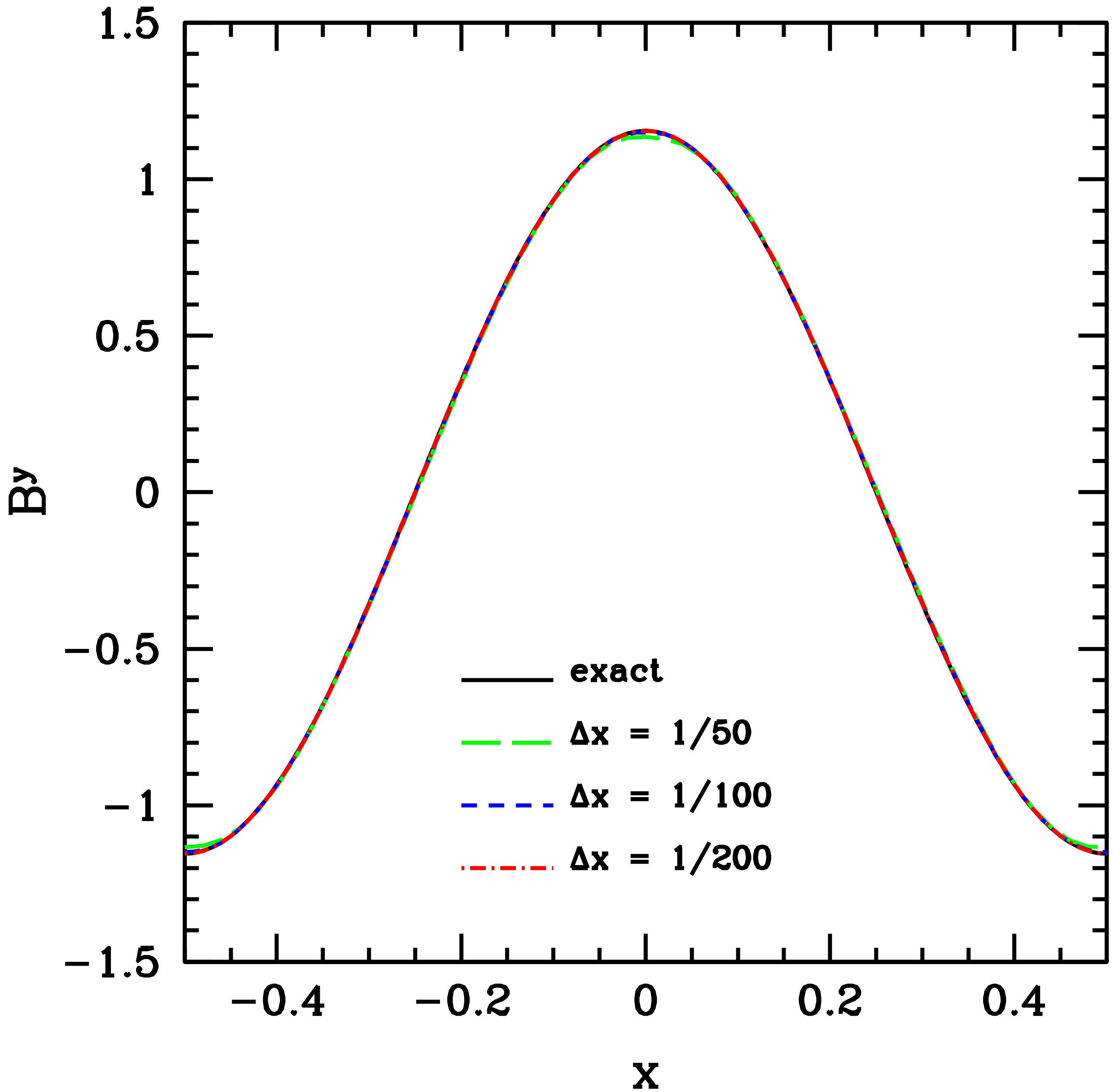


$\sigma \rightarrow 0, 10, 10^2, 10^3, 10^6$

$\sigma \rightarrow 0$ Wave-like solution

$\sigma \rightarrow \infty$ Ideal MHD solution

Large Amplitude Alfven waves

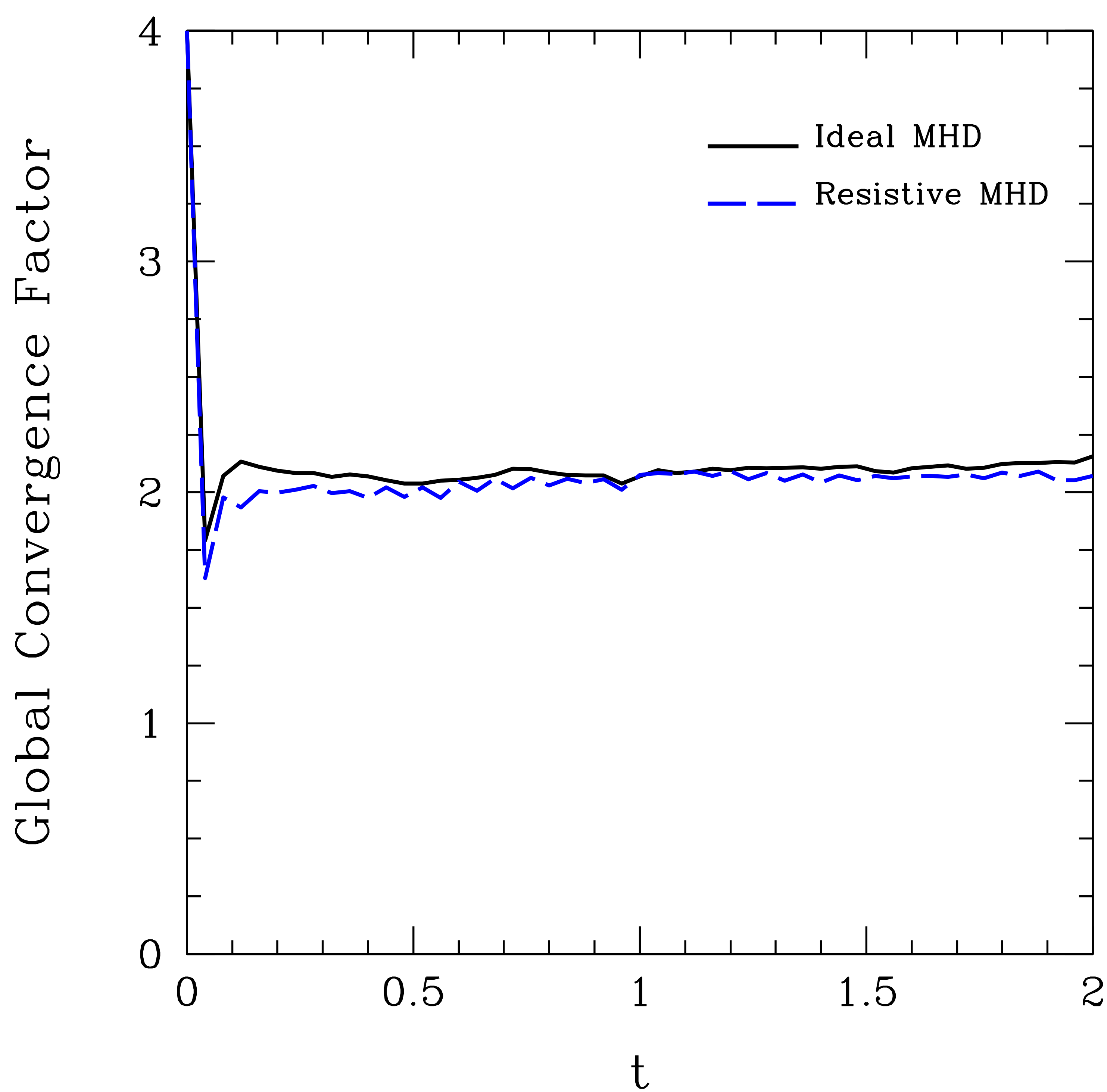


$\sigma \rightarrow 10^6$

Propagation of large amplitude Alfven waves in a uniform background field B, in a domain with periodic boundary conditions.

The numerical solution after one full period converges to the exact one.

Large Amplitude Alfven waves

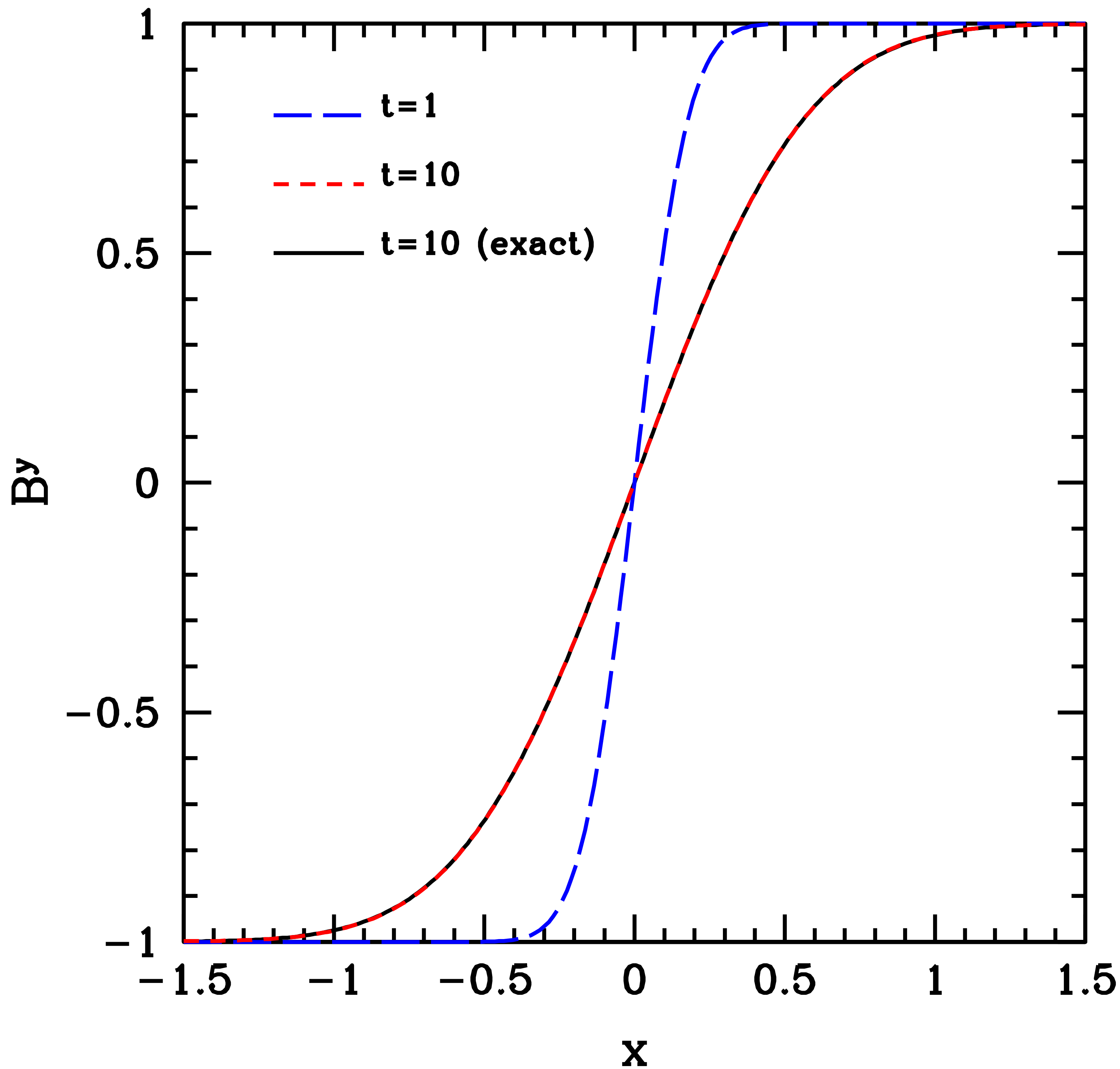


$\sigma \rightarrow 10^6$

Propagation of large amplitude Alfven waves in a uniform background field B, in a domain with periodic boundary conditions.

The numerical solution after one full period converges to the exact one.

Test 1D: Self-similar current-sheet



$\sigma \rightarrow 100$

Slow diffusive expansion of the layer due to the resistivity. The width of the layer becomes larger and it evolves in a self-similar fashion.

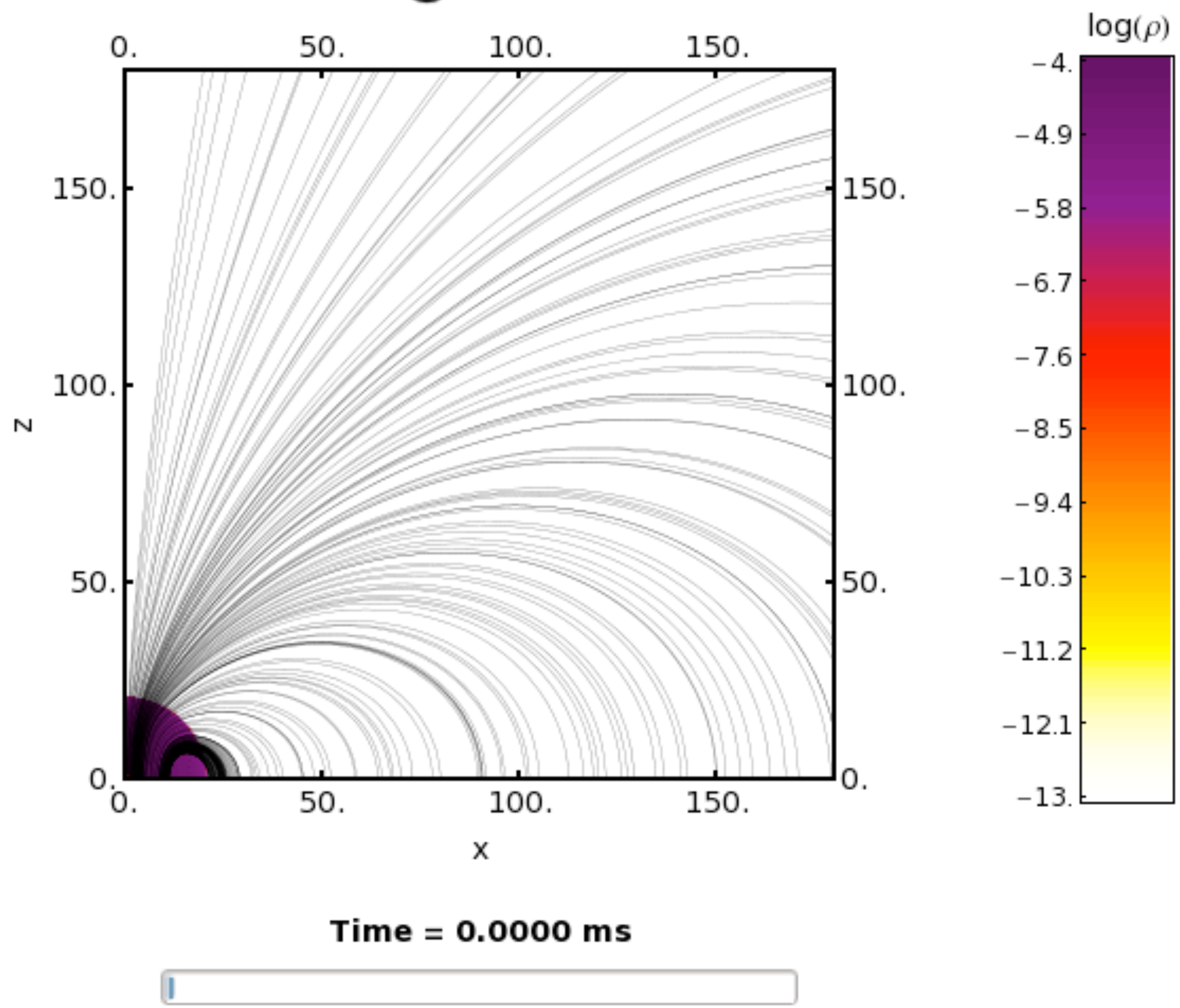
Numerical solution converges to the exact one.

Numerical Evolution of Stars

Non-rotating Stable Star with Magnetic field extending outside the star

The magnetic field lines and the rest mass density, at times t = 0 and t = 37 ms. As expected, the magnetic field remains stable after several tens of ms.

Non-rotating Stable Star with Magnetic field extending outside the star



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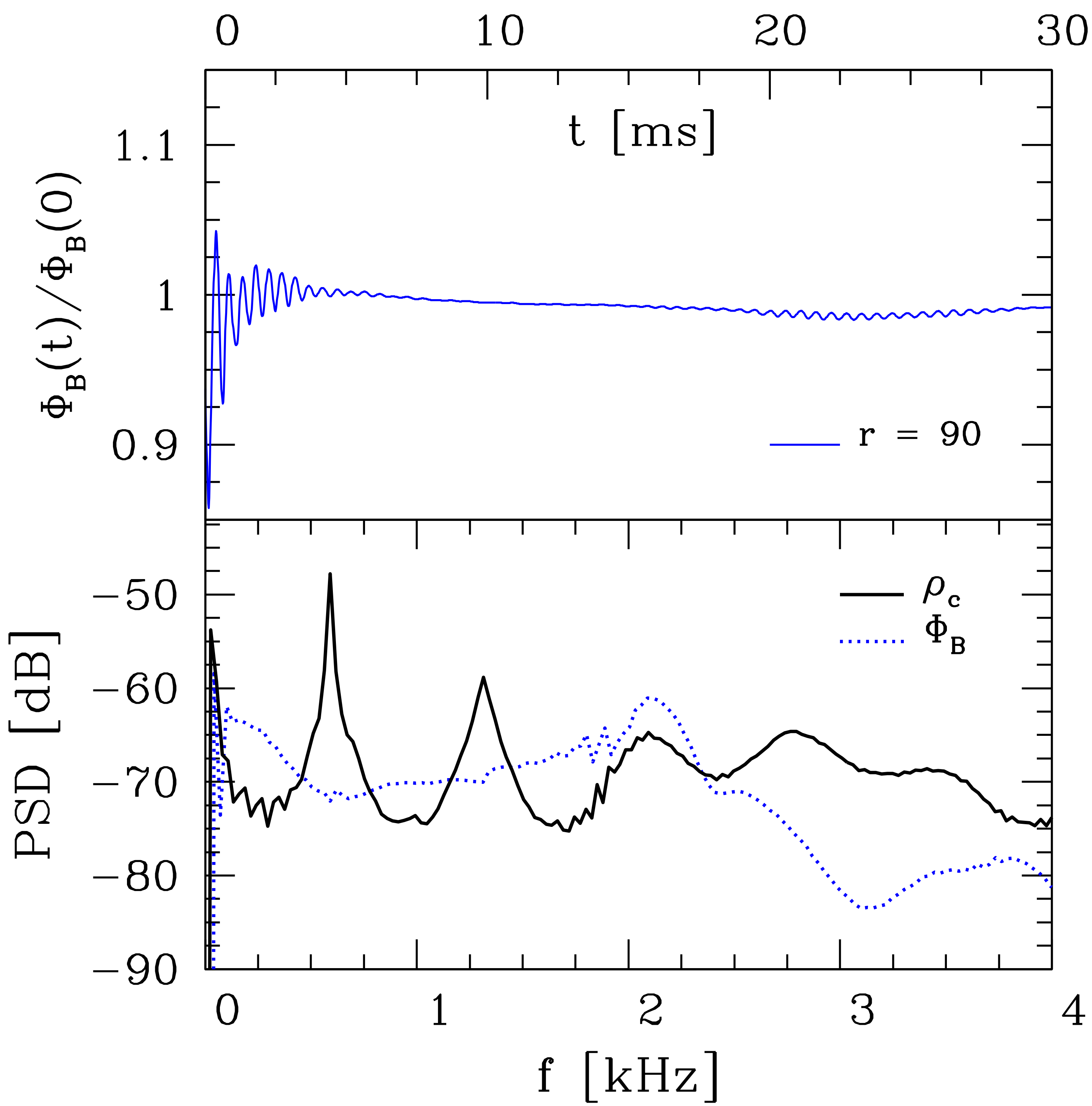
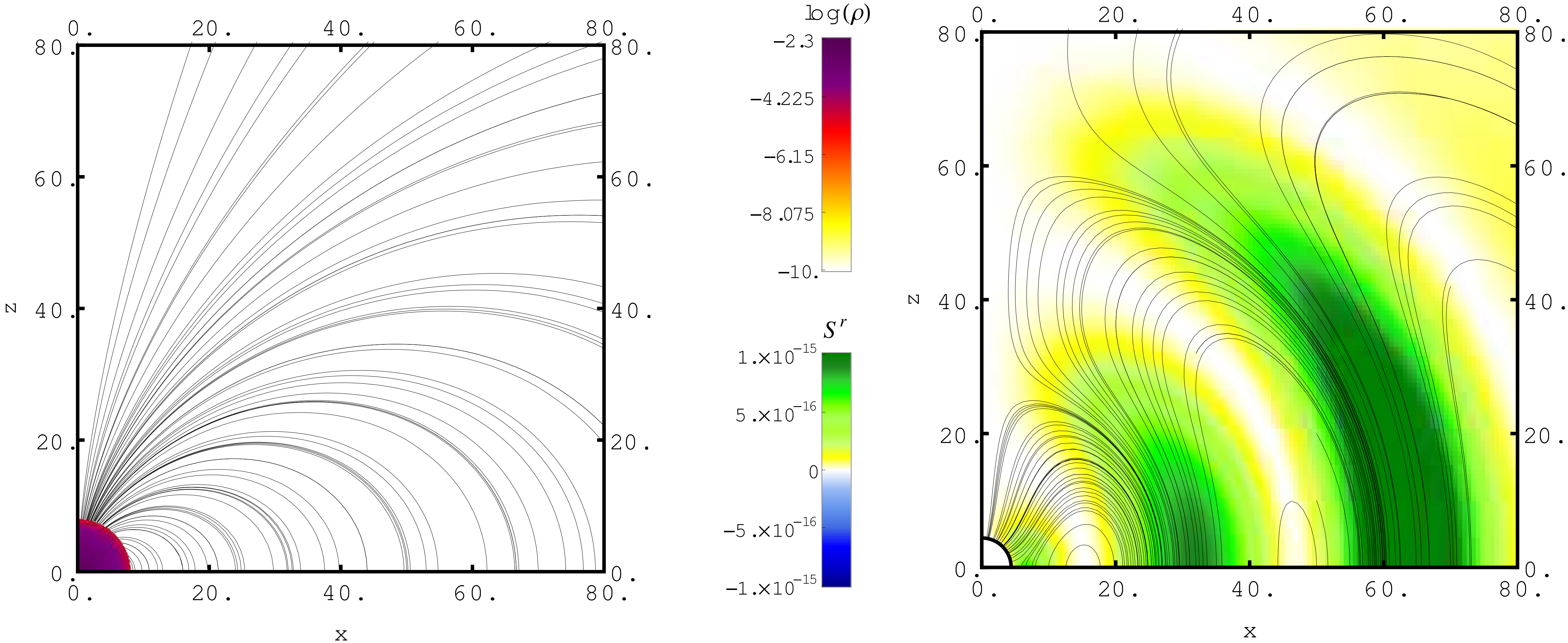


The Power Spectral Density of the magnetic flux (computed at r = 90 $M_\odot$) and of the rest mass density.

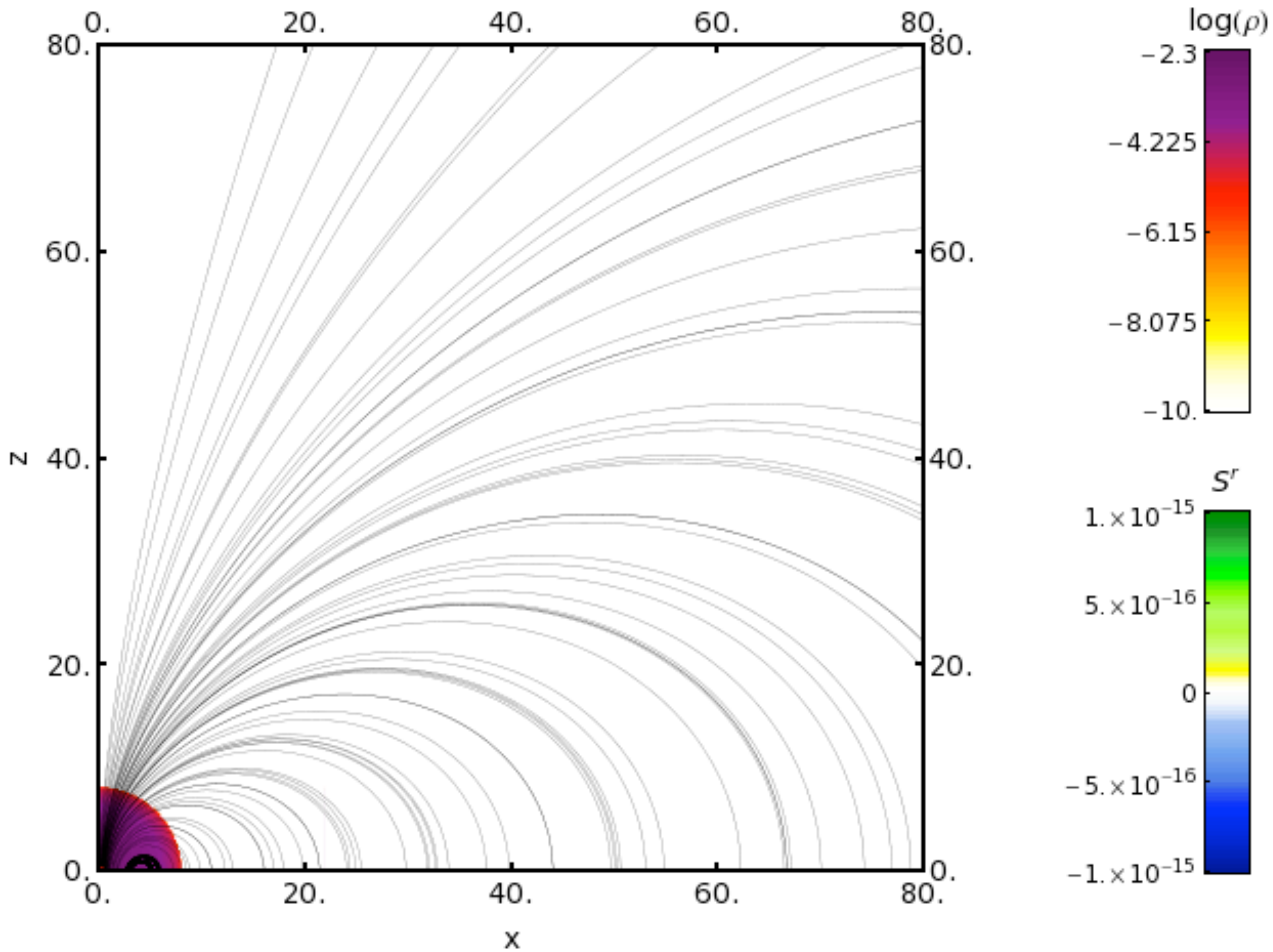
Gravitational Collapse of a Magnetized star



The fluid rest mass density, the radial Poynting vector and the magnetic field lines at times t = 0 ms and t = 1.1 ms. The perturbed electromagnetic field radiates a part of its energy through electromagnetic waves.

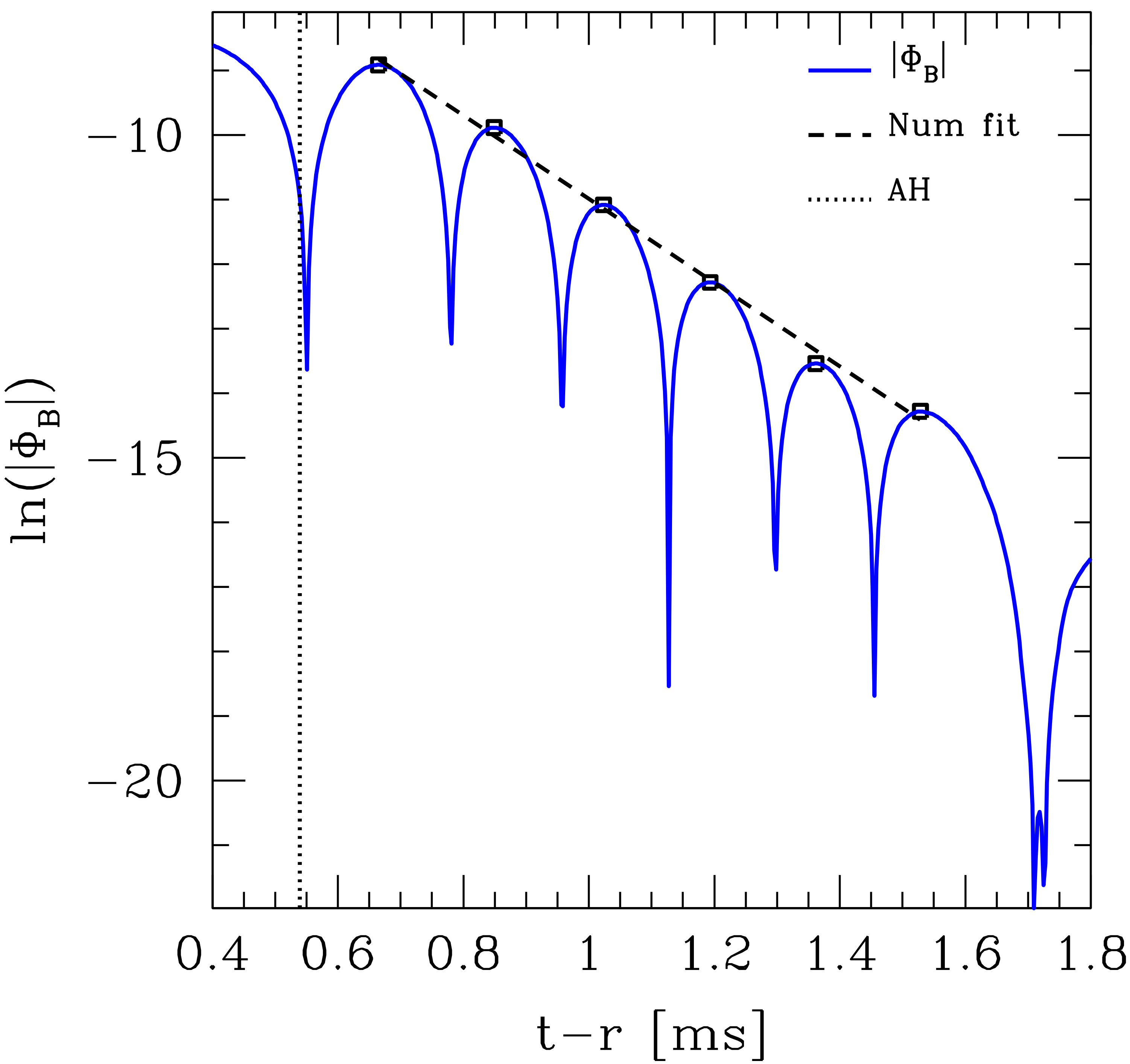
Gravitational Collapse of a Magnetized star

Gravitational Collapse of a Magnetized star



Time = 0.0000 ms

Gravitational Collapse of a Magnetized star

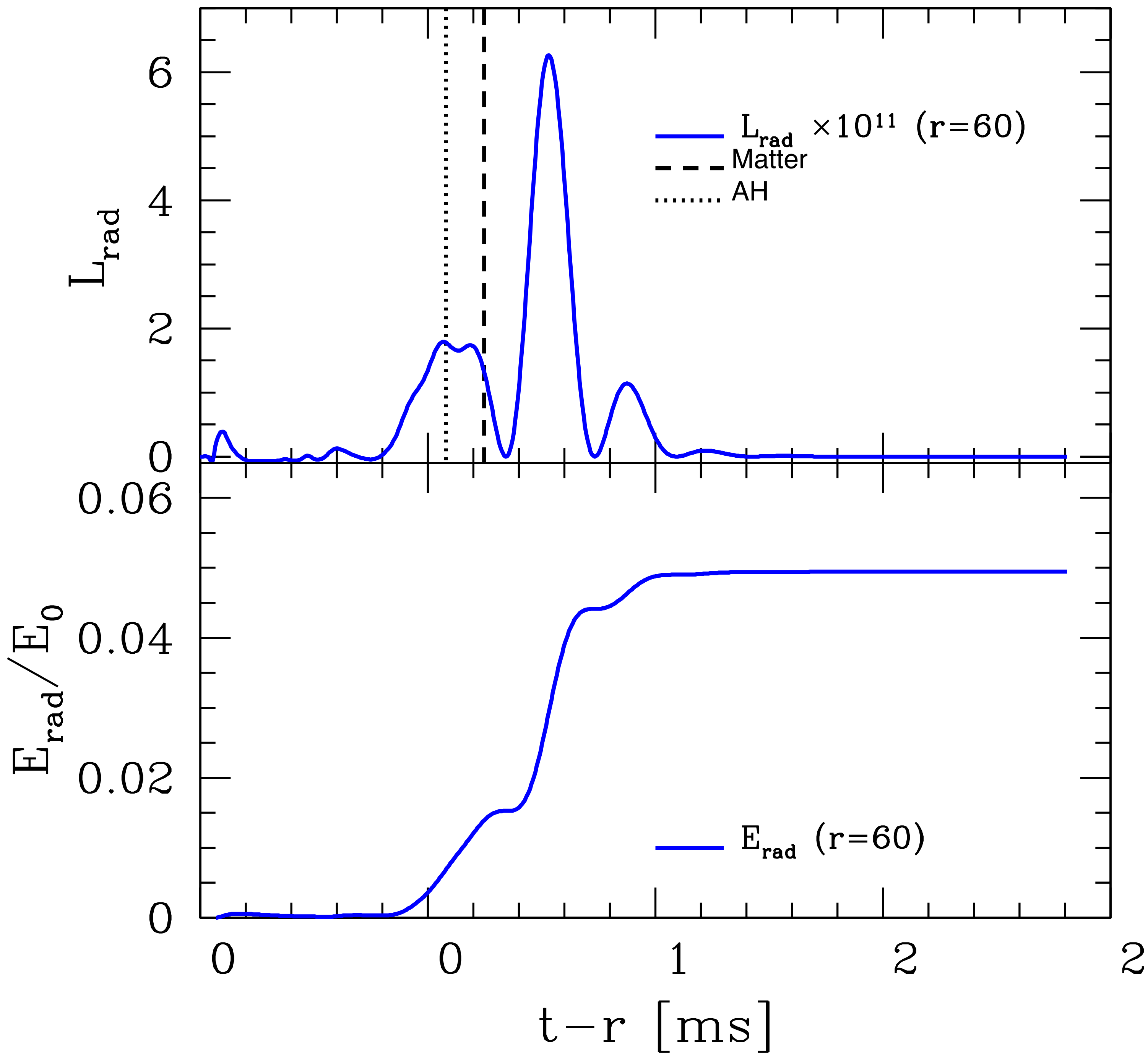


Imprint of the quasi-normal modes of the final perturbed black hole on the magnetic flux. The frequency of the ringing magnetic field is:

ω = 0.343384(±0.5%) - i 6.48543(±5%) kHz

corresponding to a non-rotating black hole of 2.7M$_\odot$.

Gravitational Collapse of a Magnetized star



- The luminosity and radiated energy as a function of time, calculated on a surface at r = 60 $M_\odot$.
- The value of the radiated energy is normalized to the initial energy of the magnetosphere, in order to estimate the efficiency of the emission.

Conclusions

- We present an alternative approach to the numerical treatment of the GR-Resistive MHD formalism based on IMEX methods.

- We provide a Cactus+Whisky full 3D GR-RMHD implementation, robust in all regimes of conductivity and able to accurately follow the evolution of regions where shocks occur.

- The accuracy of our code has been verified against exact solutions in 1D tests.

- We can successfully perform evolutions of magnetized stars, using non-constant conductivity profiles in order to capture both the ideal MHD regime (inside the star) and the electro-vacuum regime (in the magnetosphere).

Thank you!