Charged black holes in scalar-tensor theories of gravity

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- One of the most natural generalizations of the Einstein theory of gravity.

- Scalar fields appear in the reduction of Kaluza-Klein theories to four dimensions, in string theory and in higher dimensional gravity.

- Fit the observational data very well

- They are an essential part of some alternative dark energy and dark matter models.

Jordan Frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left(F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right) + S_m[\Psi_m; \tilde{g}_{\mu\nu}]$$

+

- This is the true physical frame
- The scalar field doesn't interact directly with matter

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- Very complicated form of the equations

Einstein Frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}]$$

+

- Much simpler equations

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- Direct coupling between the sources of gravity and the scalar field
- This is not the physical frame

Do the black holes in scalar-tensor theories (STT) differ from these in GR?

- **“No-scalar-hair” conjecture**

The black-hole solutions is STT (static or rotating) with Maxwell electromagnetic field can be completely characterized by only three parameters (asymptotic charges): mass, electric charge, and angular momentum => the scalar field is trivial.

- If we consider nonlinear matter sources such as nonlinear electrodynamics then the trace of the energy momentum tensor is not zero, unlike in the Maxwell electrodynamics, and acts as a source for the scalar field.

$\Rightarrow$ **Black hole solutions with nontrivial scalar field may exist!**

Nonlinear electrodynamics

- **Basic concept –** to use a more complicated form of the Lagrangian which leads to nonlinear field equations.

- **Example**: The Euler-Heisenberg nonlinear electrodynamics is proven experimentally to be a more accurate classical approximation of QED than Maxwell's theory when fields have high intensity.

Born-Infeld nonlinear electrodynamics

- First introduced by Born and Infeld in 1934 to obtain finite energy density model for the electron.

- Very popular nowadays because it plays an important role in string theory.

- It is invariant under electric-magnetic duality rotation

The metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)e^{-2\delta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$f(r) = 1 - \frac{2m(r)}{r}$$

For Reissner-Nordström:
(GR + Maxwell ED)

$$\delta_{\mathrm{RN}}(r) = 0,$$

$$f_{\mathrm{RN}}(r) = 1 - \frac{2M_{\mathrm{E}}}{r} + \frac{Q^2}{r^2},$$

For STT black holes + Born-Infeld ED:

- A system of coupled nonlinear ODE

- for the metric functions

$$\frac{d\delta}{dr} = -r\left(\frac{d\varphi}{dr}\right)^2,$$

$$\frac{dm}{dr} = r^2\left[\frac{1}{2}f\left(\frac{d\varphi}{dr}\right)^2 - \mathcal{A}(\varphi)^4 L(X)\right]$$

- for the scalar field

$$\frac{d}{dr}\left(r^2 f\frac{d\varphi}{dr}\right) = r^2\left\{-4\alpha(\varphi)\mathcal{A}^4(\varphi)\left[L - X\partial_X L(X)\right] - rf\left(\frac{d\varphi}{dr}\right)^3\right\}$$

Input for the STT:

• Choice of the coupling function (when β=0 the GR is restored)

$$\mathcal{A}(\varphi) = e^{\frac{1}{2}\beta\varphi^2}$$

• The potential of the scalar field $V(\varphi)$ is zero

Results:

• More that one black hole solution for fixed values of the asymptotic charges (mass and charge) exist. (Stefanov, Yazadjiev, Todorov (2007))

$\Rightarrow$ **Contradiction to the "no-scalar-hair" theorems**

• We can introduce a new parameter to label the different solutions – the scalar charge

$$\mathcal{D} = -\lim_{r\to\infty} r^2 \frac{d\varphi}{dr}$$

- A typical $D(M)$ diagram for sequences of black hole solutions showing the non-uniqueness:

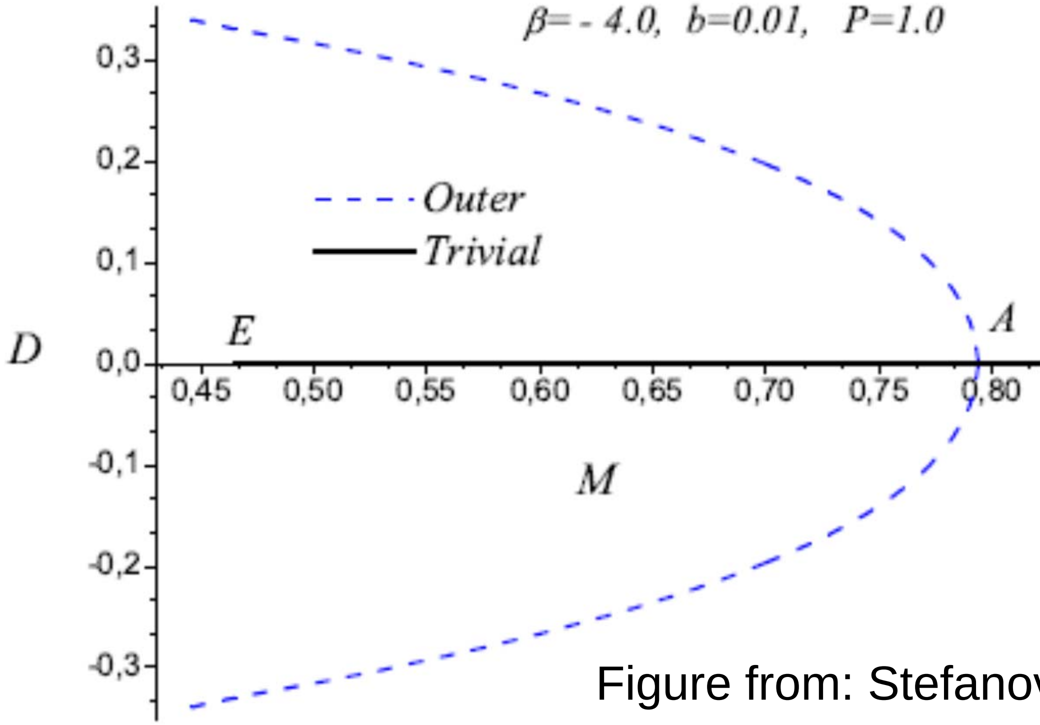


Figure from: Stefanov, Yazadjiev, Todorov (2007)

- **The solutions with zero scalar charge (i.e. with trivial scalar field) are the corresponding solution in GR and will be called “trivial solutions”**

We consider the perturbations of the trivial solution within the framework of the described class of scalar-tensor theories.

- The perturbations of the scalar field decouples
- The perturbations of the metric and the electromagnetic field are the same as in the GR and thus are not of an interest.
- The equation governing the perturbations of the scalar field is

$$f\frac{d}{dr}\left(f\frac{d\psi}{dr}\right) + [\omega^2 - U(r)]\psi = 0$$

The deviation from GR

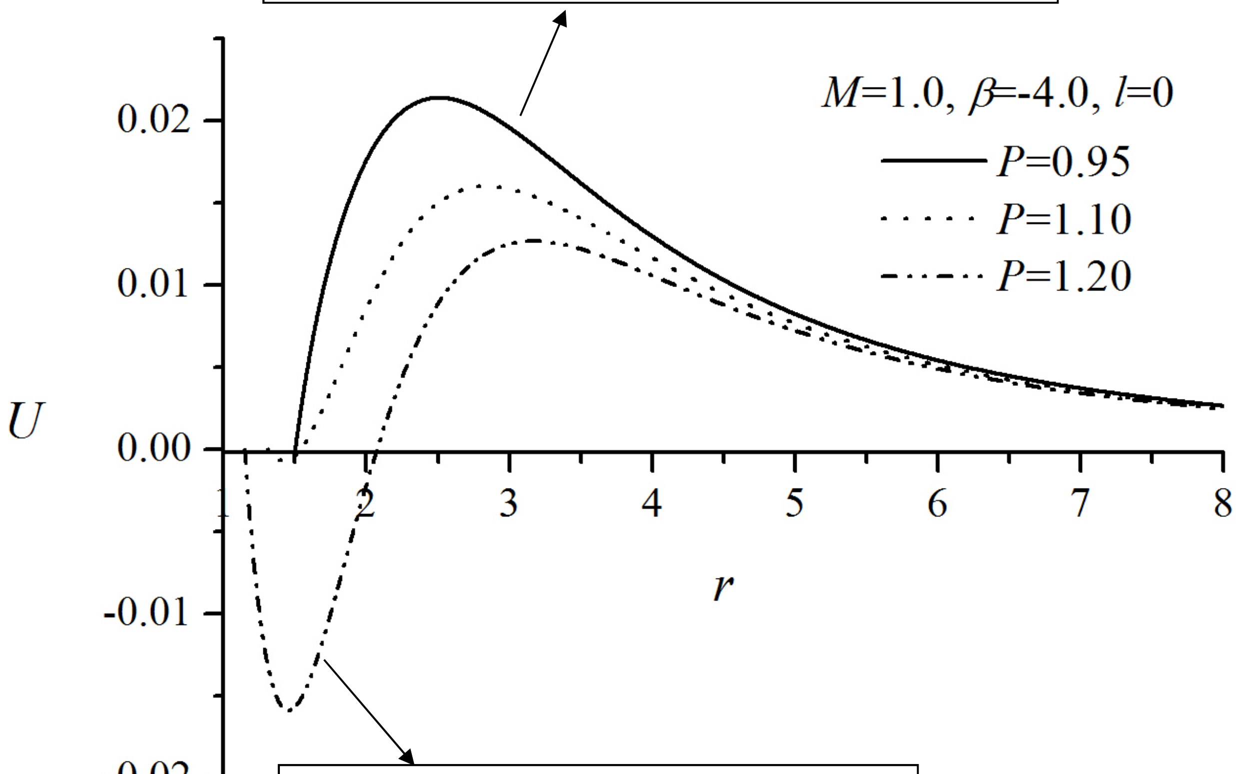
$$U(r) = f\left[\frac{1}{r}\frac{df}{dr} + \frac{l(l+1)}{r^2} + \boxed{4\beta T_{EM}}\right]$$

$$dr_* = \frac{dr}{f(r)} \qquad r_* \in (-\infty, \infty)$$

- ω is a complex number. But for the unstable modes ω is purely imaginary.

The potential $U(r)$:

Qualitative behavior for small charges P and/or large l (similar to GR)

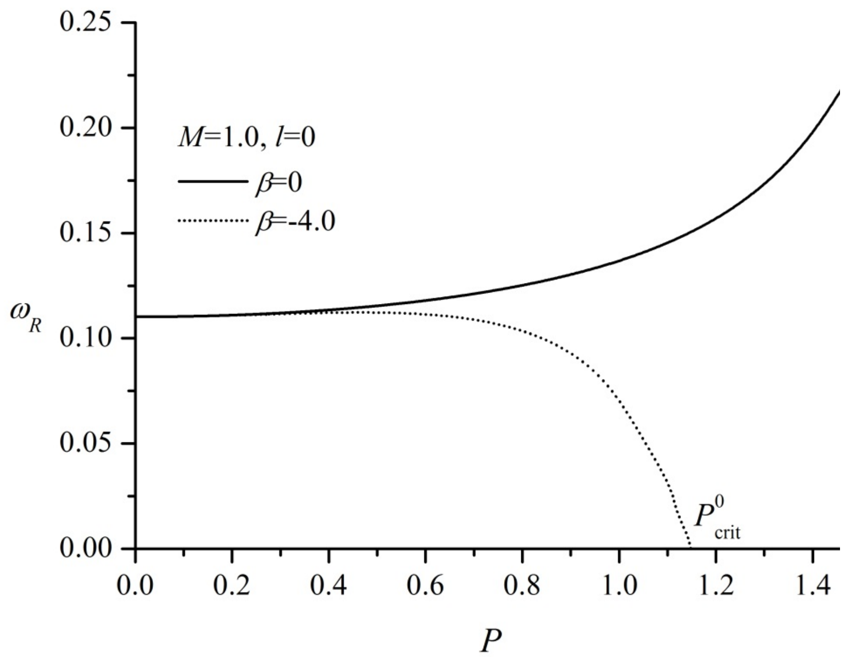


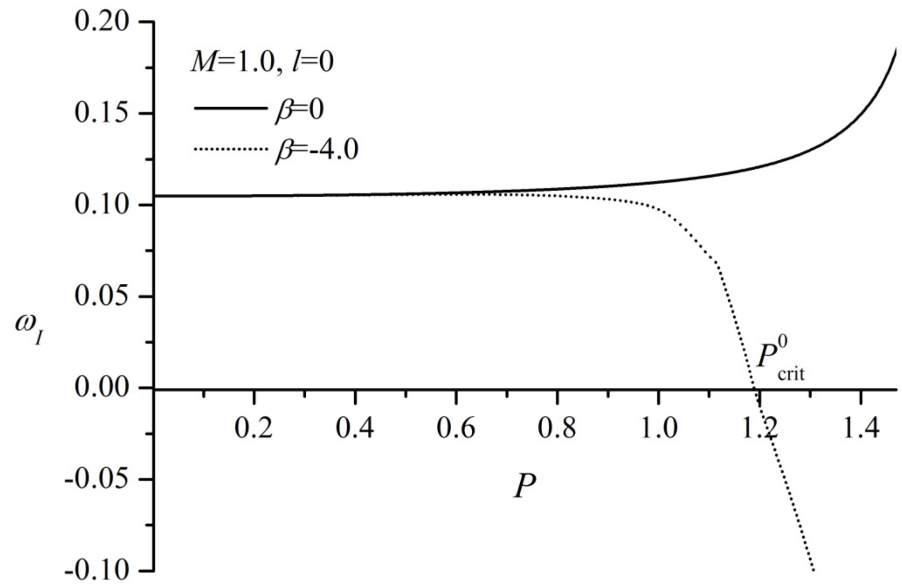
Qualitative behavior for large charges P and l=0

A negative minimum exists which is a signal for unstable modes.

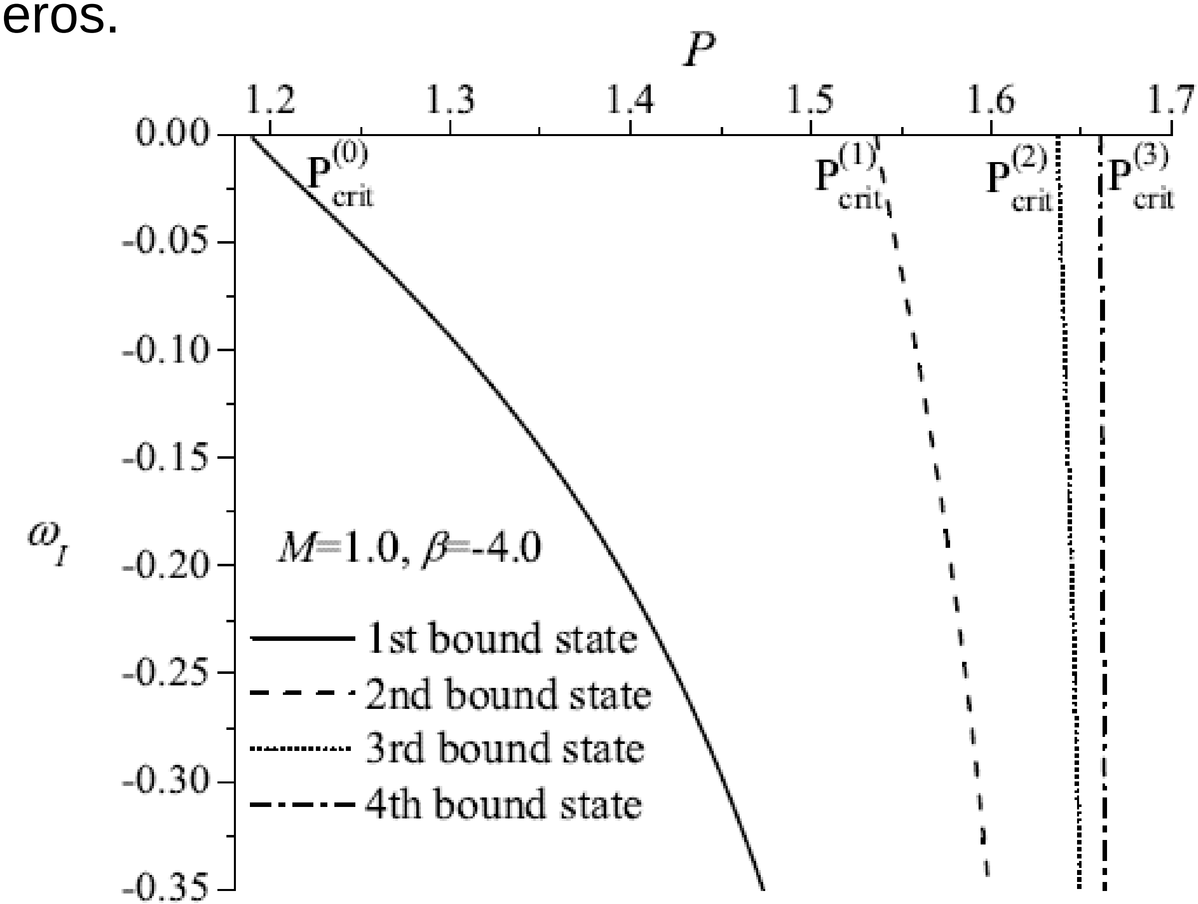
Quasi-normal Modes

- The fundamental (n=0) QNM frequencies as a function of the charge *P* for *l*=0
- Only the unstable modes corresponding to the first bound state are shown





- The unstable modes corresponding to the first few bound states.
- The k-th unstable mode originates at $P(k)_{crit}$, where $P(0)_{crit} < P(1)_{crit} < P(2)_{crit} < \ldots$, and the radial part of the corresponding perturbation function has k zeros.



New Hairy Black Holes

- **The static zero-modes (with ω=0 and l=0) are perturbative solutions of the field equations.**

- The presence of unstable modes (for $l = 0$) gives us a reason to expect that new black-hole solutions with nontrivial scalar field exist.

- The number of the nontrivial solutions that bifurcate from the trivial solution should be equal to the number of the unstable modes.

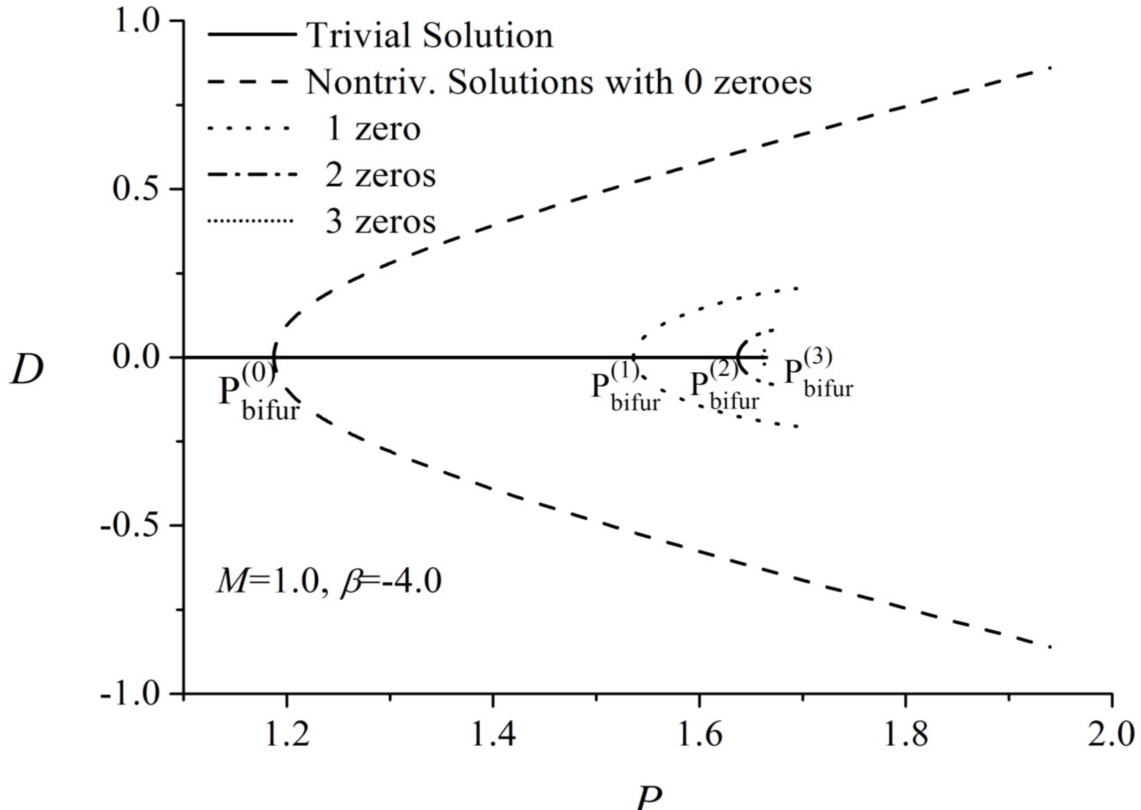
- The scalar field of the non-trivial black-hole solutions should have the same number of zeros as a function of r as the corresponding unstable zero-mode.

- Some examples of such black-hole solutions have already been found in Stefanov et al (2007). In the phase diagrams there, however, only one bifurcation point is present and our numerical results show that it is just the point where the $n = 0$ mode reaches zero.

- In order to check our hypothesis we should solve the full system of field equations presented above.

- **Such new black hole solutions where found indeed.**

The values of the charge $P(k)_{crit}$ where the unstable modes corresponding to the k-th bound state becomes zero and the values of the charge $P(k)_{bifur}$ corresponding to the k-th bifurcation point on the phase diagram $D(P)$ coincide!



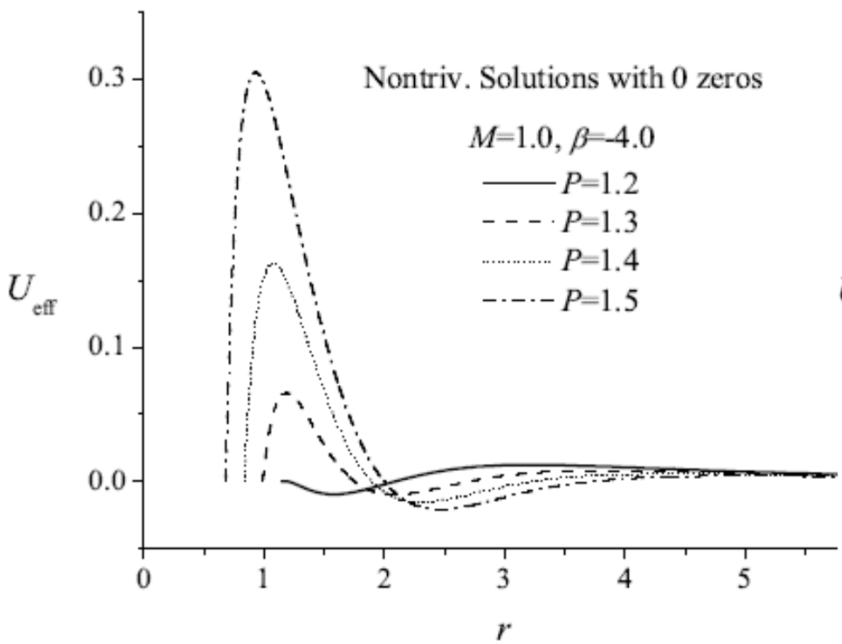
The stability of the obtained new hairy black-hole solutions

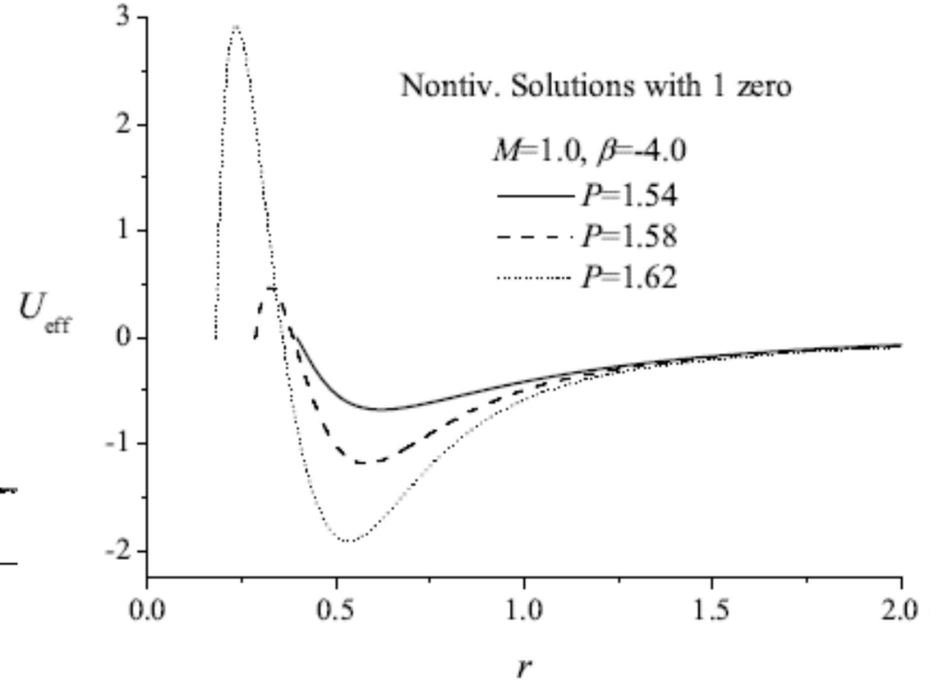
- We study the QNMs associated with the pure radial perturbations of these solutions.
- Only the instability of a black-hole solution can be proven rigorously within this approach.
- The perturbations of the metric and the electromagnetic field are functions of the perturbation of the scalar field.

- The equation governing the radial perturbations of the scalar field

$$\frac{d^2\psi(r_*)}{dr_*^2} + \omega^2\psi(r_*) = U_{\rm eff}(r_*)\psi(r_*)$$

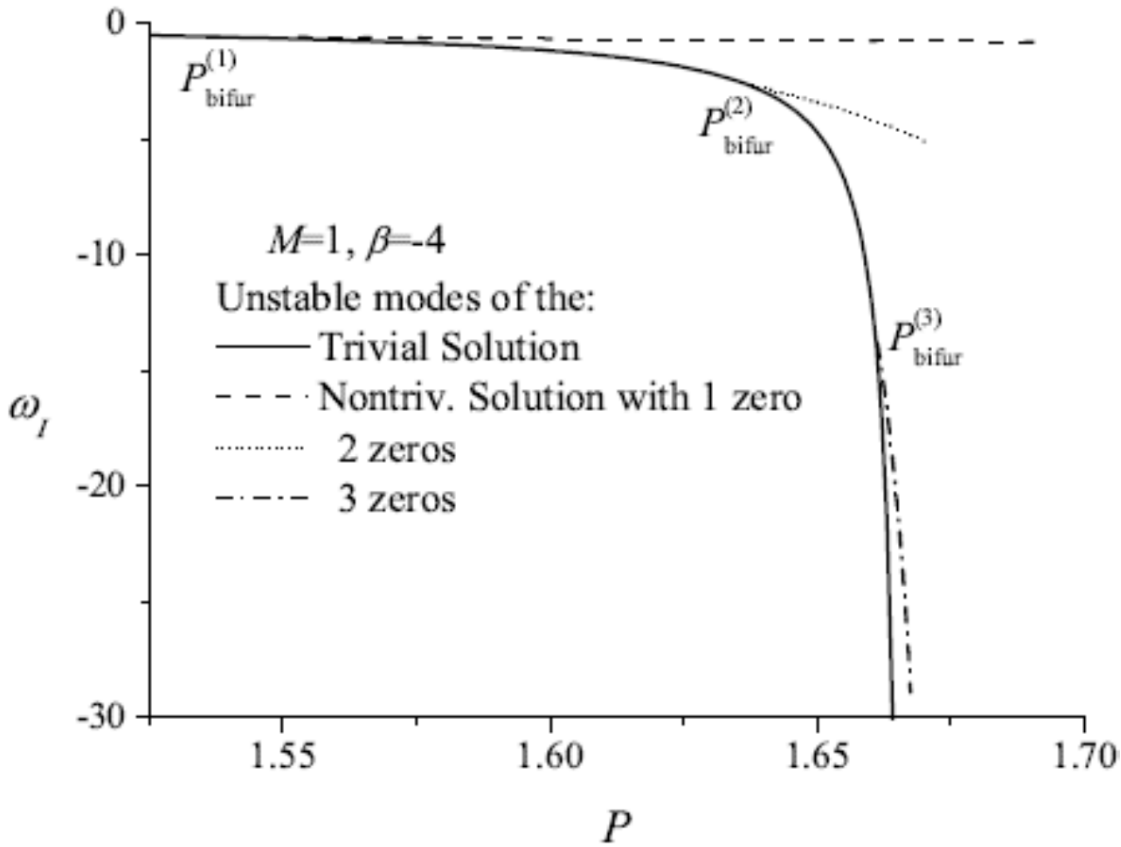
The potential has negative minimum for all of the solutions with nontrivial scalar field.





The results show that:

1. All of the new nontrivial black hole branches characterized by scalar field which has one or more zeros, are UNSTABLE
2. The branches characterized by monotonous scalar field are STABLE against radial perturbations.



Conclusions:

- The scalar QNMs of the Einstein-Born-Infeld black-hole solution (the trivial solution) embedded in a certain class of scalar-tensor theories are studied
- The investigation of the spectrum of the QNMs shows that for l=0 there are unstable modes which signal the presence of new nontrivial scalar-tensor black holes.
- Such black-hole solutions were constructed numerically by solving the full system of static and spherically symmetric field equations and the number of nontrivial solutions that bifurcate from the trivial one is equal to the number of the bound states of the potential governing the scalar perturbations of the trivial solution
- We have shown that the scalar-tensor black holes possessing scalar field with one or more zeros are unstable. It seems that only the scalar-tensor black holes having scalar field without any zeros are stable solutions.

T H A N K Y O U !