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# *Lagrangian Bias in the Local Bias Model*

Noemi Frusciante

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*In Collaboration with Ravi K. Sheth (in preparation)*

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# Outline


1. Introduction
   - Motivations
   - Biasing
2. The Lagrangian Bias in The Local Bias Model
   - The Local Bias Model
   - Normalizing density rather than overdensity
   - Renormalized Bias
3. Conclusions

# Motivations

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# Motivations

*An important part of Cosmology is the Large Scale Structure, today numerical simulations/observations allow us to test*

- *The formation/evolution of structures*
- *Initial spectrum*
- *Biasing*
- *Dark Energy models (among which Modified Gravity Models)*
- *Whatever other parameters come into describing the Universe*

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*In modelling the structures' clustering a crucial step is to understand how the spatial distribution of galaxies is related to the underlying (dark) matter distribution*

# The Bias Factor I

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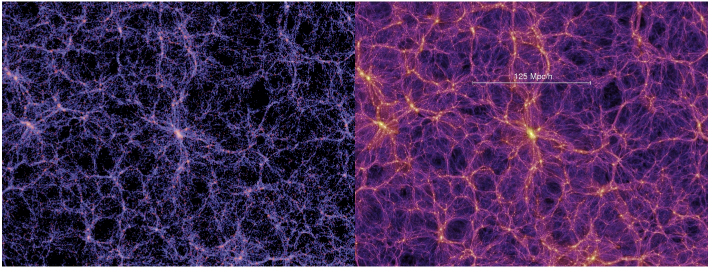
- *Light does not trace the mass* $\rightarrow$ **BIAS**
- *The matter distribution is highly clustered*




*Springel et al. (2005)*

# The Bias Factor II



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*REAL BIAS is non-linear, stochastic and quite deterministic see Dekel & Lahav (1999), Sheth & Lemson 1999, Tegmark & Bromley (1999)*

# Scale dipendence of the Bias factor

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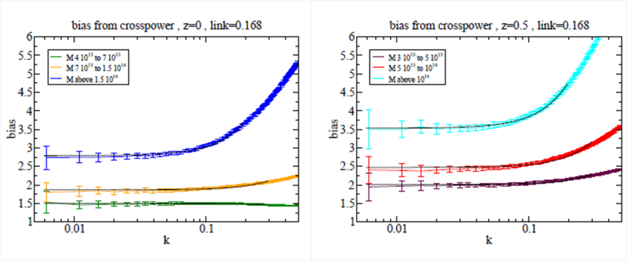


*Manera, Sheth & Scoccimarro (2010)*

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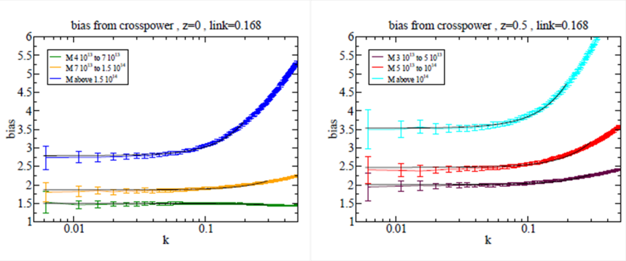


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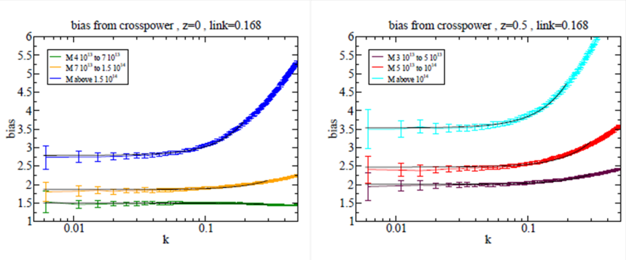


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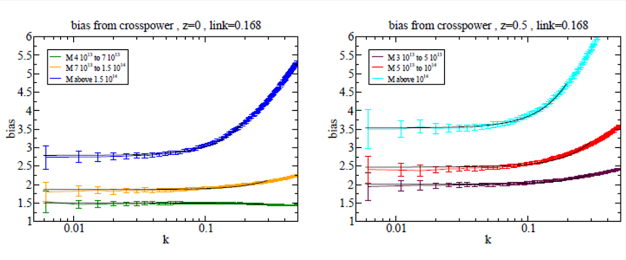


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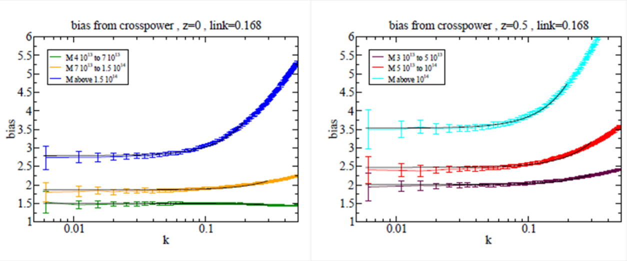


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2. *MG: The bias depends on scale also at small k:*
   - $D(k,t)$
   - *Curved barrier:* $\delta_{sc}(m(R_s))$

# The Local Bias Model

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## The Model

*On large enough smoothing scales,where the fluctuations are small, this relation can be expanded in a Taylor series*

$$\delta_b = f(\delta_m) = \sum_{i>0} \frac{b_i}{i!} (\delta_m^i - \langle \delta_m^i \rangle),$$

*Note that this model ensures* $\langle \delta_b \rangle = 0$ *by subtracting-off the* $\langle \delta_m^i \rangle$ *terms.*

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- *It is used to describe the bias with respect to the Eulerian field BUT it is often assumed to describe the bias with respect to the initial Lagrangian field:* $\delta_L$.

# Problem


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- *PROBLEM: The cross correlation between the biased tracers and the initial field involves higher order terms*

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- *We expect (e.g. Peaks and Patches which forms halos)*

$$\langle \delta_b \delta_L \rangle = b \langle \delta_L^2 \rangle$$

# Normalizing density rather than overdensity

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*The correctly normalized bias field is defined by*

**SOLUTION**

$$\delta_B \equiv \frac{1+\delta_b - \langle 1+\delta_b\rangle}{\langle 1+\delta_b\rangle} = \frac{\sum_{k=1}^{\infty}(b_k/k!)(\delta_L^k - \langle\delta_L^k\rangle)}{\sum_{k=0}^{\infty}(b_k/k!)\langle\delta_L^k\rangle}$$

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- *The cross correlation is*

$$\langle \delta_{L'} \delta_B | r \rangle = \xi_{LL'}(r)\, \frac{\sum_{k=1}^{\infty}(b_k/k!)\langle \delta_L^{k+1} \rangle / \langle \delta_L^2 \rangle}{\sum_{k=0}^{\infty}(b_k/k!)\langle \delta_L^k \rangle} \equiv \xi_{LL'}(r)\, B_L,$$

**Note:this is an exact statement, valid for any $r$, $L$ or $L'$, and for any local deterministic bias function.**

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- *The auto-correlation function of the biased tracers will reduce to a series of the form [Similar result by Szalay (1988)]*

$$\langle\delta_{B'}\delta_B|r\rangle = B_L^2\,\xi_{LL}(r) + \frac{C_L}{2}\,[\xi_{LL}(r)]^2 + \dots$$

# Renormalized Bias I



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*What do you do when this cannot be done analytically?*

*Mc Donald (2006) suggests redefining the mean density, and hence all bias factors order by order*

$$\delta_B^{(j)} = \frac{1+\delta_b - \langle 1+\delta_b\rangle_j}{\langle 1+\delta_b\rangle_j} = \frac{\sum_{k=1}^{j}(b_k/k!)(\delta_L^k - \langle\delta_L^k\rangle)}{\sum_{k=0}^{j}(b_k/k!)\langle\delta_L^k\rangle}.$$

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- *To 4th order in $\delta_L$, this is*

$$\langle\delta_{L'}\delta_B^{(4)}|r\rangle = \left[b_1 + \left(\frac{b_3}{2} - \frac{b_1 b_2}{2}\right)\langle\delta_L^2\rangle\right]\xi_{LL'}(r) \equiv b_\times^{(4)}\,\xi_{LL'}(r)$$

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*The Bias coefficient is defined*

$$b_\times^{(4)} \equiv b_1 + \left( \frac{b_3}{2} - \frac{b_1 b_2}{2} \right) \langle \delta_L^2 \rangle$$

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*Examples:*

- *Lognormal field :* $1 + \delta_b = \exp(b\delta_L) \exp(-b^2 \langle \delta_L^2 \rangle / 2)$

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- *Peaks* : $1 + \delta_p = \exp(b\delta_L - c\delta_L^2/2)\, \sqrt{1 + c\langle \delta_L^2 \rangle}\, \exp\left( -\frac{b^2 \langle \delta_L^2 \rangle /2}{1 + c\langle \delta_L^2 \rangle} \right)$

$$b_3 \neq b_1 b_2 \quad \rightarrow \quad b_\times^{(4)} = b(1 - c\langle \delta_L^2 \rangle) \quad \rightarrow \quad B_L = \frac{b}{1 + c\langle \delta_L^2 \rangle}$$

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*Assuming that the Lagrangian bias is local and deterministic with respect to the initial Gaussian field, we showed that with a CORRECT NORMALIZATION*

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- *These relations allow for simple tests of whether or not halo bias is indeed local in Lagrangian space.*

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- *The auto correlation function of locally biased tracers can be written as a Taylor series in the auto-correlation function of the mass.*
- *These relations allow for simple tests of whether or not halo bias is indeed local in Lagrangian space.*
- *Our results should also hold for MG models. Of course, for MG, it is not obvious that the assumption that the Lagrangian bias should be local is a good one (needs to be tested).*