Quantum particles around near black hole objects:

Resonant particle capture, spectrum collapse, and the smooth transition to black hole absorption

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References:

"Energy levels of a scalar particle in a static gravitational field close to the black hole limit"
G. H. Gossel, J. C. Berengut and V. V. Flambaum
Gen. Relativ. Gravit. (2011)

"Dense spectrum of resonances and particle capture in a near-black-hole metric"
V. V. Flambaum, G. H. Gossel and G. F. Gribakin
Phys. Rev. D (2012)

Overview

- Introduction to the system (finite sized spherical body, $R > r_s$)

Part 1: Elastic Scattering

- Emergence of long lived meta-stable scattering states (resonances)
- Effective absorption cross section for 'near black hole bodies'

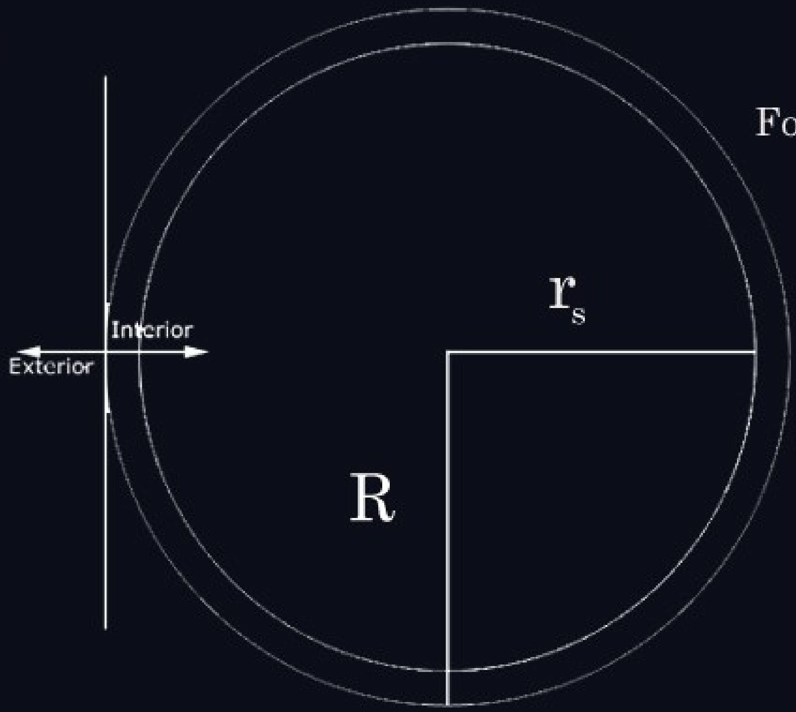
Part 2: Bound states

- Energy level spectrum and pair production

Physical system

$$r_s = \frac{2GM}{c^2}$$

We consider a quantum particle in the gravitational field of a finite sized body with radius $R > r_s$.



For a given interior metric, we can construct and solve (numerically and using analytic approximations) the wave equations for spin 0, 1/2 and 1 particles.

Example: Klein-Gordon equation

metric in

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Psi \right) = 0$$

wavefunction out

Once the wavefunction is calculated, scattering and bound states (energy spectrum etc.) can be computed.

Physical system

We consider the limiting approach $r_s/R \to 1$ (for black holes $r_s/R = 1$)

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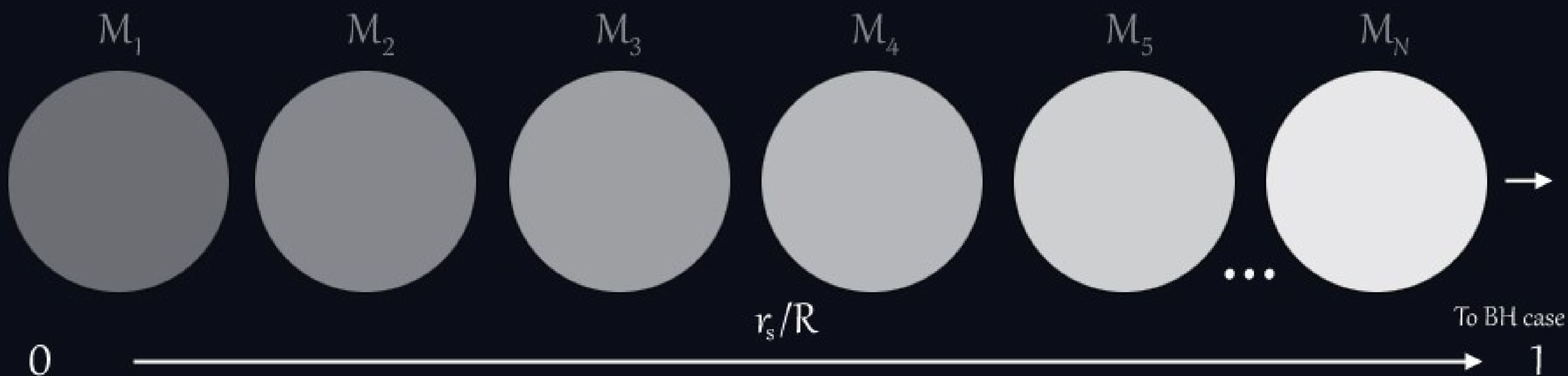

Part I:

Elastic scattering and resonant capture

Resonances

A smooth transition to black hole absorption

We find a dense spectrum of increasingly narrow resonances. The resulting energy average cross section equals that of a black hole in the low energy limit.

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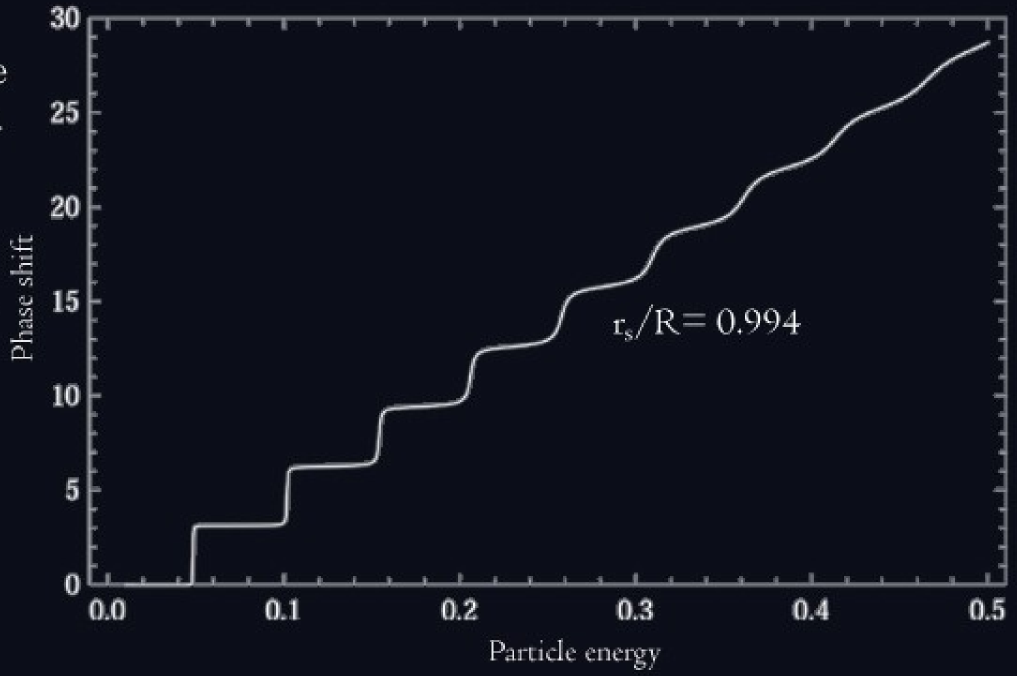
- The phase shift parameterizes the strength of the scattering process.
- Steps in pi of phase represent resonances; narrower resonances correspond to longer lived states.



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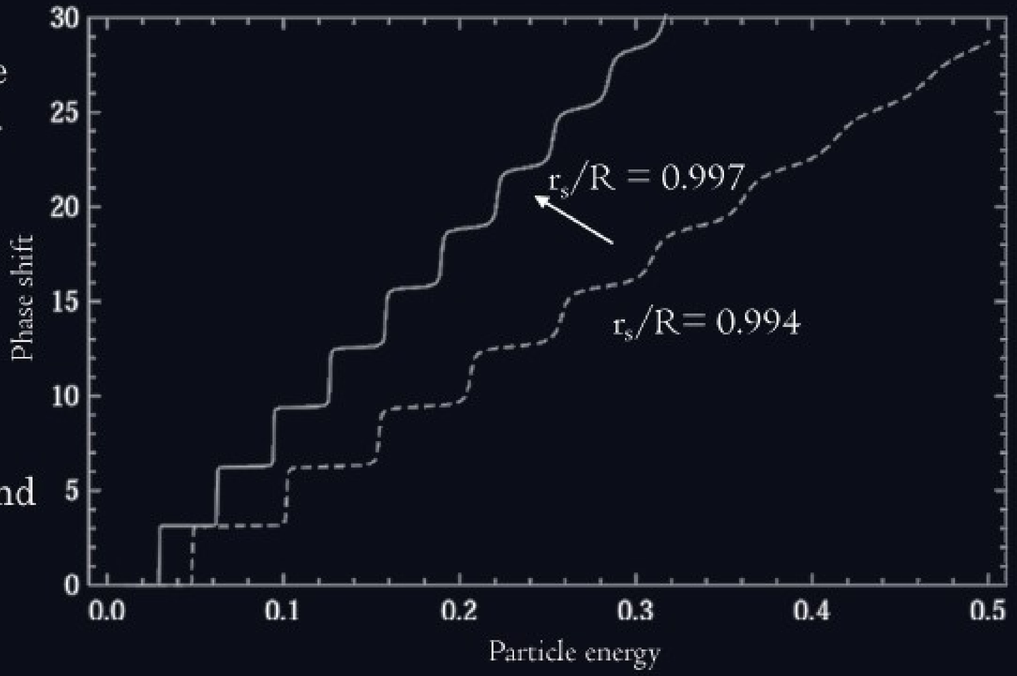
- The phase shift parameterizes the strength of the scattering process.
- Steps in pi of phase represent resonances; narrower resonances correspond to longer lived states.
- The resonance widths, energies and spacings tend to zero as $r_s/R \rightarrow 1$



Resonances

A smooth transition to black hole absorption

We analytically calculate the resonance widths and spacings by searching for complex poles of the scattering matrix; e.g. S-matrix for s-wave spin-0:

$$S_0 = \frac{1+\mathcal{R}-\varepsilon^2 C^2(1-\mathcal{R})}{1+\mathcal{R}+\varepsilon^2 C^2(1-\mathcal{R})} e^{2i\delta_C}$$

Low energy s-wave scattering matrix from Kuchiev, Flambaum PRD70,044022 (2004)

Here $\mathcal{R} = -\exp[2i\epsilon r_s B(\xi)]$ is the reflection coefficient at the boundary.

The complex poles correspond to resonances with positions and widths given by

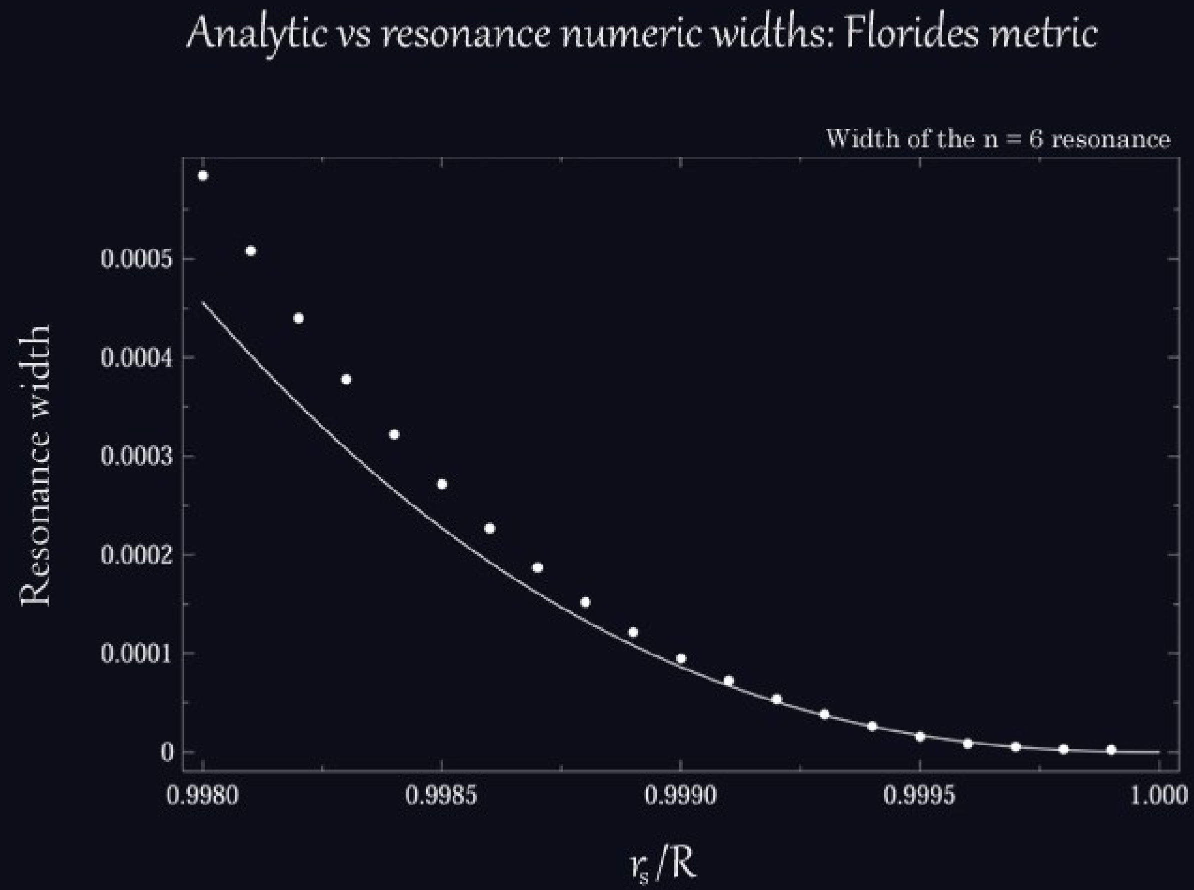
$$\epsilon_n = \frac{n\pi}{B(\xi)} \qquad \Gamma_n = \frac{2\epsilon_n^2 C^2}{B(\xi)}$$

where $B(\xi)$ is a known function of $\xi = 1 - r_s/R$.

These analytic calculations are in good agreement with the numerics in all cases.

Resonances

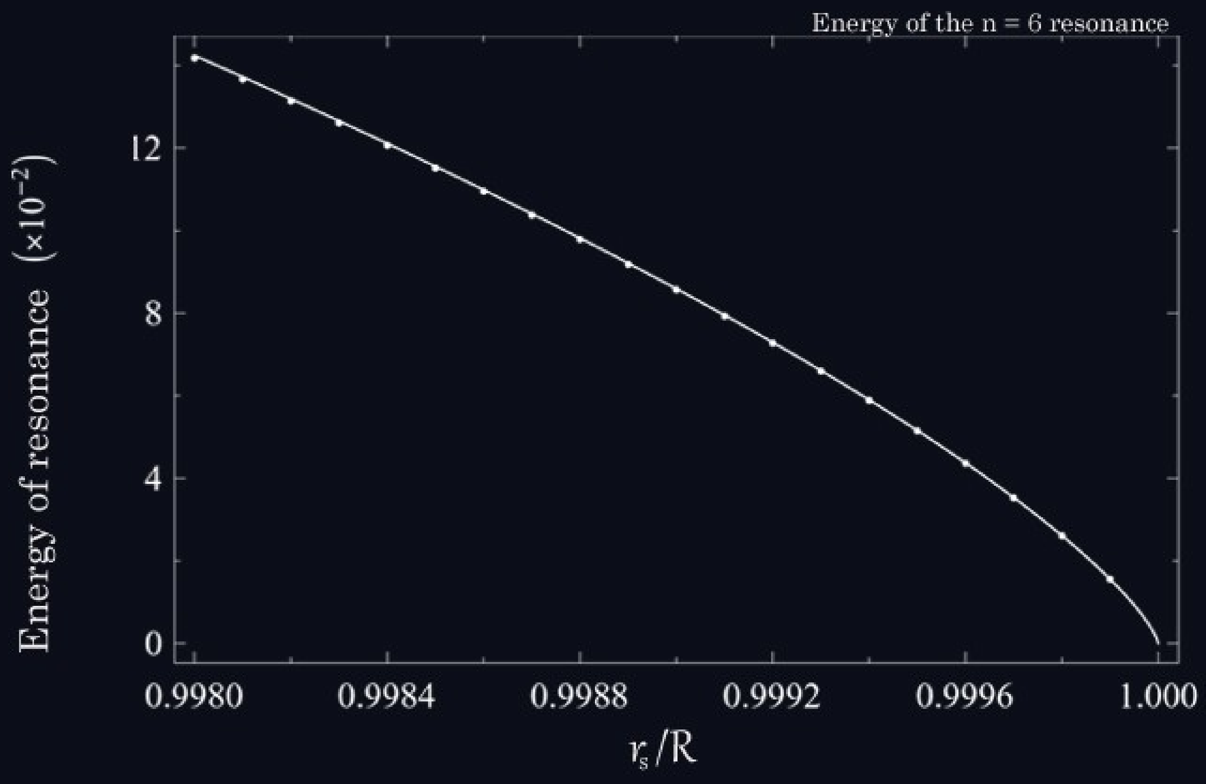
A smooth transition to black hole absorption



Resonances

A smooth transition to black hole absorption

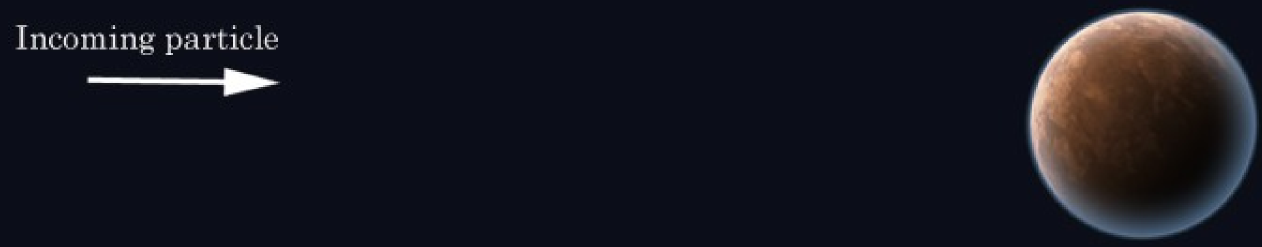
Analytic vs numeric resonance energies: Florides metric



Resonances

A smooth transition to black hole absorption

Long lived resonant capture gives rise to effective absorption during pure elastic scattering.



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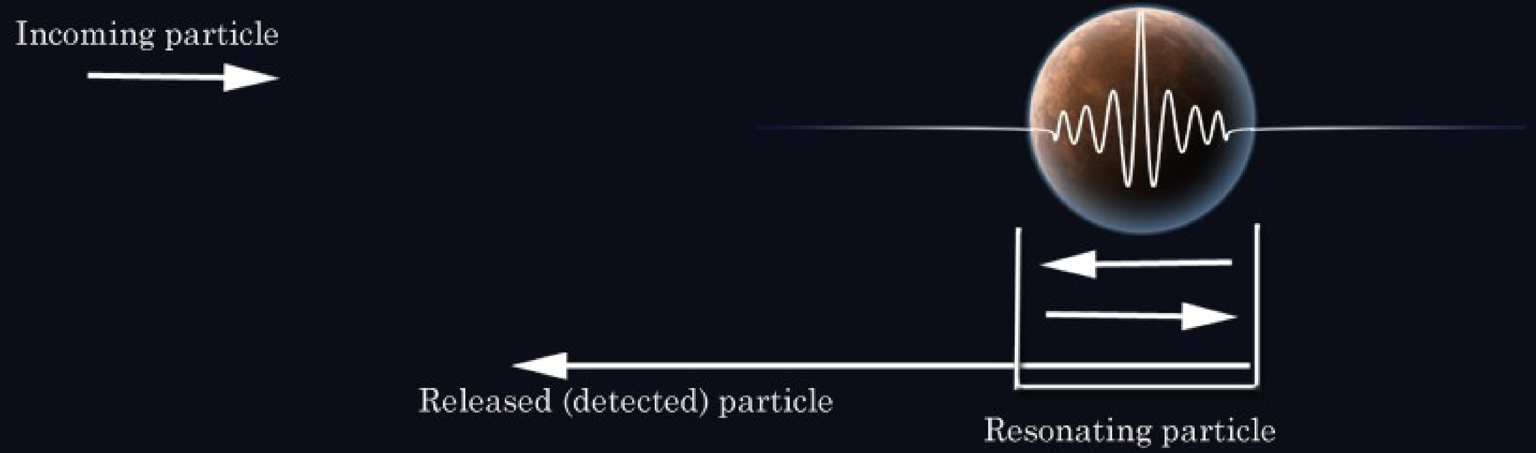

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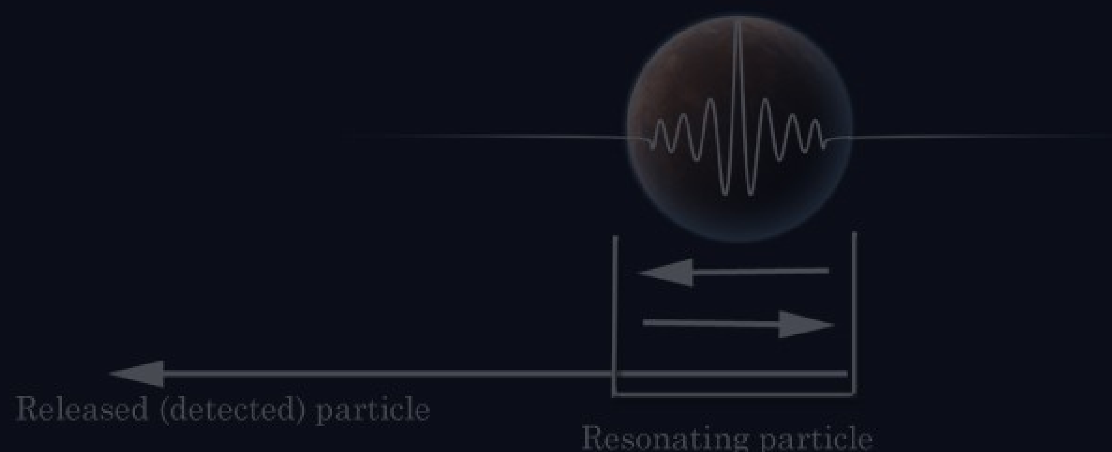

Energy averaged absorption cross sections (low energy):

$$\sigma_0^a = 4\pi r_s^2$$

$$\sigma_{1/2}^a = \frac{1}{4}\pi r_s^2$$

$$\sigma_1^a = \frac{4}{3}\epsilon^2 \pi r_s^4$$

Bodies that are not formally black holes can have black hole-like absorption.

Part II:

Bound states and particle pair production

Bound states

We find a dense spectrum of bound states quantum particles of non-zero mass. This spectrum collapses in the black hole limit. Only positive energy states are found in prior to the BH case..

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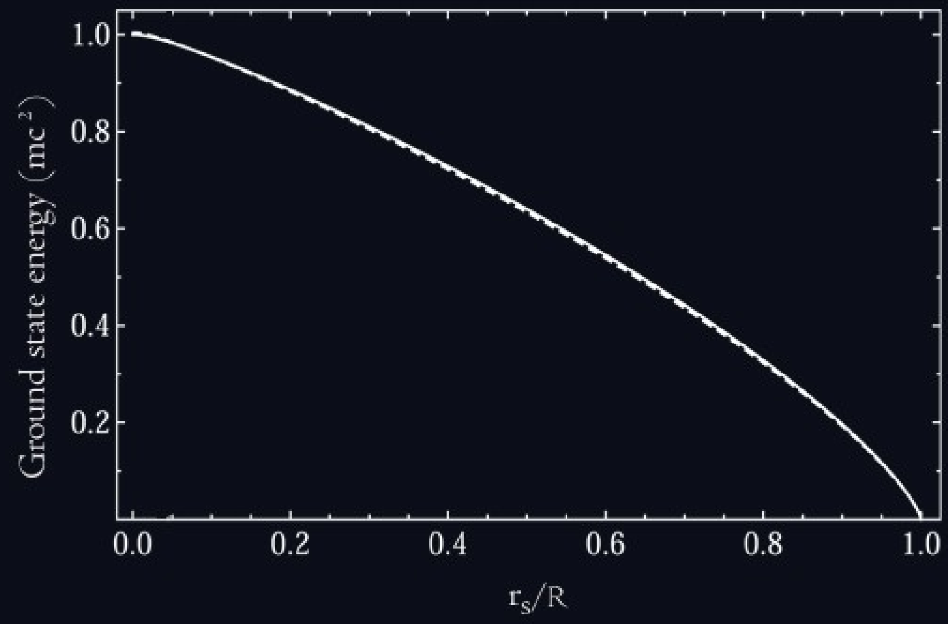

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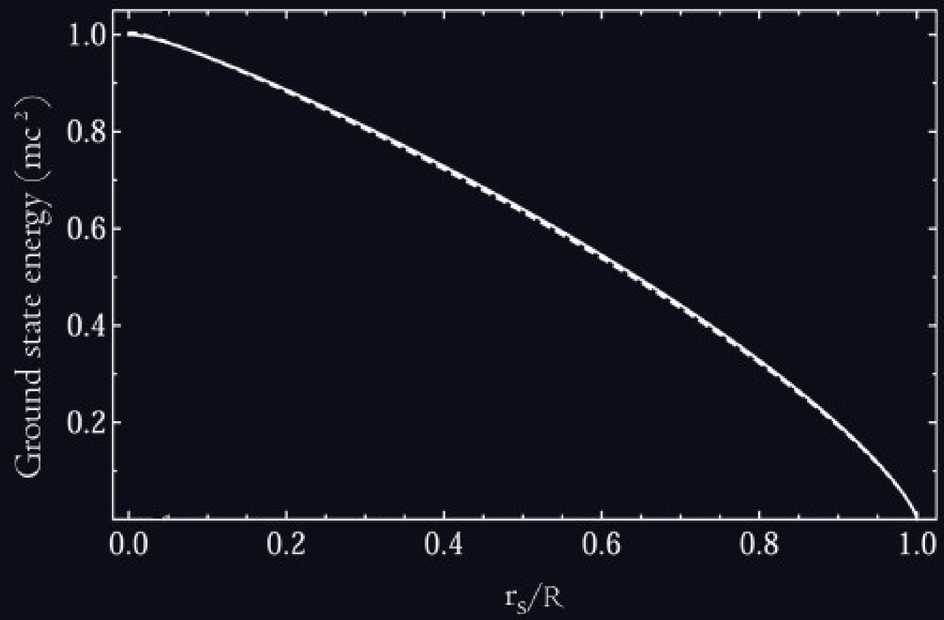

Bound states

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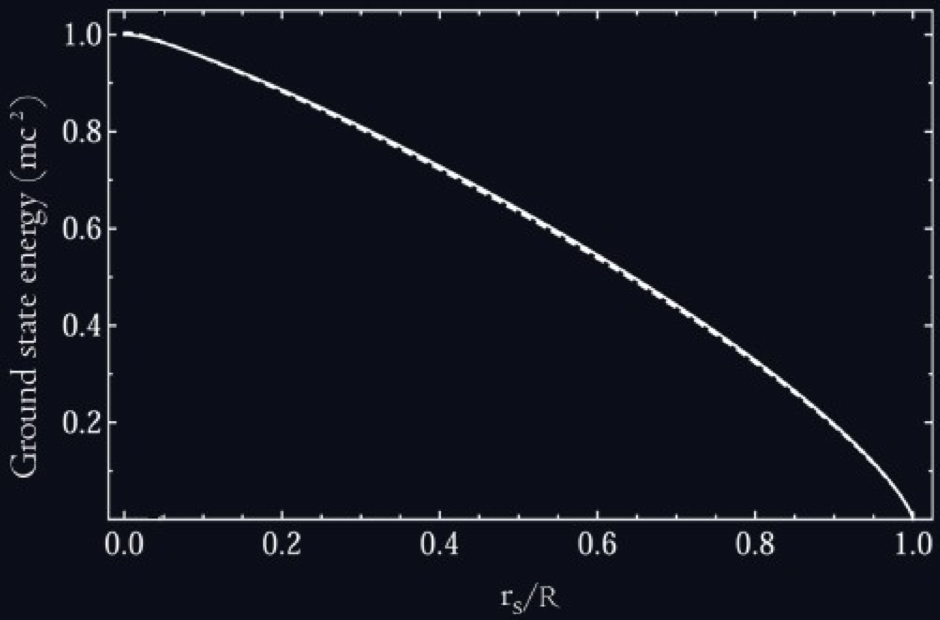
- As the metrics singular point is approached, all the bound levels collapse to zero energy.
- A zero energy bound state neccessary, but not sufficient, for pair production and thus Hawking radiation to occur.
- Thus such phenomena are only possible in the singular metric limit



Example: ground state energies for Florides interior shown with:

$$\epsilon = (1 - r_s/R)^{3/4}$$

Bound states

The non-existence of negative energy states is evident directly from the wave equation. The term in the radial wave equation containing energy is

GR Klein-Gordon energy term

$$\psi(r)\frac{r^2\epsilon^2\sqrt{B(r)}}{\sqrt{A(r)}}$$

Components of metric

Potential form Klein-Gordon energy term

$$\psi(r)(\epsilon - V(r))^2$$

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Components of metric

- No linear energy terms
- Cannot move from positive to negative energies
- Can only tend to zero

Potential form Klein-Gordon energy term

$$\psi(r)(\epsilon - V(r))^2$$

- Contains linear energy terms
- Can pass from positive energy through zero to negative energy levels
- For example, the ground state of an electron orbiting a finite-size nucleus reaches the lower continuum when Z ~ 170 (for scalar particle Z~>90)

Conclusions

In considering the case of quantum particles in the gravitational field of finite-size ($R > r_s$) bodies we find the following:

- Part I: Elastic scattering

  A dense spectrum of narrow (long lived) resonances emerges. By averaging over several resonances we can compute the cross section for absorption into these states.

  In black hole limit, this cross section (for low ϵ) matches the black hole absorption cross section for spin 0, 1/2 and 1 particles.

- Part II: Bound states

  The bound state spectrum collapses and becomes quasi continuous in the limit the metric becomes singular.

  Only in this limit are $\epsilon = 0$ states possible. Thus, prior to this point no particle pair production is possible.

Appendix: mathematical formulae

Exterior (Schwarzschild) metric:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Florides interior metric:
(constant density fluid sphere with vanishing radial stresses)

$$ds^2 = -\frac{\left(1 - \frac{r_s}{R}\right)^{\frac{3}{2}}}{\sqrt{1 - \frac{r^2 r_s}{R^3}}} dt^2 + \left(1 - \frac{r^2 r_s}{R^3}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Radius of body

Schwarzschild interior metric:
(constant density fluid sphere)

$$ds^2 = -\left(\frac{3}{2}\sqrt{1 - \frac{r_s}{R}} - \frac{1}{2}\sqrt{1 - \frac{r_s r^2}{R^3}}\right)^2 dt^2 + \left(1 - \frac{r_s r^2}{R^3}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Separation of variables used:

$$\Psi(\mathbf{r}) = e^{-i\epsilon t} \psi_l(r) Y_{lm}(\theta, \phi)$$

Appendix: mathematical formulae

Exterior wave equation (unique: Birkhoff's theorem)

$$\psi_l''^E(r) + \left(\frac{1}{r-r_s} + \frac{1}{r}\right)\psi_l'^E(r) + \left(\frac{r^2\epsilon^2}{(r-r_s)^2} - \frac{m^2 r}{r-r_s} - \frac{l(l+1)}{r(r-r_s)}\right)\psi_l^E(r) = 0$$

For the interior we use the scaled coordinates:

$$\rho = r/R,\; s = r_s/R,\; \mu = mR,\; \varepsilon = \epsilon/m$$

Florides interior:

$$\psi_l''^I(\rho) + \frac{1}{2\rho}\left(5 - \frac{1}{1-\rho^2 s}\right)\psi_l'^I(\rho) + + \frac{1}{1-\rho^2 s}\left(\frac{(1-\rho^2 s)^{\frac{1}{2}}\varepsilon^2\mu^2}{(1-s)^{\frac{3}{2}}} - \mu^2 - \frac{l(l+1)}{\rho^2}\right)\psi_l^I(\rho) = 0$$

Appendix: mathematical formulae

Analytic expression for energy levels in the Florides interior case

Under the conditions
$$\mu \gg \sqrt{\frac{2}{s}}(n+3/2)$$
$$l \ll \sqrt{\frac{2}{s}}\mu - \frac{1}{2}$$

The bound state energy in the Florides case, through matching to the QHO, is given by

$$\varepsilon = (1-s)^{3/4}\left(1 + (n+3/2)\sqrt{\frac{s}{2\mu^2}} + O\left(\frac{s}{\mu^2}\right)\right)$$

Analytic expression for energy levels in the Schwarzschild interior case

Under the conditions

$$\delta \ll \frac{1}{10\mu^2}\,,\ \delta \ll \mu^2$$

The l = 0 ground state is: $\varepsilon = C\frac{\sqrt{\delta}}{\mu} = C\frac{\sqrt{8/9-s}}{\mu}$

Appendix: mathematical formulae

The low energy s-wave scattering matrix from Kuchiev, Flambaum PRD70,044022 (2004) is given as

$$S_0 = \frac{1+\mathcal{R}-\varepsilon^2 C^2(1-\mathcal{R})}{1+\mathcal{R}+\varepsilon^2 C^2(1-\mathcal{R})} e^{2i\delta_C}$$

where $(\varepsilon = \epsilon r_s)$

Euler Gamma function

Coulomb phase shift

$$\delta_C = arg\, \Gamma(l+1-Z/p)$$

$$C^2 = \frac{2\pi Z/p}{1-\exp[-2\pi Z/p]}$$

'Coulomb factor'

$$Z = \frac{r_s}{2}(\epsilon^2 + p^2)$$

Effective charge

Momentum of particle at infinity