Black Holes in Higher Dimensions

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NEB15 Recent Developments in Gravity, Chania

Thanks to:

Jutta Kunz
(Oldenburg University)

Eugen Radu
(Oldenburg University)

Maria Rodriguez
(Harward Univeristy)

Motivation

4D Spacetimes

- Black Holes are a consequence of Einstein gravity
- Astrophysical evidence
- Well studied und understood nowadays

Higher Dimensional Spacetimes

- Understand what is special in 4D spacetimes
- Superstring theories are candidates for Quantum Gravity
  - require higher dimensional spacetimes
- Gravity in spacetimes with compact dimensions
- What is the role of topology?

Not Included in this Talk

- AdS Black Holes, Braneworld Black Holes, Supersymmetric Black Holes
- Higher order curvature gravity

Outline

1. Reminder: Black Holes in $D = 4$ Dimensions
2. Black Holes in $D > 4$
3. Black Rings in $D = 5$
4. Black Rings in $D = 6$
5. Black Strings
6. Conclusions

Reminder: Black Holes in $D = 4$ Dimensions

	Vacuum	Charged
static	Schwarzschild (M)	Reissner-Nordström (M, Q, P)
rotating	Kerr (M, J)	Kerr-Newman (M, Q, P, J)

Properties

- Global charges:

  Mass M, e.-m. charges Q, P, angular momentum J

  - define uniquely the Black Hole spacetime (Uniqueness)
- Topology: Spherical horizon topology S^2
- 1^{st} Law: $dM = TdS + \Omega_{\text{H}} dJ + \Phi_{\text{H}} dQ$
- Smarr formula: $M = 2TS + 2\Omega_{\text{H}} J + \Phi_{\text{H}} Q$
- Gyromagnetic ratio: $g = \dfrac{2M\mu_{\text{mag}}}{QJ} = 2$

Static Black Holes in $D \geq 4$

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2_{D-2}$$

Vacuum [Tangherlini (1963)]

$$N(r) = 1 - \left(\frac{r_{\rm H}}{r}\right)^{D-3}, \qquad \text{Mass } M = \frac{r_{\rm H}}{2G}$$

Electrically charged EM [Myers & Perry (1986)]

$$N(r) = 1 - \frac{\mathcal{C}}{r^{D-3}} + \frac{\mathcal{D}^2}{r^{2[D-3]}}$$

$$\text{Mass } M = \frac{(D-2)\mathcal{C}A_{D-2}}{16\pi G}, \qquad \text{Charge } Q^2 = \mathcal{D}^2 \frac{(D-2)(D-3)}{8\pi G}$$

$$\text{Outer Horizon } r_{\rm H}^{D-3} = \frac{\mathcal{C}}{2} + \left(\frac{\mathcal{C}^2}{4} - \mathcal{D}^2\right)^{1/2}$$

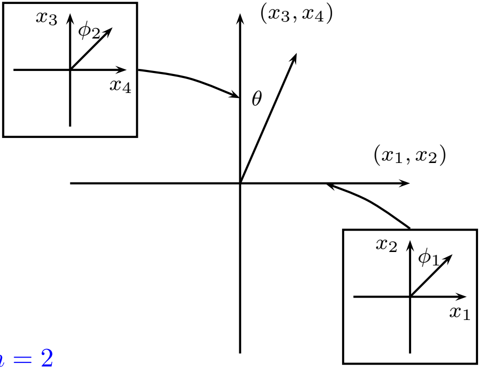
Rotating Black Holes in $D > 4$

Solutions in closed form [Myers & Perry (1986)]

$D-1$: dimension of space

n: number of independent planes: $n \equiv \left[\frac{D-1}{2}\right]$

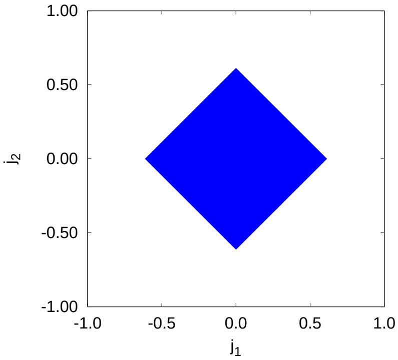
n: number of independent angular momenta J_i



example: $D = 5$, $n = 2$

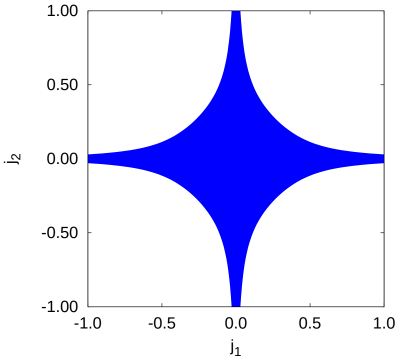
Rotating Black Holes: Domain of Existence

- $D = 5$:
  - domain of existence is bounded



scaled angular momenta

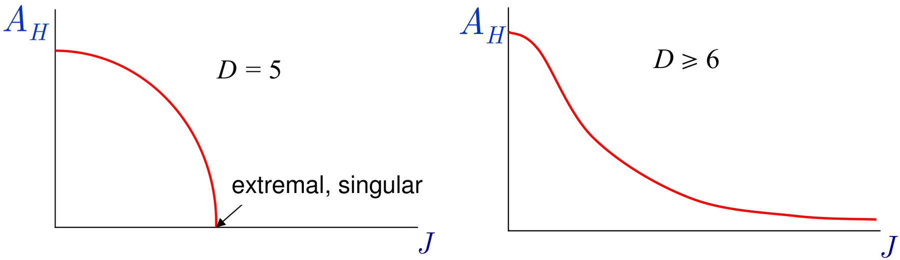
- $D = 6$:
  - domain of existence is unbounded on axes:



scaled angular momenta

Rotating Black Holes: Domain of Existence

a single angular momentum $J_1 = J$ ($J_i = 0$, $i > 1$)



scaled horizon area A_H versus scaled angular momentum J

Charged EM Black Holes in $D > 4$

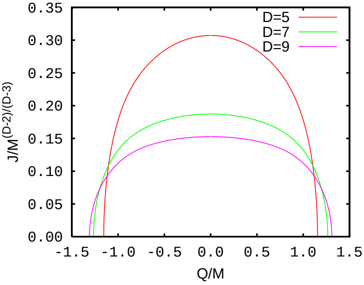
No charged rotating EM Black Holes in closed form

Special case: D odd, equal angular momenta $J_i = J$, $i = 1, \ldots, N$

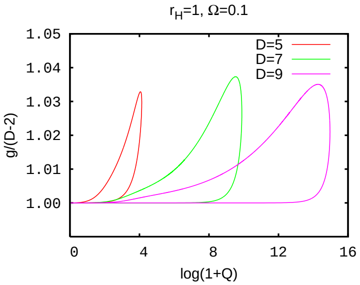
[Kunz, Navarro & Petersen (2005); Kunz, Navarro & Viebahn (2006)]

- numerical solutions - ordinary differential equations
- Smarr relation $\frac{D-3}{D-2}M = \frac{\kappa A_{\rm H}}{8\pi G} + N\Omega J + \frac{D-3}{D-2}\Phi_{\rm H} Q$

Domain of existence



Gyromagnetic ratio



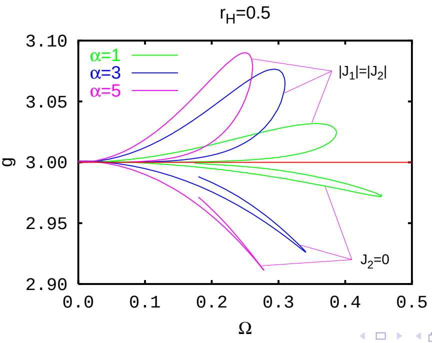
Charged EM Black Holes in $D > 4$

Special case: $D = 5$, a single angular momentum $J_1 = J$, $J_2 = 0$

[Kunz, Navarro & Petersen (2005)]

- numerical solutions - partial differential equations
- Smarr relation $\frac{2}{3}M = \frac{\kappa A_{\rm H}}{8\pi G} + \Omega_1 J_1 + \frac{2}{3}\Phi_{\rm H} Q$
- gyromagnetic ratio $g = 3$ (perturbative)

$$\alpha = \frac{Q}{\pi^{3/2}}$$



The Show So Far:

$D > 4$	Vacuum	Charged
static	M	M, Q
rotating	M, J_i	M, Q, J_i

Properties

- Global charges:
  Mass M, e.-m. charge Q, angular momenta J_i
- Smarr formula: $$\frac{D-3}{D-2}M = \frac{\kappa A_{\mathrm{H}}}{8\pi G} + \sum \Omega_i J_i + \frac{D-3}{D-2}\Phi_{\mathrm{H}} Q$$
- Topology: Spherical horizon topology S^{D-2}

The Show So Far:

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- Topology: Spherical horizon topology S^{D-2}

New Solutions: non-spherical topology

- $D-1$ dim Black Hole $+$ flat direction $\rightsquigarrow$ Black String
- Bend to a ring $\rightsquigarrow$ Black Ring
- Horizon topology: $S^1 \times S^{D-3}$

Black Rings in $D = 5$

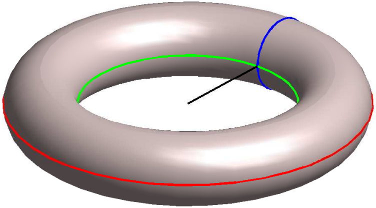
[Emparan & Reall 2002]

Black Ring

horizon topology $S^1 \times S^2$

Static Ring

- attraction:
  gravity/string tension
  shrink ring
- repulsion:
  conical singularity
  inside: push
  outside: pull
- unbalanced ring



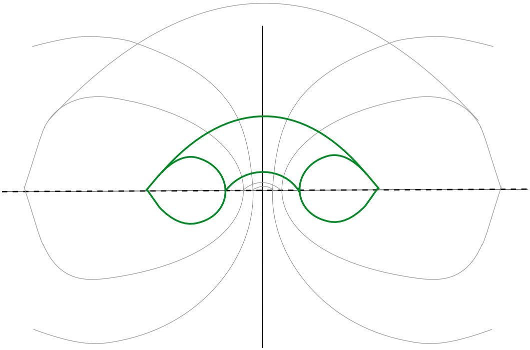
rotating ring

- attraction:
  gravity/string tension
- repulsion:
  rotation along S^1
  centrifugal force
- balanced ring

Black Rings in $D = 5$

[Emparan & Reall 2002]

static Black Ring: string pulling from outside the ring (shown)

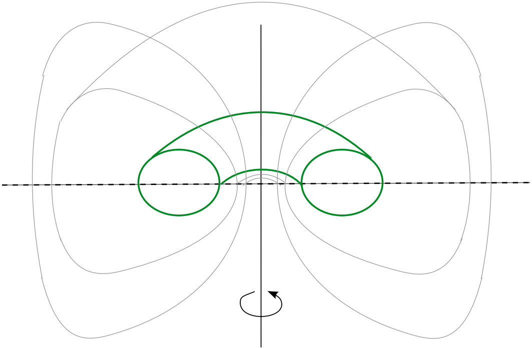


static Black Ring: strut pushing from inside the ring (not shown)

Black Rings in $D = 5$

[Emparan & Reall 2002]

rotating Black Ring:

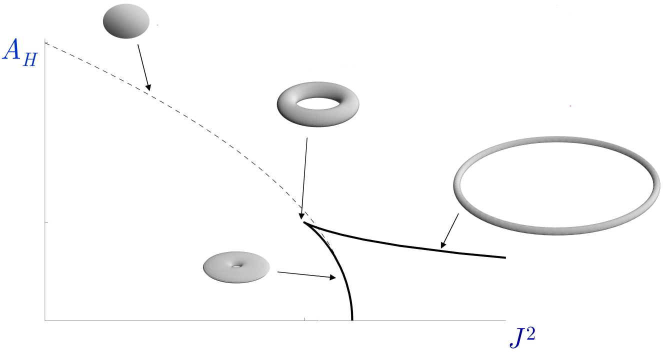


no conical singularity: appropriate horizon velocity

Black Holes & Black Rings in $D = 5$

[Emparan & Reall 2002]

phase diagram



nonuniqueness

region with

- MP Black Holes
- fat Black Rings
- thin Black Rings

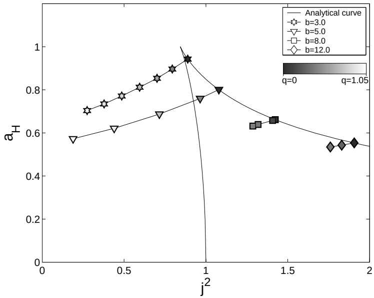
scaled horizon area A_H vs. scaled angular momentum J^2

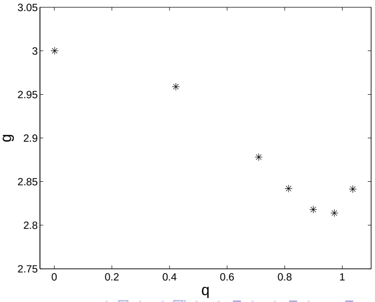
Electrically Charged EM Black Rings in $D = 5$

[Kleihaus, Kunz & Schnülle (2011)]

Properties

- Numerical solutions - partial differential equations
- a single angular momentum
- rotating balanced Black Rings
- Smarr relation $M = \frac{3}{16\pi G}\kappa A_H + \frac{3}{2}\Omega_H J + \Phi_H Q$





Black Objects in $D = 5$

More black objects with non-spherical topology

- Black rings with 2 angular momenta [Pomeransky et al. (2006)]
- Black di-rings [Iguchi et al. (2007)]
- Black saturn [Elwang et al. (2007)]
- Black bi-rings [Elwang et al. (2008)]
- Black rings with dipole charge [Yazadiev (2006), Emparan (2004)]
- ...

Black Objects in $D > 5$

- Powerful method in $D = 4$ and 5: Generalized Weyl coordinates [Emparan & Reall (2002)]
- Fails in higher dimensions

The Blackfold Approach

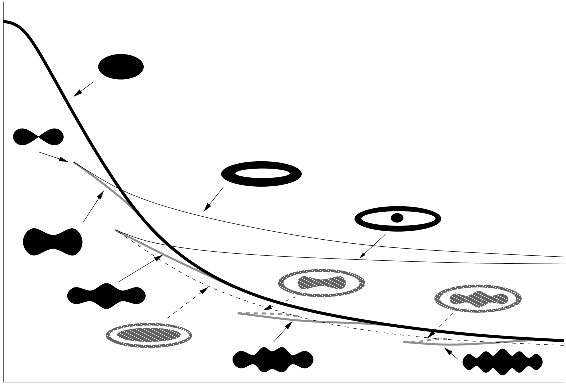
[Emparan et al. (2007), (2009)]

- Matched asymptotic expansion
- Two different lengthscale
- Approximate analytical solutions
- Thin Black Rings in $D \geq 5$
- Fails for fat Black Rings
- Predicted new Black objects with non-spherical horizon topology

Black Objects in $D=6$

[Emparan, Harmark, Niarchos, Obers, Rodriguez (2007)]

phase diagram (proposed)



scaled area A_H vs. scaled angular momentum J^2

nonuniqueness

region with

- MP Black Holes
- pinched Black Holes
- Black Rings
- Black Saturns
- …

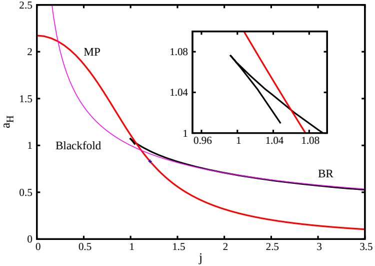
Black Rings in $D = 6$

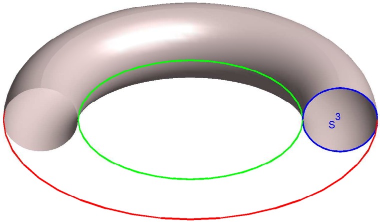
[Kleihaus, Kunz & Radu (2012)]

Properties

- Horizon topology: $S^1 \times S^3$
- rotation in S^1 $\rightsquigarrow$ balanced Black Rings

Numerical results



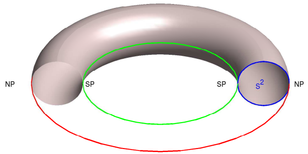


- Pinched Black Holes?

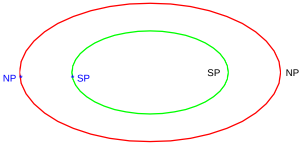
Generalized Black Ring: $S^2 \times S^2$

[Kleihaus, Kunz & Radu (2009)]

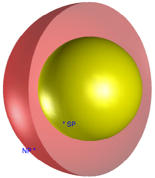
$S^1 \times S^2$ Black Ring



Suppress S^2 dimensions



$S^2 \times S^2$



Properties

- Numerical solutions - similar to Black Rings
- No rotation - Non-balanced solutions
- conical deficit/excess
- Higher dimensions: $S^3 \times S^2$, $S^4 \times S^2$, etc.

More solutions in [Kleihaus, Kunz, Radu & Rodrigues (2011)]

And now . . .

. . . for something completely different

BLACK STRINGS

Black Strings

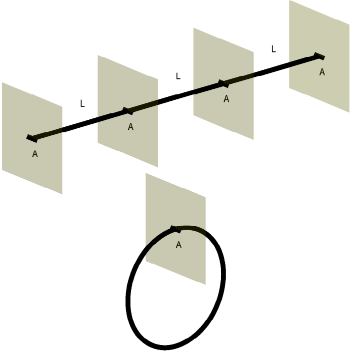
String Theories

- Higher dimensional spacetimes
- Gravity in $D > 4$
- Compact extra dimensions

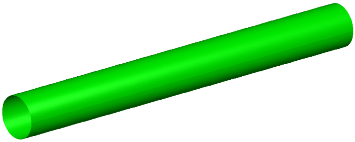
Consequences for Gravity?

- Uniform Black Strings
- Non-uniform Black Strings
- Caged Black Holes

$(D-1)$ spacetime $\times\ 1$ comp. dim.



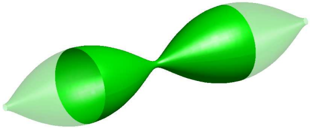
L length of compact dimension



Uniform Black String

- $(D-1)$-dim Black Hole stretched in extra dim.
- horizon $S^{D-3} \times S^1$
- asympt. $\mathcal{M}^{D-1} \times S^1$
- exist for all L, M
- unstable for small M

[Gregory & Laflamme (1993)]

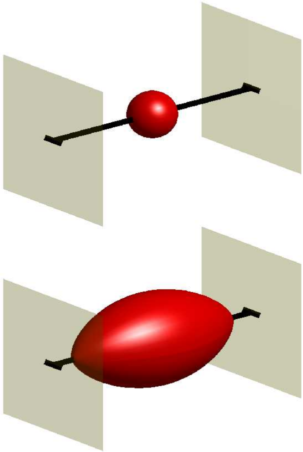


Non-uniform Black String

- emerge from uniform BS
- depend on extra coord.
- horizon $S^{D-3} \times S^1$
- belly ($R_{\rm max}$) and waist ($R_{\rm min}$)
- Non-uniformity $\lambda = \frac{1}{2}\left(\frac{R_{\rm max}}{R_{\rm min}} - 1\right)$

[Wiseman (2003), Kleihaus et al. (2006)]

Caged Black Hole

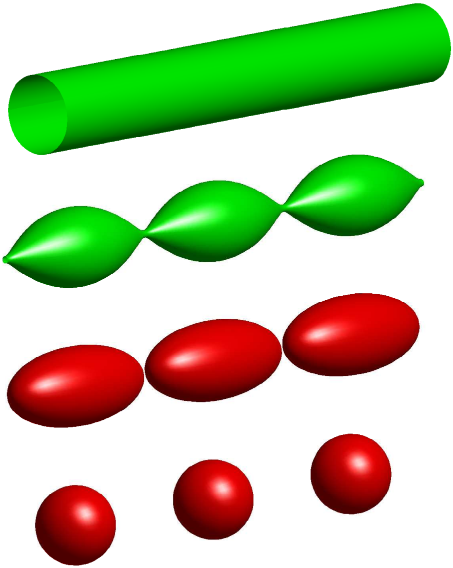


caged Black Hole

- horizon S^{D-2}
- exist for small masses

[Sorkin et al. (2003), Kol (2005, 2006), Kudoh & Wiseman (2005)]

Topology changing transition?



uniform Black String
$S^{D-3} \times S^1$

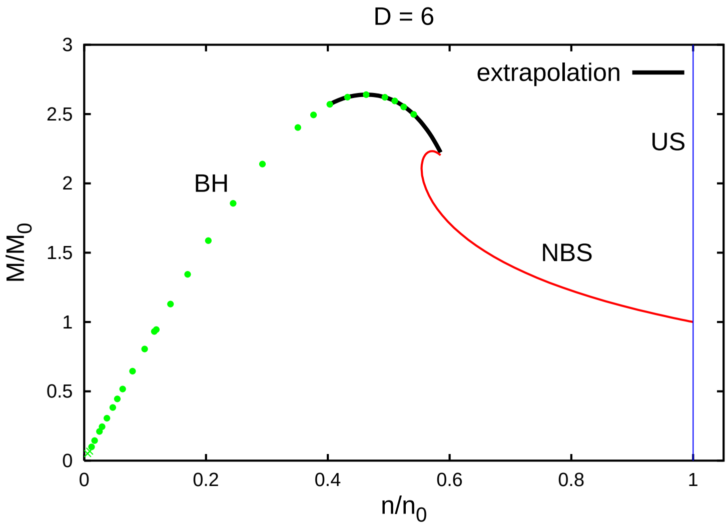
non-uniform Black String
$S^{D-3} \times S^1$

pinch off

caged Black Hole
S^{D-2}

Topology changing transition?

[Kleihaus, Kunz & Radu (2006)]



[Black Hole data from Kudoh & Wiseman, PRL94(2005)161102]

Rotating Black Strings

[Kleihaus, Kunz & Radu (2007)]

$D = 5$

- Uniform Black String

  Kerr Black Hole (r, θ) + compact dimension (z)

- Non-uniform Black String

  Metric functions depend on (r, θ, z)

$D = 6$

- Uniform Black String

  Myers-Perry Black Hole (r, θ) + compact dimension (z)

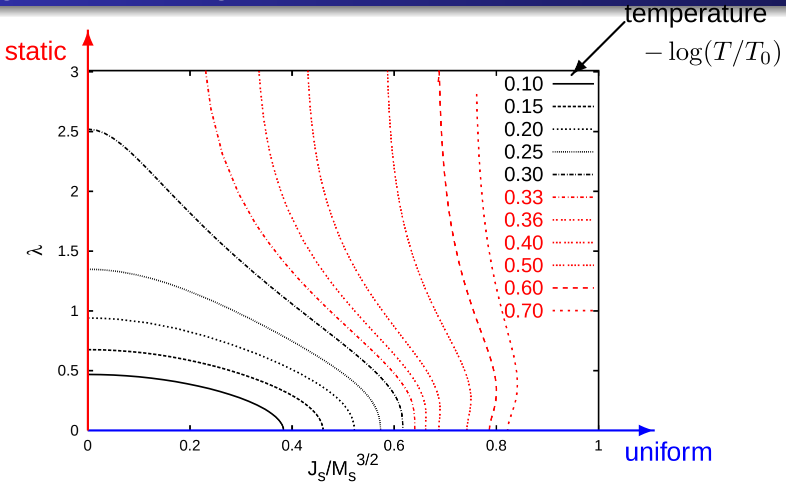
- Equal angular momenta $J_1 = J_2 = J$

  Myers-Perry Black Hole (r) + compact dimension (z)

- Non-uniform Black String

  Metric functions depend on (r, z)

Rotating Black Strings



No non-uniform Black Strings below T^*

Conclusions

- Static and spherically symmetric Black Holes
  - Schwarzschild and Reissner-Nordström Black Holes generalized to D-dim counterparts
- Stationary rotating Black Holes
  - Several independend angular momenta
  - Kerr Black Holes generalized to D-dim Myers-Perry Black Holes
  - Electrically charged Black Holes: Numerical investigations
- Black Holes with non-spherical horizon topology
  - $D = 5$: Black Rings, Black Saturn, Black Di-Rings, ... known in closed form
    Numerical studies on electrically charged Black Rings
  - $D > 5$: Blackfold approach (approximate solutions)
  - $D = 6$: Black Rings
    $S^2 \times S^2$ horizon topology
    Pinched Black Holes?
- Black Strings
  - Long (or thin) uniform Black Strings are unstable
  - non-uniform Black Strings
  - caged (or localized) Black Holes
  - Topology changing transition?
  - Stationary rotating non-uniform Black Strings in $D = 6$

Thank you very much!