Aspects of horizon entropy in Lanczos-Lovelock gravity and action principle for the fluid/gravity correspondence

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Outline of the talk

1. Two aspects of Black hole entropy in Lanczos-Lovelock gravity

   Sanved Kolekar, Dawood Kothawala and T. Padmanabhan, Phy. Rev. D 85, 064031 (2012) [arXiv:1111.0973]

   We look at two approaches which calculate the black hole entropy in GR and compare them in the case of Lanczos-Lovelock gravity. Only one approach leads to the expression of Wald entropy.

2. Action principle for the Fluid-Gravity correspondence

   Sanved Kolekar and T. Padmanabhan, Phy. Rev. D 85, 024004 (2011) [arXiv:1109.5353]

   Starting from an extremum principle we obtain the Damour-Navier-Stokes equation which describes the membrane properties of a black hole horizon.

I. Two aspects of Black hole entropy: Intrinsic v/s Extrinsic

What is the origin of Black entropy?.

Various approaches to explain Black hole entropy

• Extrinsic origin

1. from the entropy of matter forming the black hole
2. entanglement entropy of matter fields propagating in the background metric, etc

• Intrinsic origin - microscopic degrees of freedom corresponding to underlying statistical theory of quantum gravity which are different depending on the approach.

• Within the context of Einstein's gravity, it is very difficult to discriminate between these two approaches since all the different approaches (intrinsic as well as extrinsic) lead to $S \propto A$

Degeneracy??

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Degeneracy??

• The proportionality between horizon entropy and area does not extend to more general class of gravitational theories. Many of the approaches which correctly reproduces $S \propto A$ in the context of GR cannot be generalized in a natural fashion to more general class of theories

• The possibility of generalization beyond GR acts as an acid test in discriminating between the different approaches for obtaining the horizon entropy both intrinsic and extrinsic.

Lanczos-Lovelock theory

• The Lanczos-Lovelock lagrangian L_m is given by completely anti-symmetrised product of m curvature tensors

$$L_m = \frac{1}{16\pi}\frac{1}{2^m}\delta^{a_1b_1\ldots a_mb_m}_{c_1d_1\ldots c_md_m}R^{c_1d_1}_{\quad a_1b_1}\cdots R^{c_md_m}_{\quad a_mb_m}$$

For $m = 1$, the L_m reduces to R, the Einstein-Hilbert lagrangian.

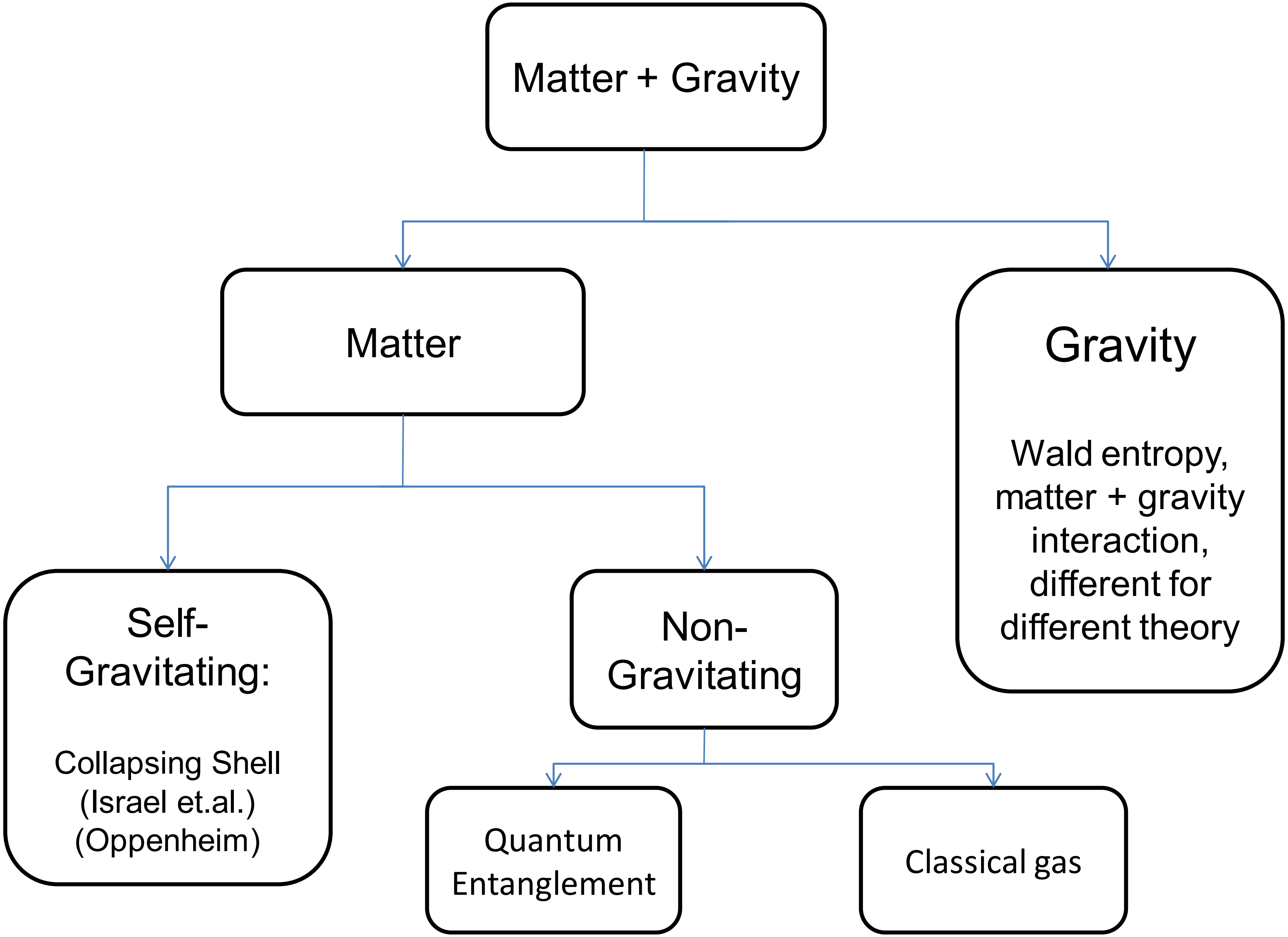
• Second order quasi-linear field equations.

Another motivation

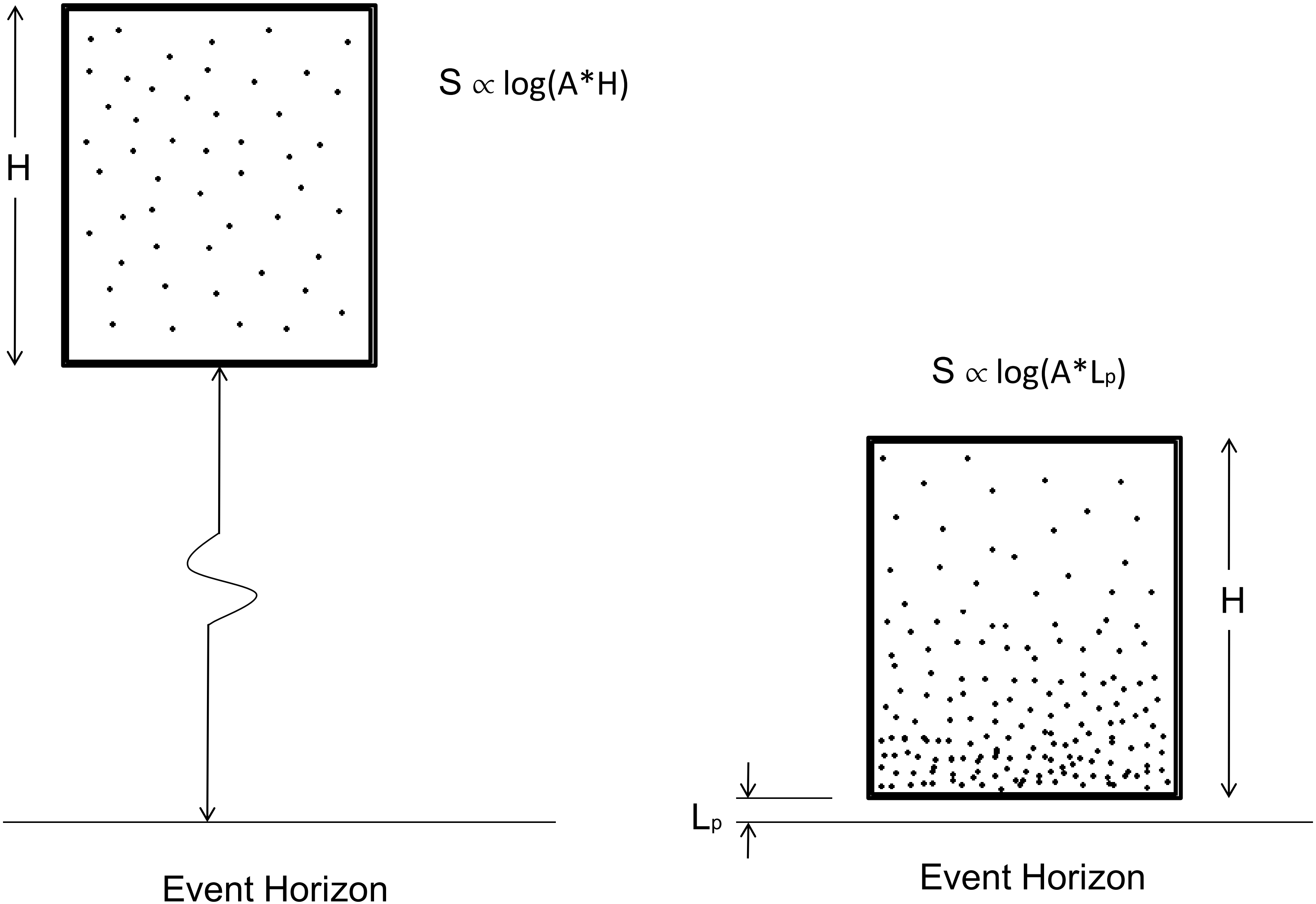
Facts about horizons

(i) Temperature of horizons does not depend on the field equations of the theory and is just an indication that spacetimes, like matter, can be hot, but in a observer-dependent manner

(ii) Entropy of horizons depends on the field equations of gravity and cannot be determined by using just QFT in a background metric



Matter + Gravity
Matter
Gravity
Wald entropy, matter + gravity interaction, different for different theory
Self-Gravitating:
Collapsing Shell (Israel et.al.) (Oppenheim)
Non-Gravitating
Quantum Entanglement
Classical gas



H
S ∝ log(A*H)
Event Horizon
S ∝ log(A*Lp)
H
Lp
Event Horizon

Minkowski spacetime

- The entropy of the gas is:

$$S = N\left[\log\left(\frac{D!\pi^{D/2}}{\Gamma(\frac{D}{2}+1)N}\frac{V_D}{\beta^D}\right)+D\right] \longrightarrow N\left[\log\left(\frac{8\pi}{N}\frac{V_3}{\beta^3}\right)+3\right]$$

- The corresponding equation of state obtained is the ideal gas equation:

$$P = \frac{N}{\beta V}$$

- The total energy of the system is

$$E = \frac{DN}{\beta} \longrightarrow \frac{3N}{\beta}$$

Rindler spacetime

- The entropy is

$$S \approx N \left[\log \left(\frac{8\pi}{N} \frac{\mathbf{A}_\perp(\mathbf{L_p/2})}{(\beta_{loc}(x_a))^3} \right) + 3 \right]$$

- Average energy of the system

$$U = \frac{3N}{\beta_{loc}}$$

- Transverse pressure

$$P_\perp = \frac{N}{\beta_{loc} V}$$

- Longitudinal pressure

$$P_{x_a} \approx \frac{N}{\beta_{loc} \mathbf{A}_\perp(\mathbf{L_p/2})}$$

• The contribution to the entropy comes from only a fraction $\mathcal{O}(\mathsf{L_p}/\mathsf{K})$ of the matter degrees of freedom near to the horizon while the rest are suppressed.

• This effect is purely kinematical in its origin and is independent of spacetime curvature

Gravitating systems ??

Extrinsic Approach

• The most natural extrinsic origin of black hole entropy would be to consider the entropy of the matter which formed the black hole.

• In GR, it is known $S_{\mathsf{matter}} = A/4$

- W. Israel et. al. (1998), Oppenheim (2001)

The Setup

• A system of n densely packed spherically symmetric shells in D dimensions, assumed to be in thermal equilibrium, and supporting itself against its own gravity.

• What is the entropy of this system when the outermost shell is close to the event horizon of the system?

The Setup

- The metric outside the ith shell is

$$ds^2 = -c_i f_i(r) dt^2 + b_i(r)^{-1} dr^2 + r^2 d\Omega^2$$

- The junction conditions - the surface stress energy tensor gives us E_i and P_i

$$\begin{aligned} 8\pi \left(t_{(m)}{}^{\nu}_{\mu} \right)_i &= \frac{m!}{2^{m+1}} \alpha_m \left\langle \sum_{s=0}^{m-1} C_{s(m)} \left(\pi_{s(m)} \right)^{\nu}_{\mu} \right\rangle_i \\ \left(\pi_{s(m)} \right)^{\nu}_{\mu} &= \delta^{[\nu\nu_1 \cdots \nu_{2m-1}]}_{[\mu\mu_1 \cdots \mu_{2m-1}]} \hat{R}^{\mu_1\mu_2}_{\nu_1\nu_2} \cdots \hat{R}^{\mu_{2s-1}\mu_{2s}}_{\nu_{2s-1}\nu_{2s}} K^{\mu_{2s+1}}_{\nu_{2s+1}} \cdots K^{\mu_{2m-1}}_{\nu_{2m-1}} \end{aligned}$$

- Each shell satisfies the Gibb's Duhem relation

$$E_i = T_i S_i - P_i A_i + \mu_i N_i$$

The Result

• Only the outermost shell contributes to the total matter entropy !

$$S_{\text{matter}} \underset{r_n \to r_H}{\longrightarrow} S_n$$

• For a mth order Lanczos-Lovelock theory: A system of n self- gravitating shells in thermal equilibrium on the verge of forming a black hole: $r_n \to r_H$

$$S^{(m)}_{\text{matter}} = \frac{(D-2m)}{(D-2)} S^{(m)}_{\text{Wald}}$$

where $S^{(m)}_{\text{Wald}}$ is the Wald entropy of the horizon

• $S_{\text{matter}} < S_{\text{Wald}}$!

• The relation between pressure and entropy

$$\frac{S_{\text{matter}}}{A_H} \equiv \frac{P}{T}$$

- Sanved Kolekar, Dawood Kothawala and T. Padmanabhan, Phy. Rev. D 85, 064031 (2012)

Intrinsic Approach

- The path integral of the Euclideanised action

$$Z(\beta) = \sum_{g\epsilon\mathcal{S}} \exp\left(-\int_0^\beta d\tau \int d^3x\sqrt{g_E}L_m[f(r)]\right)$$

where the sum $g\epsilon\mathcal{S}$ is over a mini space of metrics of the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^n$$

with the boundary condition that $f(r)$ vanishes at some surface $r = a$, say, with $f'(a)$ remaining finite.

- The Lagrangian $L_m[f(r)]$ becomes a total derivative

$$\begin{aligned} r^{D-2}L_m &= \frac{d}{dr}\left[\frac{(D-2)!}{(D-2m-1)!}(1-f)^m r^{D-2m-1}\right] \\ &\quad -\frac{d}{dr}\left[\frac{(D-2)!}{(D-2m)!}mf'(1-f)^{m-1}r^{D-2m}\right] \end{aligned}$$

- No contribution from the classical path! Hence independant of the field equations

• The final result can be written in a very suggestive form:

$$Z(\beta) \propto \exp\left[S(a) - \beta E(a)\right]$$

with the identifications for the entropy and energy being given by:

$$S = \frac{(D-2)!}{(D-2m)!}\frac{m\Omega}{4}a^{D-2m} = S_{\mathrm{Wald}}$$

$$E = \frac{(D-2)!}{(D-2m-1)!}\frac{\Omega}{16\pi}a^{D-2m-1} = E_{\mathrm{quasi-local}}$$

Summary

• Our motivation goes beyond a mere generalization of certain known results to modified actions and higher dimensions. (We do not presume that the class of modified actions we have considered would have any practical relevance in realistic situations.)

• discriminating between different approaches to calculate entropy in GR is difficult because an entropy proportional to area can arise in completely different contexts

• An acid test for the validity of any particular approach for calculating horizon entropy is that it should reproduce the Wald entropy in, say, Lanczos-Lovelock models.

• if one evaluates the entropy of matter which is on the verge of collapsing to a black hole, then this matter entropy does not necessarily match with Wald entropy.

• Euclidean action for Lanczos-Lovelock models, in spherical symmetry, becomes a total derivative and can be written as an expression for free-energy from which entropy S and energy E can be read-off. The expression for S then matches with Wald entropy

II. Action for fluid-gravity correspondence

• Damour-Navier-Stokes equation: Projected Einstein field equations on a black hole horizon behave as Navier-Stokes like equation

$$(\partial_0 + v^B\, D_B)\omega_A + \Omega_B\, D_A v^B + \theta\omega_A + D_B\sigma_A^B - D_A\left(\kappa + \frac{1}{2}\theta\right) = 8\pi T_{mA}\,\ell^m$$

where momemtum density is $-\omega_A/(8\pi)$, shear viscosity coefficient $\eta = (1/16\pi)$, bulk viscosity coefficient $\zeta = -1/16\pi$, pressure $(\kappa/8\pi)$, external force $F_a = T_{ma}\ell^m$

• the black hole horizon can be interpreted as a dissipative membrane satisfying a dynamical equation having a form very similar (but not identical) to the Navier-Stokes equation $\longrightarrow$ membrane paradigm.

- On any null surface in a freely falling frame

$$(\partial_0 + v^B \partial_B)\left(-\frac{\omega_A}{8\pi}\right) = \frac{1}{8\pi}\partial_B \sigma^B_A - \frac{1}{16\pi}\partial_A \theta - \partial_A\left(\frac{\kappa}{8\pi}\right) - T_{mA} l^m$$

- T. Padmanabhan, Phys. Rev. D 83, 044048 (2011) [arXiv:1012.0119]

- Dictionary between a viscous fluid variables and null normals l^a, Γ^a_{bc}

$$[l^m, \Gamma^a_{bc}, \kappa] \longleftrightarrow [\sigma_{ab}, \theta, \omega_m, p]$$

• **Question**: If gravity is indeed an emergent phenomenon, should not the action functionals contain some signature of this fact ?

Evidence!!!

Entropy functional

- Action defined in terms of normal to null surfaces:

$$S[l^a] = -\int_{\mathcal{V}} d^4x\sqrt{-g}\left(2P_{ab}^{cd}\nabla_c l^a \nabla_d l^b - T_{ab}l^a l^b\right)$$

where $2P^{abcd} = g^{ac}g^{bd} - g^{ad}g^{bc}$

- T. Padmanabhan and Aseem Paranjape, Phys. Rev. D 75, 064004 (2007) [arXiv:gr-qc/0701003]

- Extremizing* the action w.r.t null normals leads to the Einstein field equations:

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{1}{2}(T_{ij} + g_{ij}\Lambda)$$

Action for DNS equation

- Use the dictionary to obtain

$$S[l^a] = -\int_{\mathcal{V}} d^4x \sqrt{-g}\left(\left(2\eta\sigma_{ab}\sigma^{ba} + \zeta\theta^2\right) + 2\theta\kappa - T_{ab}l^a l^b\right)$$

- Extremizing w.r.t null normals leads to the DNS equation:

$$(\partial_0 + v^B D_B)\omega_A + \Omega_B D_A v^B + \theta\omega_A + D_B\sigma^B_A - D_A\left(\kappa + \frac{1}{2}\theta\right) = T_{mA}\,\ell^m$$

- Imposing additional condition: independent of l^a leads to Einstein field equation with cosmological constant.

Interpretation of the action

- When the null surface corresponds to a horizon, use the fact $T = \kappa/2\pi$ and $S = A/4$

$$
\begin{aligned}
\frac{d\mathcal{S}}{dt} &\propto -\delta A d\lambda \left(2\eta\sigma_{ab}\sigma^{ba} + \zeta\theta^2\right) + T dS_{\mathcal{H}} + P d\delta A \\
&= -dE + T dS_{\mathcal{H}} + P d\delta A
\end{aligned}
$$

where $2\eta\sigma_{ab}\sigma^{ba} + \zeta\theta^2$ has a form of viscous dissipation energy density

- Sanved Kolekar and T. Padmanabhan, Phy. Rev. D 85, 024004 (2011) [arXiv:1109.5353]

Equation of state for null surfaces

• Using the relations $T = \kappa/2\pi$, $P = \kappa/8\pi$ and $S = A/4$, it is easy to see that

$$PA = ST$$

• The form of the equation of state holds even in the case of the membrane paradigm for mth order Lanczos-Lovelock theory.

$$\frac{PA}{T} = \frac{(D-2m)}{(D-2)} S_{wald}$$

where S_{wald} is the corresponding Wald entropy of the horizon

- Sanved Kolekar and Dawood Kothawala, Membrane Paradigm and Horizon Thermodynamics in Lanczos-Lovelock gravity [arXiv:1111.1242]

• A system of n self- gravitating shells in thermal equilibrium on the verge of forming a black hole: $r_n \to r_H$

$$\frac{P_n A_n}{T} = \frac{(D-2m)}{(D-2)} S_{wald}$$

Thank You