Mind the Resonances

(Motion in “bumpy” black hole spacetime backgrounds)

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Motivation

In the case of extreme mass ratio inspirals (EMRI) systems there are mainly two reasons for studying “bumpy” black hole (non-integrable) spacetime backgrounds:

1) To take into account the perturbation of the supermassive black hole (SMBH) spacetime due to the accreting matter.

2) To check whether the super massive objects at the centre of galaxies are indeed black holes.

<u>Reviews</u>

Babak, Gair ,Petiteau & Sesana, CQG (2010)
Bambi, MPL B (2011)
Amaro-Seoane et al. ArXiv: 1202.0839

<u>Selected papers</u>

Ryan, PRD (1995), (1997)
Collins & Hughes, PRD (2004)
Glampedakis & Babak, CQG (2006)
Barausse, Rezzolla, Petroff & Ansorg, PRD (2007)
Gair, Li & Mandel, PRD (2008)

Metric from the Manko-Novikov family (CQG, 1992)

Weyl-Papapetrou metric element in prolate spheroidal coordinates:

Gair, Li Mandel, PRD (2008)

$$ds^2 = g_{tt}\,dt^2 + g_{xx}\,dx^2 + g_{yy}\,dy^2 + g_{\phi\phi}\,d\phi^2 + g_{t\phi}\,dt\,d\phi$$

$$\begin{aligned}
g_{tt} &= -f\,, \\
g_{xx} &= \frac{k^2 e^{2\gamma}(x^2-y^2)}{f(x^2-1)}\,, \\
g_{yy} &= \frac{k^2 e^{2\gamma}(x^2-y^2)}{f(1-y^2)}\,, \\
g_{\phi\phi} &= \left(\frac{k^2(x^2-1)(1-y^2)}{f} - f\omega^2\right)\,, \\
g_{\phi t} &= 2\,\omega\,f
\end{aligned}$$

$$\begin{aligned}
A &= (x^2-1)(1+a\,b)^2 - (1-y^2)(b-a)^2, \\
B &= [x+1+(x-1)a\,b]^2 + [(1+y)a+(1-y)b]^2, \\
C &= (x^2-1)(1+a\,b)[b-a-y(a+b)] \\
&\quad + (1-y^2)(b-a)[1+a\,b+x(1-a\,b)],
\end{aligned}$$

$$\begin{aligned}
\psi &= \beta\frac{P_2}{R^3}, \\
\gamma' &= \ln\sqrt{\frac{x^2-1}{x^2-y^2}} + \frac{3\beta^2}{2R^6}(P_3^2 - P_2^2) \\
&\quad + \beta\left(\sum_{\ell=0}^{2}\frac{x-y+(-1)^{2-\ell}(x+y)}{R^{\ell+1}}P_\ell - 2\right)
\end{aligned}$$

For q>0 the compact object is more oblate than Kerr and for q<0 more prolate.

For q=0 we get the Kerr metric.

$$\begin{aligned}
f &= e^{2\psi}\frac{A}{B}, \\
\omega &= 2ke^{-2\psi}\frac{C}{A} - 4k\frac{\alpha}{1-\alpha^2}, \\
e^{2\gamma} &= e^{2\gamma'}\frac{A}{(x^2-1)(1-\alpha^2)^2},
\end{aligned}$$

$$\alpha = \frac{-M+\sqrt{M^2-(S/M)^2}}{(S/M)}, \qquad k = M\frac{1-\alpha^2}{1+\alpha^2},$$

$$\beta = q\left(\frac{1+\alpha^2}{1-\alpha^2}\right)^3.$$

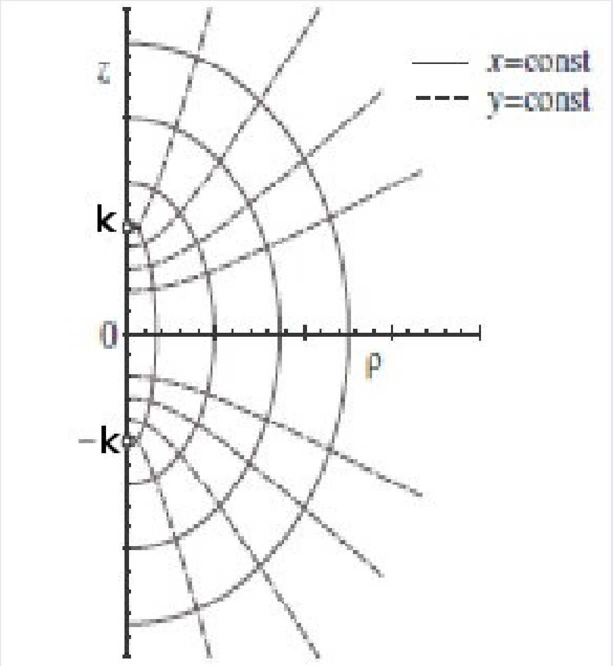
$$\begin{aligned}
a &= -\alpha\exp\left[-2\beta\left(-1+\sum_{\ell=0}^{2}\frac{(x-y)P_\ell}{R^{\ell+1}}\right)\right], \\
b &= \alpha\exp\left[2\beta\left(1+\sum_{\ell=0}^{2}\frac{(-1)^{3-\ell}(x+y)P_\ell}{R^{\ell+1}}\right)\right], \\
R &= \sqrt{x^2+y^2-1}, \\
P_\ell &= P_\ell\left(\frac{x\,y}{R}\right), \quad P_\ell(w) = \frac{1}{2^\ell \ell!}\left(\frac{d}{dw}\right)^\ell (w^2-1)^\ell
\end{aligned}$$

Coordinates transformations

From the prolate spheroidal coordinates (x,y) of the Weyl-Papapetrou metric to the Boyer-Lindquist (r,θ):

$r=x\,(M^2-(S/M)^2)^{1/2}+M$, $\cos\theta=y$, where $x \in [1,\infty)$, $y \in [-1,1]$

By the above transformation the Kerr event horizon lies on x=1.



From the prolate spheroidal coordinates (x,y) of the Weyl-Papapetrou metric to the cylindrical (ρ,z):

$$\rho = k\sqrt{(x^2-1)(1-y^2)}, \qquad z = kxy$$

Geodesic approximation

In the generic case of stationary and axisymmetric spacetime background, the energy E and the azimuthal angular momentum L_z are constants of motion:

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}$$

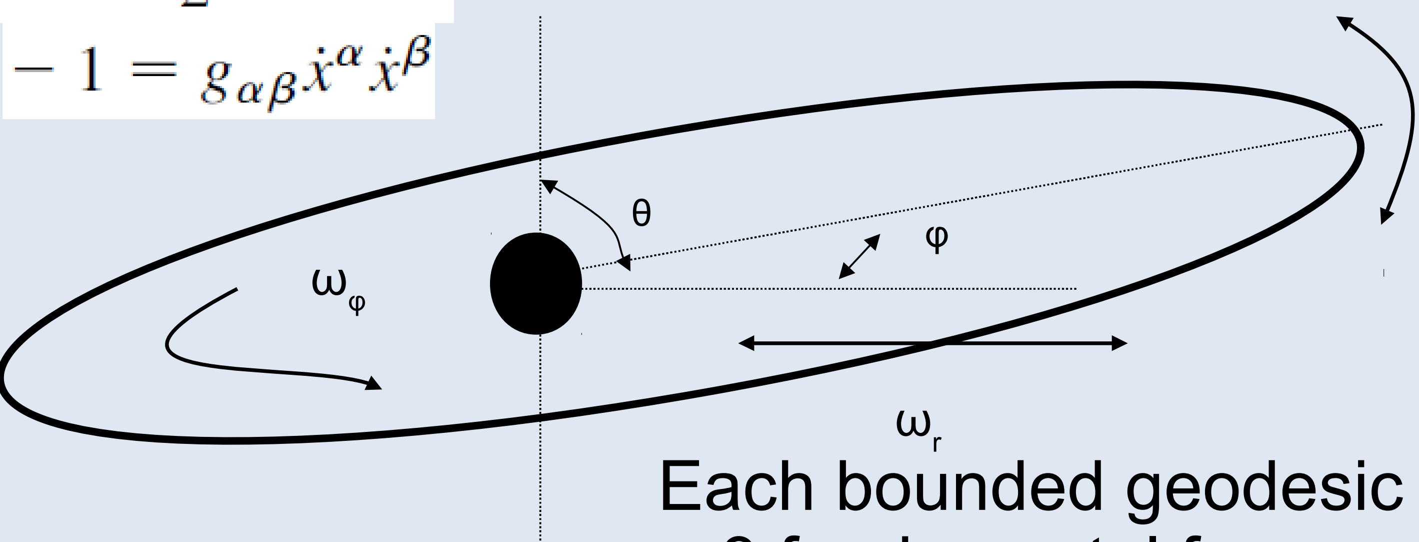
$$p_{\mu} = \frac{\partial L}{\partial \dot{x}^{\mu}} = g_{\mu\nu} \dot{x}^{\nu}$$

$$H = \frac{1}{2} g^{\mu\nu} p_{\mu} p_{\nu}$$

$$-1 = g_{\alpha\beta} \dot{x}^{\alpha} \dot{x}^{\beta}$$

$$E = -g_{tt}\dot{t} - g_{t\phi}\dot{\phi}, \qquad L_z = g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi}$$

If we had a fourth integral of motion Q, then the system would be integrable and we could express the Hamiltonian into action-angle variables $H^{(aa)}$.



$$\omega_k = \frac{\partial H^{(aa)}}{\partial J_k}$$

$J_k = J_k(H, E, L_z, Q)$

Each bounded geodesic is labelled by the 3 fundamental frequencies ω_r, ω_{θ}, ω_{φ}.

Kerr metric

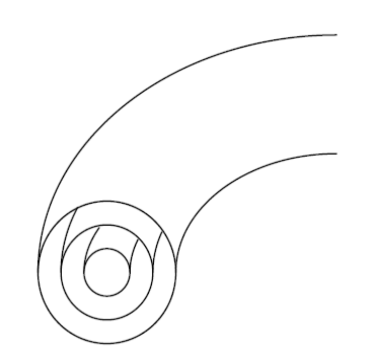
The Kerr metric has the "extra" integral of motion, the Carter constant Q. Carter, PR (1968)

Though the Hamiltonian has not been yet expressed in action-angle variables, the characteristic frequencies ω_r, ω_θ, ω_φ were found "analytically" as functions of:

the black hole spin a and mass M,

the constants of motion (E, L_z, Q) and

the geometrical characteristics of the orbit (turning points of r and θ). Schmidt, CQG (2002)



The ratio $\nu_\theta(E,L_z)= \omega_r/\omega_\theta$ seems to be strictly monotonic function along a foliation of invariant tori. (numerical indications, not proven!)

Shaken, not stirred

Consider a weak perturbation of a Kerr background that "breaks" the Carter constant, but leaves the background stationary and axisymmetric, e.g. a compact object more prolate or oblate than Kerr black hole.

$$H_{New} = H_{Kerr} + q\, H_{Perturbation} \qquad (q \ll)$$

By the two remaining integrals of motion E, L_z we can reduce the number of degrees of freedom from 4 to 2, i.e. instead of 4 coupled ODEs we can study 2 coupled ODEs.

ODEs: $$\frac{D\dot{x}^\lambda}{D\tau} \equiv \ddot{x}^\lambda + \Gamma^\lambda{}_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 0$$

Thus, we restrict the motion and the dynamics on the meridian plane ((r,θ) Boyer-Lindquist, (ρ,z) Weyl-Papapetrou).

Two theorems to rule them all

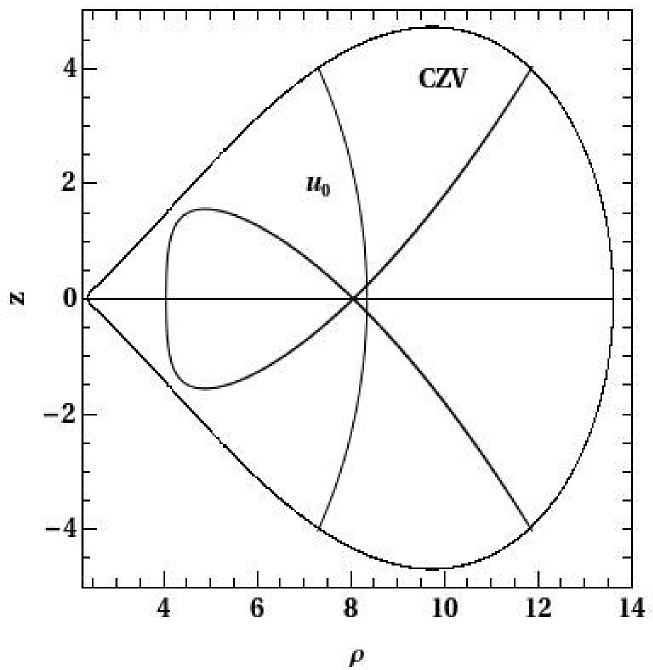
If the perturbation is weak enough, then most of the non-resonant tori survive (KAM theorem) and the resonant tori transform into the Birkhoff chain (Poincaré-Birkhoff theorem).

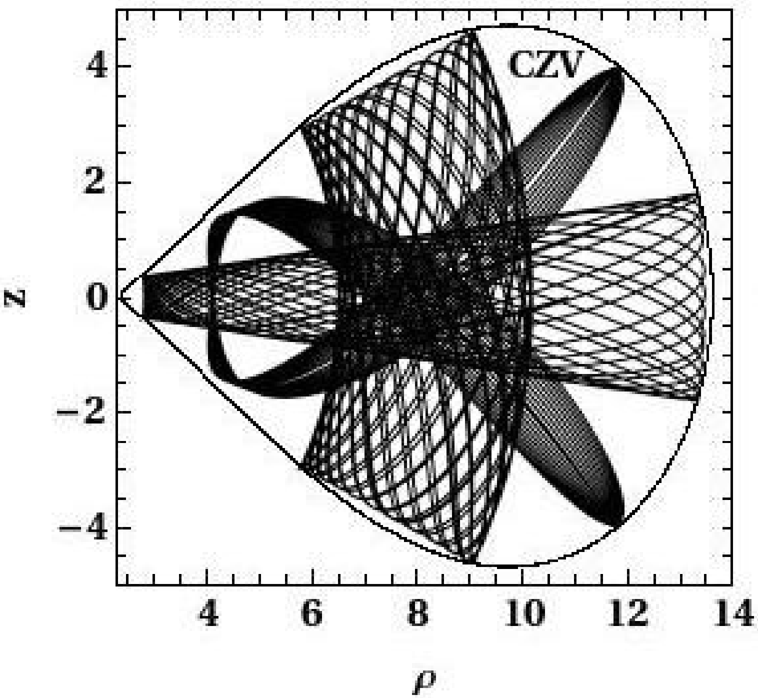
Resonance condition:

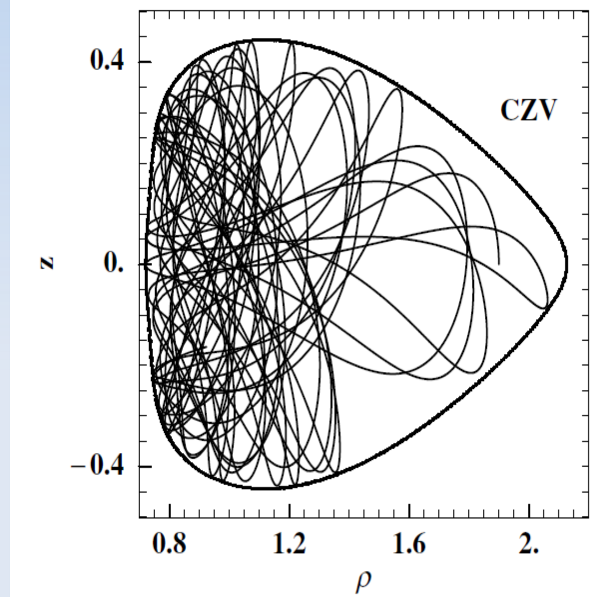
$$\sum_{i=1}^{n} k_i \omega_i = 0, \; k_i \in \mathbb{Z}, \quad |k| = \sum_{i=1}^{n} |k_i| \neq 0 \quad .$$

n=2

Orbits on a surface of section

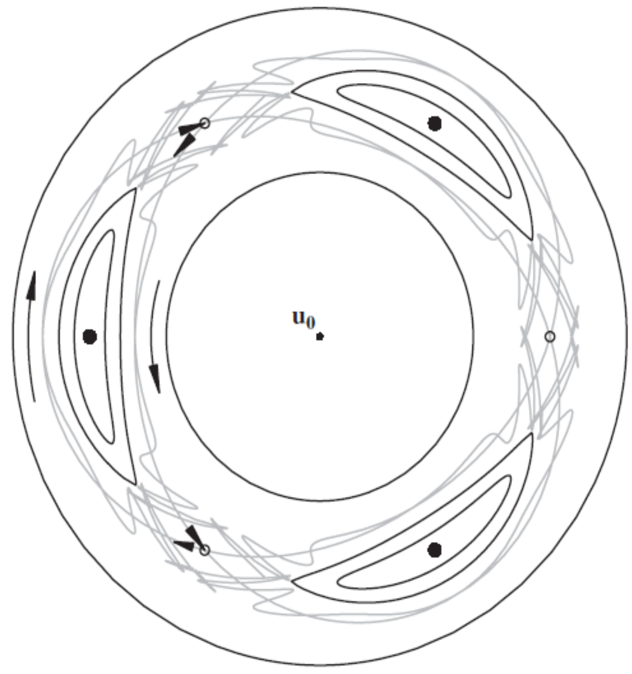


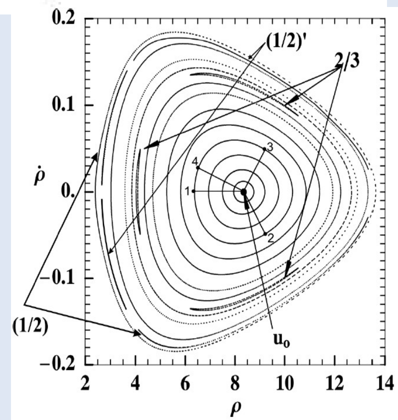


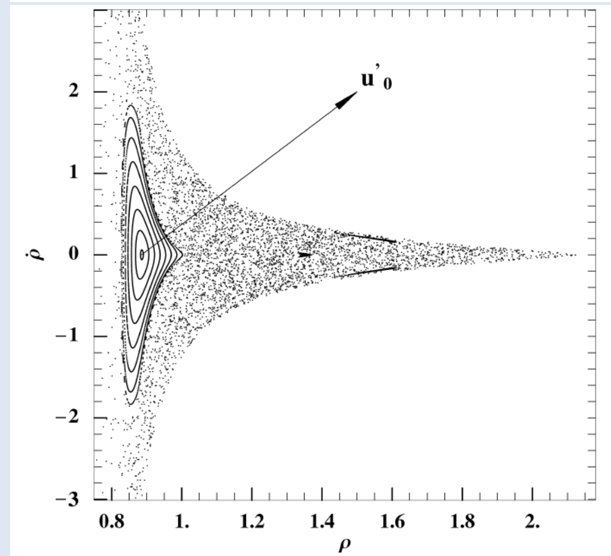


Curve of Zero Velocity (CZV)

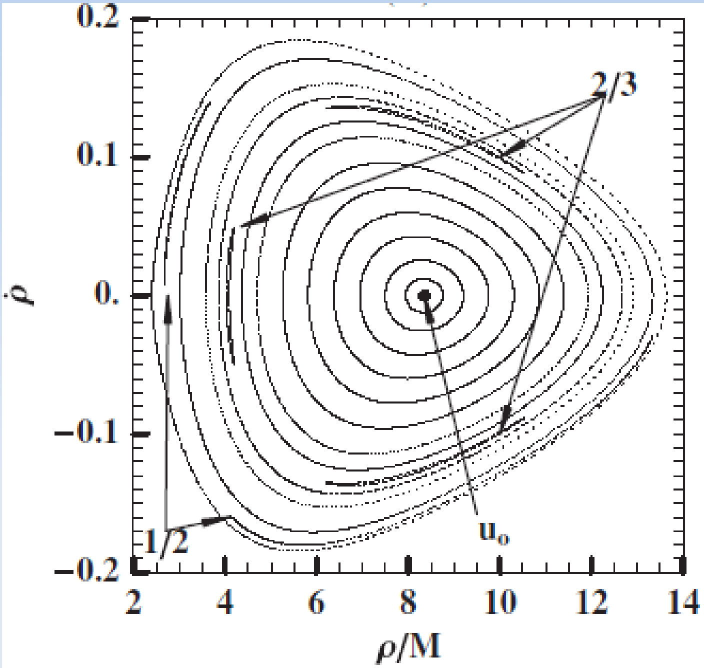
$z=0$, $dz/d\tau>0$

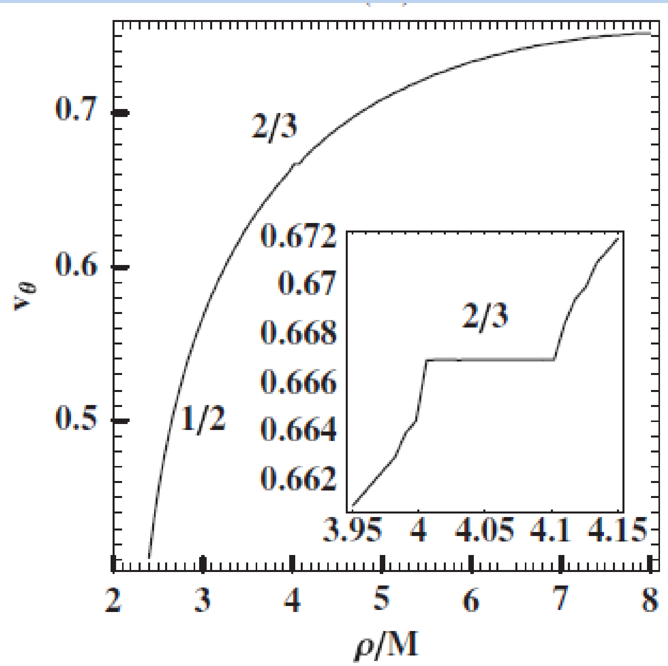






Effect of resonances



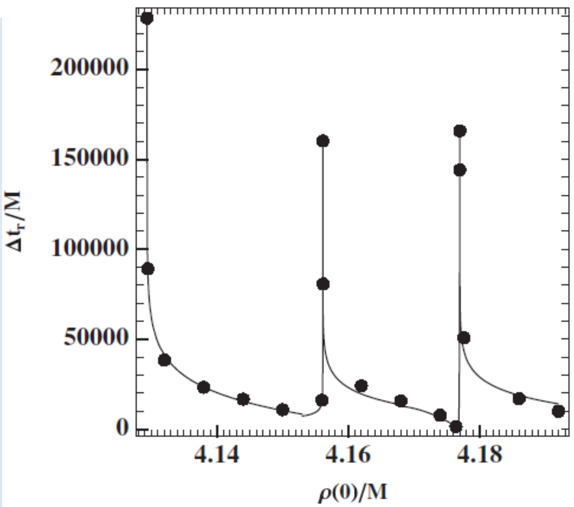


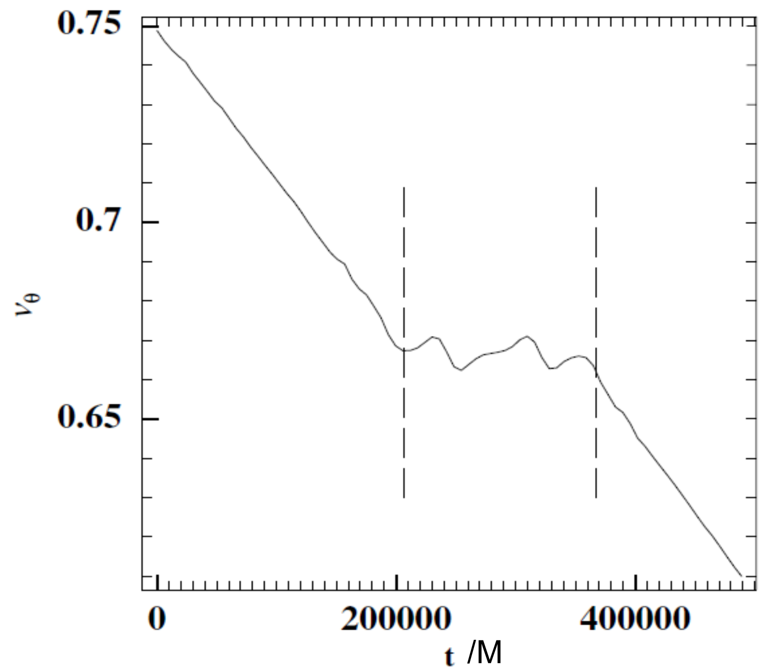
We modified the rates of energy and angular momentum loss proposed by Gair & Glampedakis (PRD, 2006) for a Kerr black hole according to the prescription given in Gair, Li & Mandel, PRD (2008)

$$E(t) = E(0) + \left.\frac{dE}{dt}\right|_0^t$$

$$L_z(t) = L_z(0) + \left.\frac{dL_z}{dt}\right|_0^t$$

The adiabatic approximation (radiation reaction included)

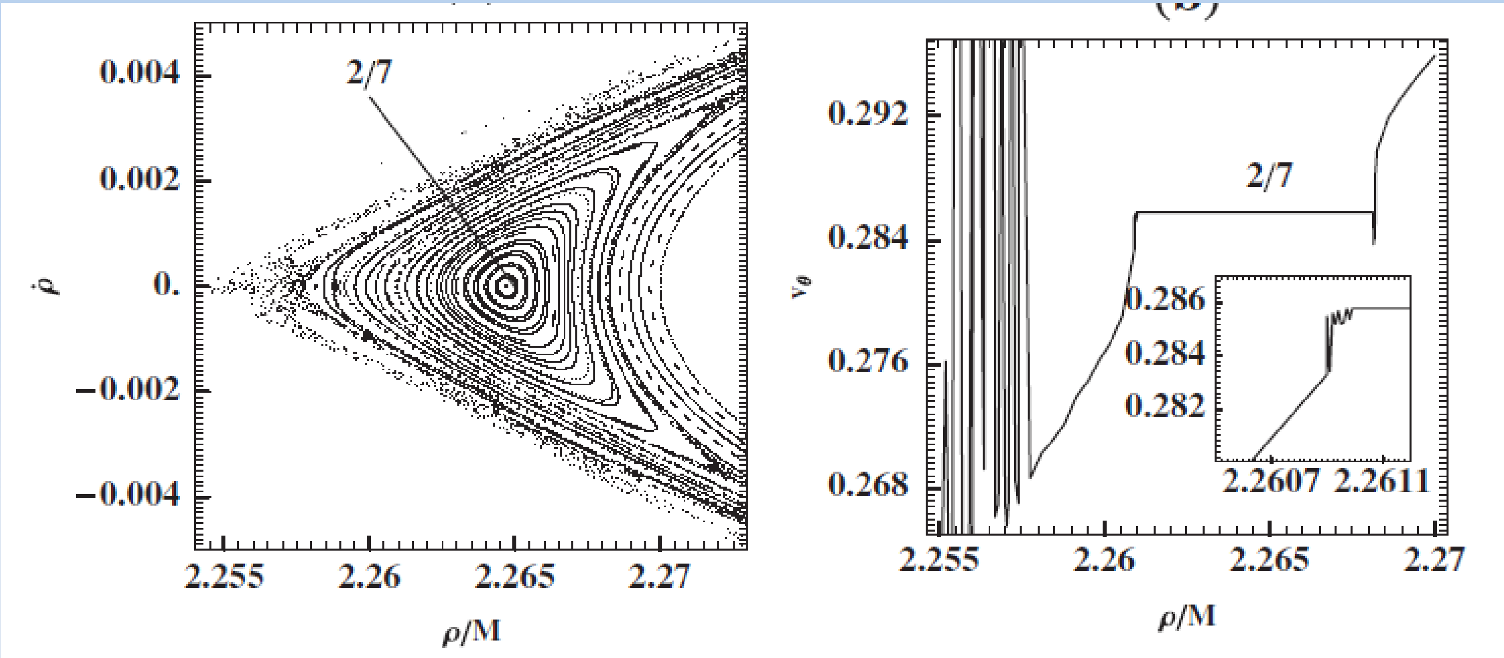




The unit of time t is 5 M/M_{Sun} μs. For $M=10^6 M_{Sun}$ a resonance that endures $\Delta t=5\ 10^4$ M corresponds approximately to a week.

Apostolatos, L-G & Contopoulos, PRL (2009)
L-G, Apostolatos & Contopoulos, PRD (2010)
Contopoulos, L-G & Apostolatos, IJBC (2011)

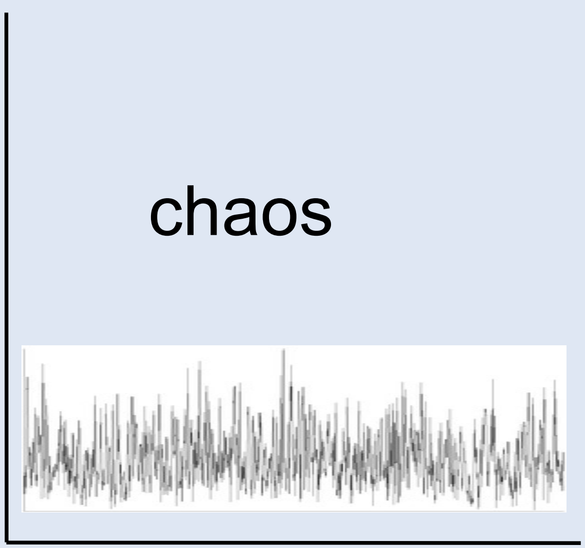
Effect of stickiness



The <u>stickiness</u> concerns chaotic orbits which for various reasons stick for a long time interval in a region, close to an invariant curve, so that their behaviour may resemble that of regular orbits, before extending further away.

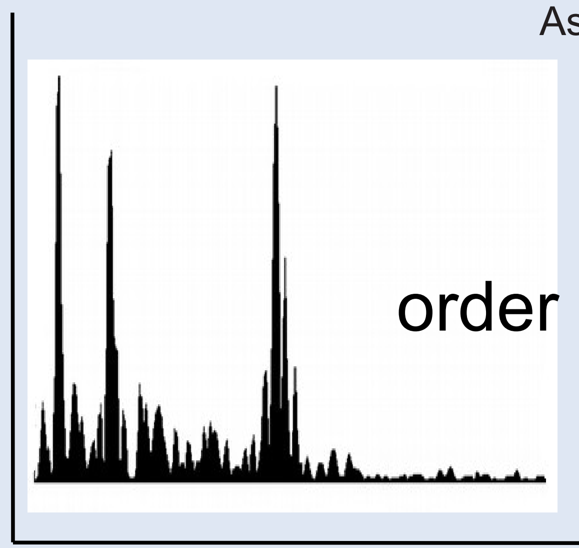
Contopoulos, Order and Chaos in Dynamical Astronomy, Springer (2002)

S(ω)

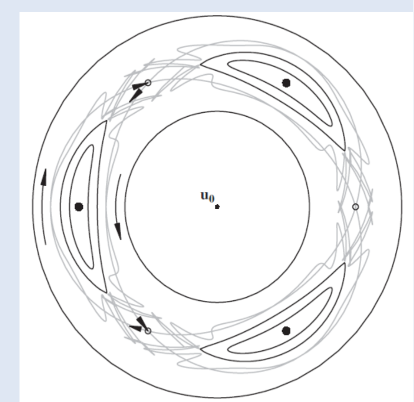


ω

S(ω)



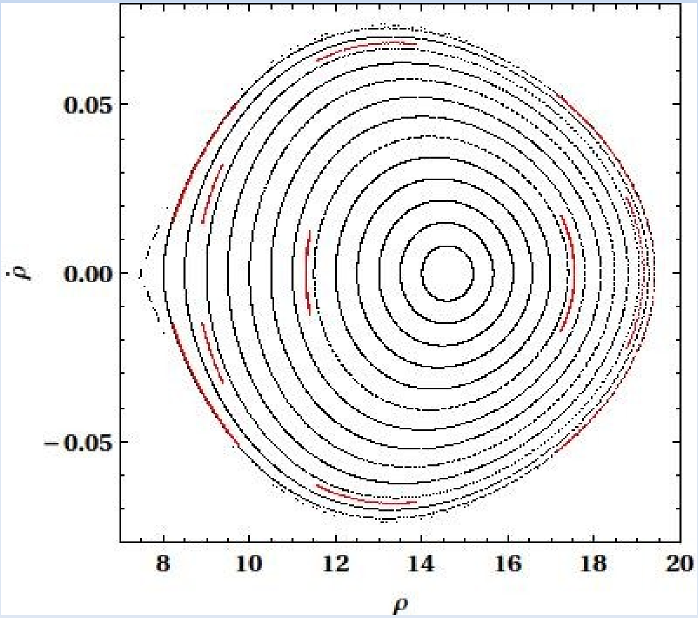
ω

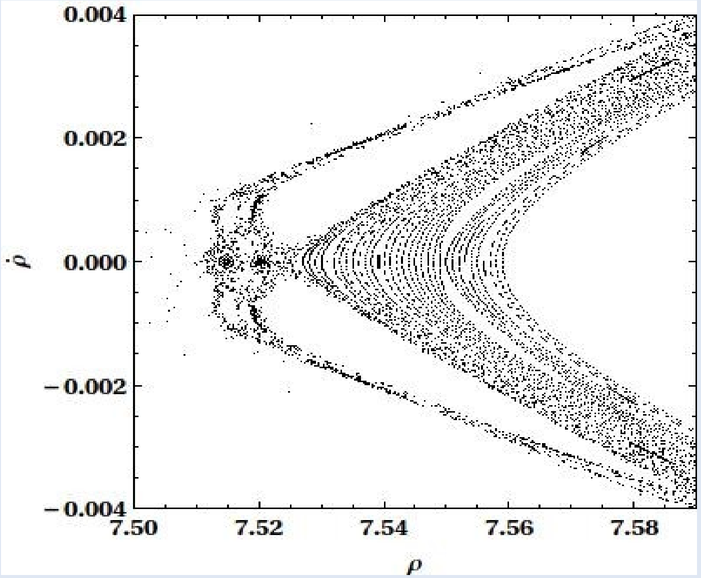


L-G, Apostolatos & Contopoulos, PRD (2010)

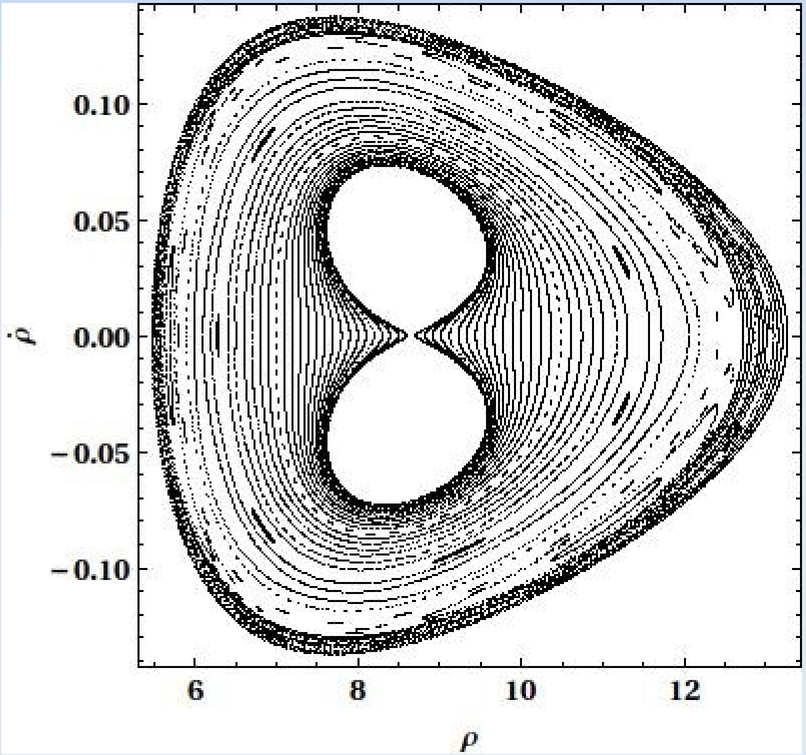
Different metrics similar structures

Zipoy, JMP (1966) Voorhees, PRD (1970) metric





Manko, Sanabria-Gómez, Manko, PRD (2000) metric



Seyrich, L-G, in preparation

L-G, arXiv:1206.0660:

End of part 1: Conclusions

The resonance and the stickiness effect are generic characteristic of the geodesic motion in <u>any</u> non-integrable Hamiltonian describing a stationary and axisymmetric background, i.e. in <u>any</u> (weak) perturbation of Kerr spacetime which remains stationary and axisymmetric.

These phenomena should be taken into account when templates are produced for EMRI systems.

Part two:
Final stages of accretion onto bumpy black holes

G. Contopoulos, M. Harsoula and G. L-G,
“Periodic Orbits and Escapes in Dynamical Systems”,
Celestial Mechanics and Dynamical Astronomy (2012)
arXiv:1203.1010

Final stages of accretion onto non-Kerr compact objects

Bambi & Barausse (PRD, 2011)

Here MN is the class of the Manko-Novikov (CQG, 1992) metric family introduced by Gair, Li & Mandel, PRD (2008).

The MN metric deviates from the Kerr metric by a parameter:

$$q = (Q_{Kerr} - Q)/M^3)$$

Innermost Stable Circular Orbit (ISCO)

The accretion process in our MN spacetimes can be qualitatively different than in a Kerr spacetime. In particular, we find that when the accreting gas reaches the ISCO (i.e. the inner edge of the Novikov-Thorne disk model) there are four qualitatively different possibilities:

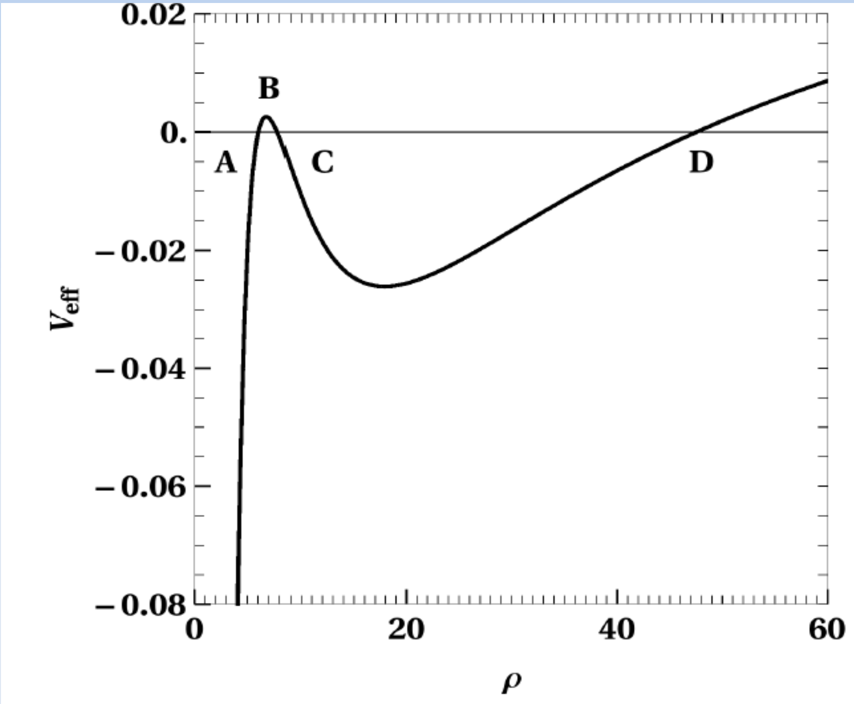
(1) The ISCO is *radially* unstable, and the gas plunges into the compact object remaining roughly on the equatorial plane. This is the same scenario as in the Kerr case.

(2) The ISCO is *radially* unstable and the gas plunges, but does not reach the compact object. Instead, it gets trapped between the object and the ISCO and forms a thick disk.

(3) The ISCO is *vertically* unstable, and the gas plunges into the compact object *outside* the equatorial plane.

(4) The ISCO is *vertically* unstable and the gas plunges, but does not reach the compact object. Instead, it gets trapped between the object and the ISCO and forms two thick disks, above and below the equatorial plane.

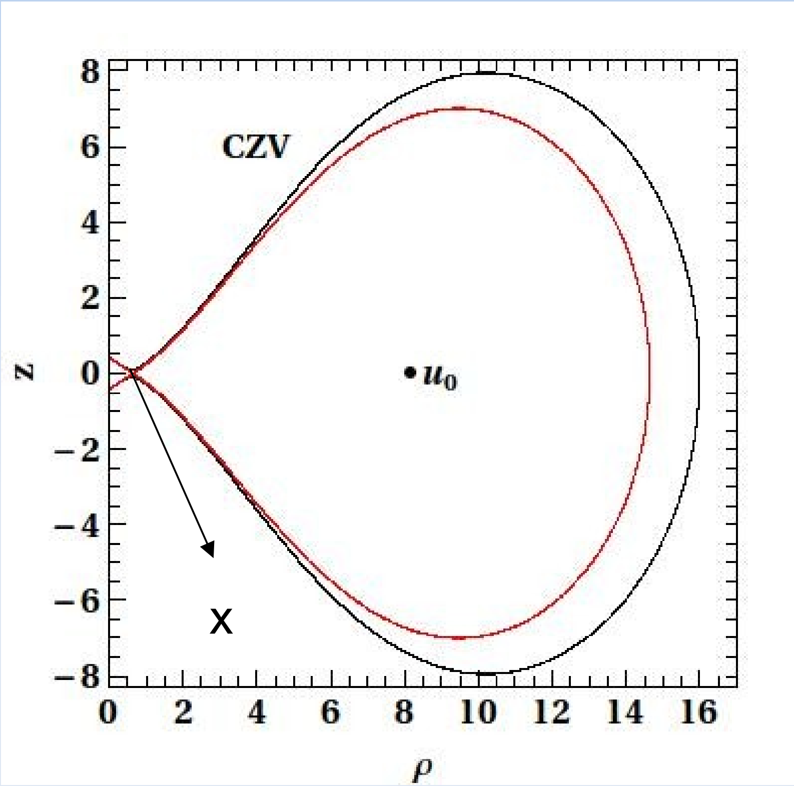
The x-factor in Kerr metric



$$\frac{1}{2}(\dot{\rho}^2 + \dot{z}^2) + V_{\text{eff}}(\rho, z) = 0,$$

$$V_{\text{eff}}(\rho, z) = \frac{1}{2} e^{-2\gamma}\left[f - E^2 + \left(\frac{f}{\rho}(L_z - \omega E)\right)^2\right]$$

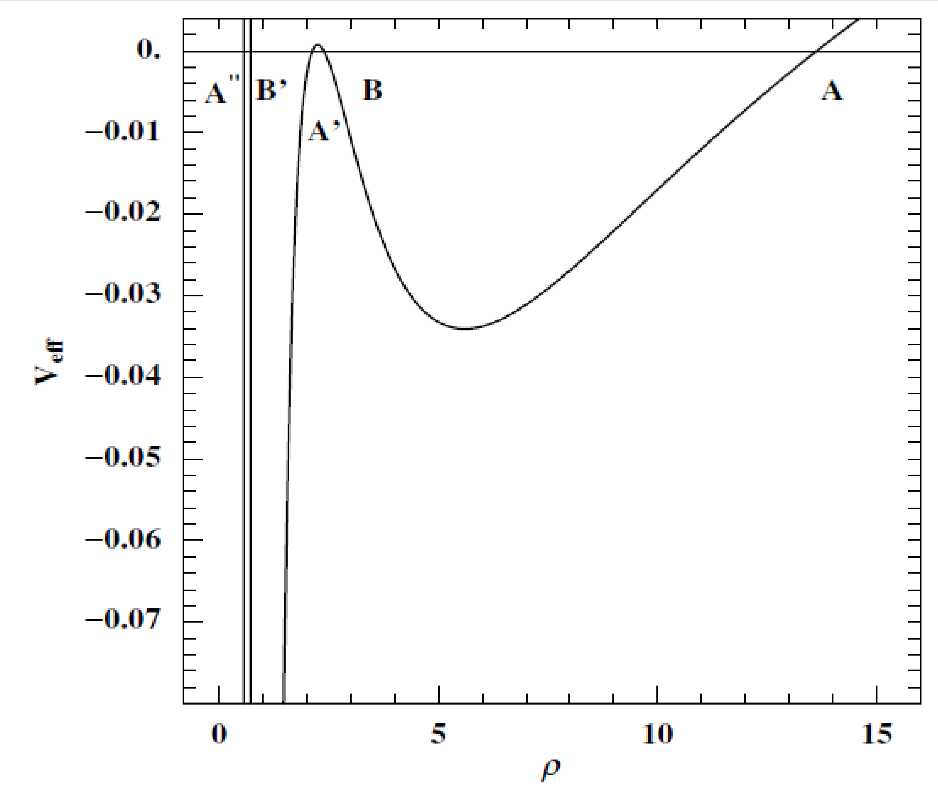
x: denotes an unstable periodic orbit which for proper energy and angular momentum becomes the Innermost Stable Circular Orbit ISCO.

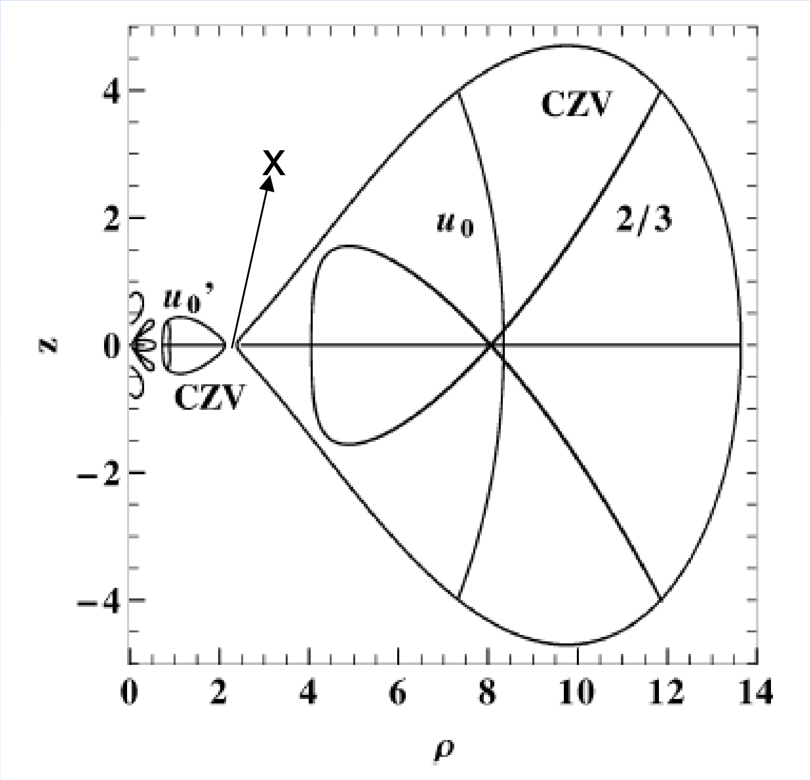


In the case of the Kerr metric from the “x” orbit emanates the separatrix manifold which is the border between the plunging and the bounded orbits.

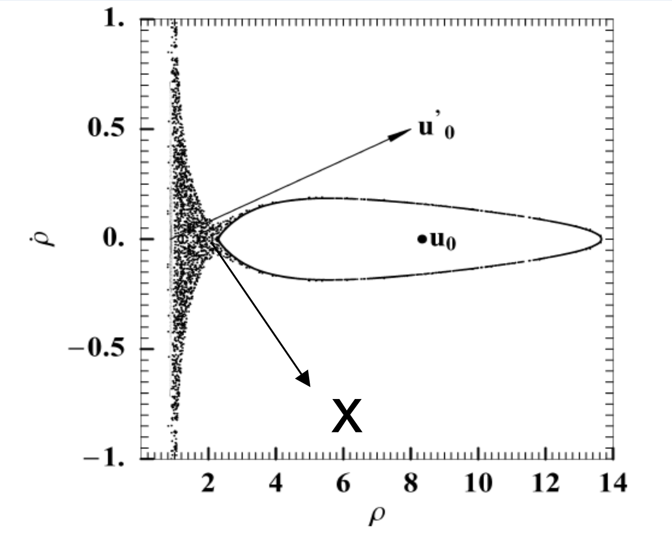
x is a Lyapunov orbit.

The x factor in MN





For MN from “x” emanate the asymptotic manifolds, the structure of chaos underneath the phenomenology of scattered points.
The x is not a Lyapunov orbit.



A simplistic “dust” accretion disk model

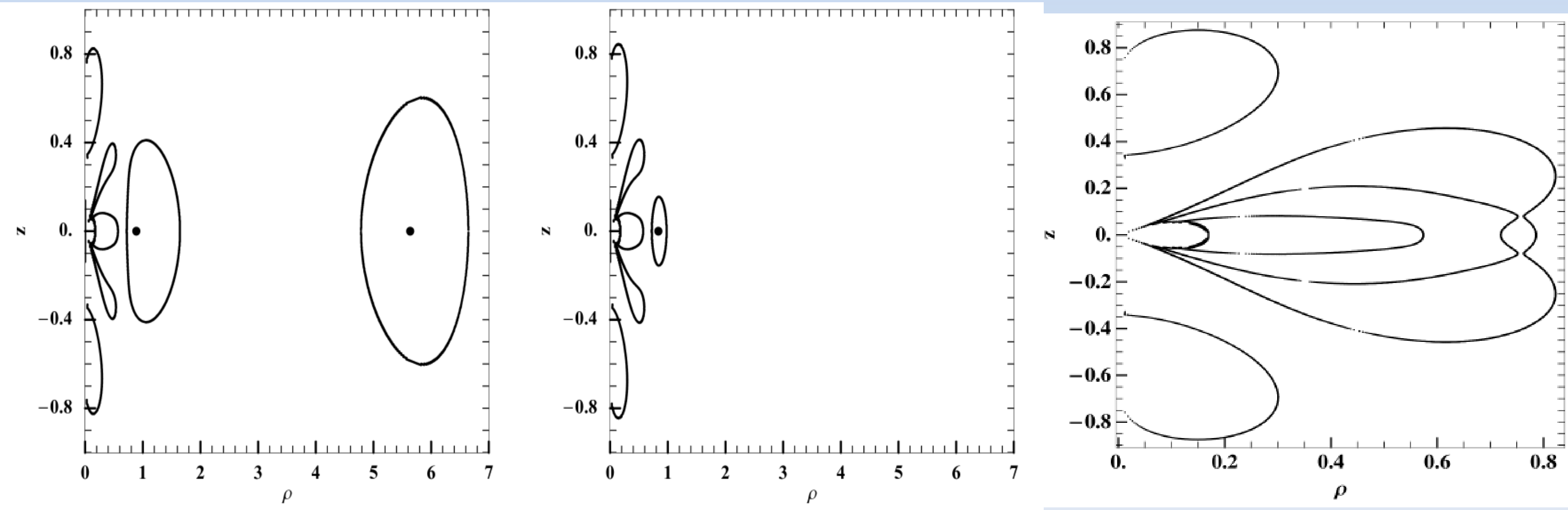
We suppose a accretion disk consisted from a noninteracting collisionless “test” particle fluid.

The accretion disk particles “share” a constant angular momentum L_z and loose for an “unknown” reason energy E.

The particles move approximately along geodesic orbits.

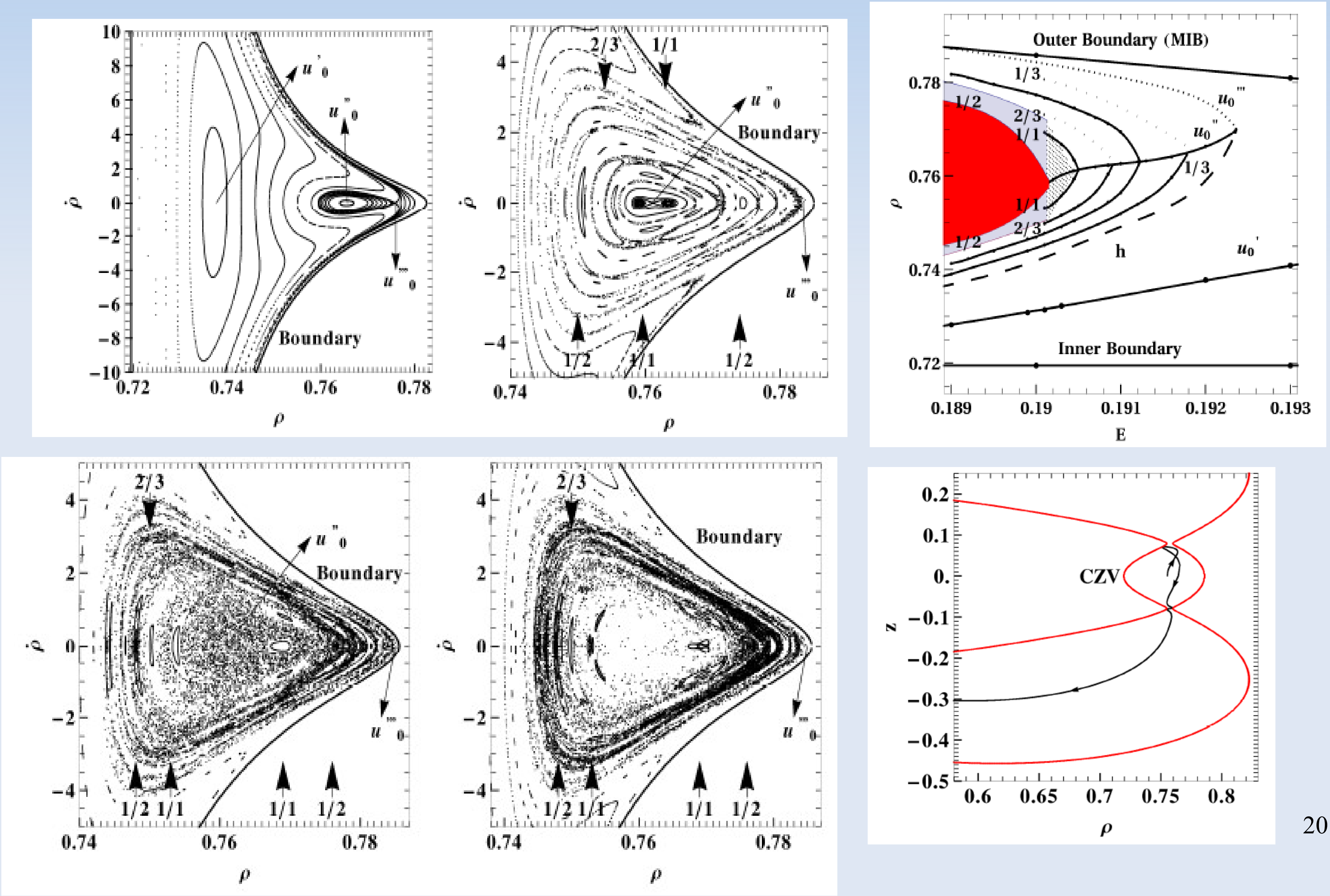
An approximation to the final stage of accreting matter around a “bumpy” black hole or of a black hole background perturbed by the accreting matter itself.

CZV for different E

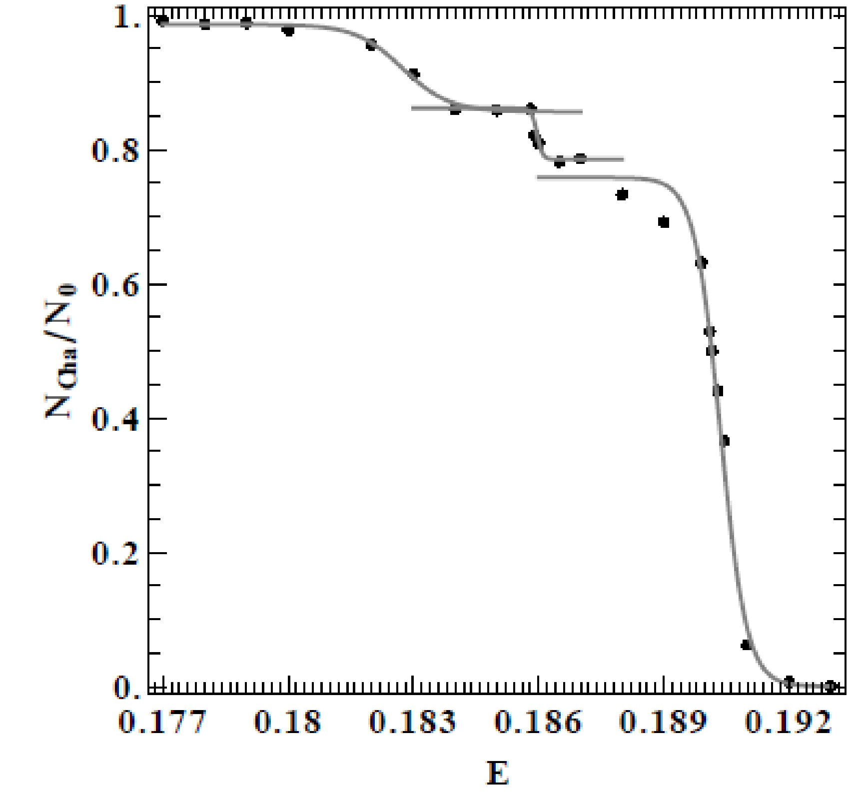


Reducing energy

The final countdown

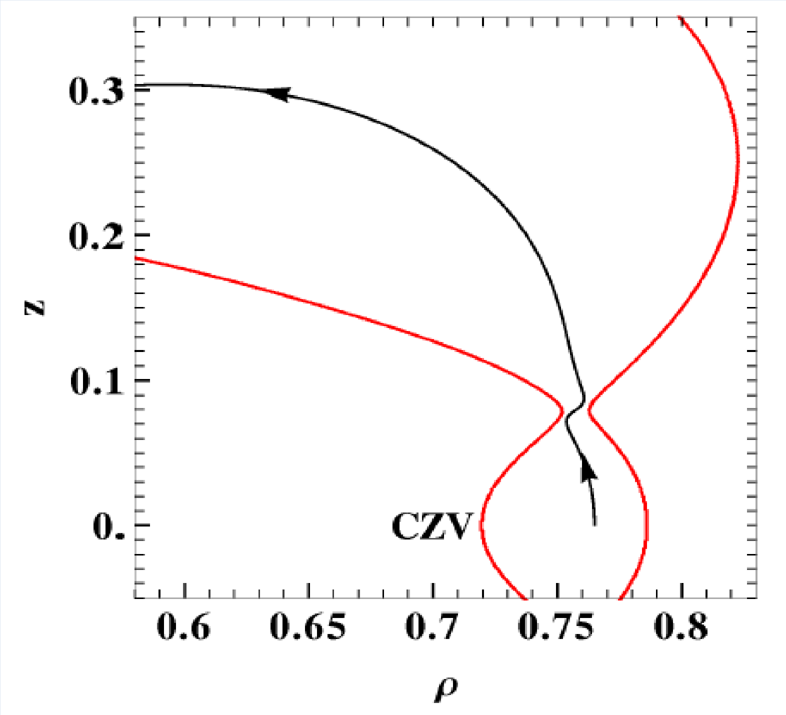


Phase Transition from order to escaping chaos



The percentage of test particles following the chaotic plunging geodesic orbits as energy reduces.

If there is a MN compact object, the vertical outflow of matter should be visible at certain electromagnetic wavelengths



Thank you!