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Generalized Misner-Sharp mass and dynamical black holes in Lovelock gravity

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References

[1] HM, Willison & Ray, CQG28, 165005 (2011)
[2] HM & Nozawa, PRD77, 064031 (2008)
[3] Nozawa & HM, CQG25, 055009 (2008)
[4] HM, PRD73, 104004 (2006)

Motivation

- Lovelock gravity research could clarify
  - (1) Generic properties of gravity
  - (2) Why 4 is so special
- The most general quasi-linear 2nd-order theory for arbitrary n
  - Ghost-free nature
  - 0^{th} order: Λ, 1^{st} order: GR, 2^{nd} order: Gauss-Bonnet

Lovelock action

Coupling constants

$$I = \frac{1}{2\kappa_n^2}\int \mathrm{d}^n x\sqrt{-g}\sum_{p=0}^{[n/2]}\alpha_{(p)}\mathcal{L}_{(p)}$$

$$\mathcal{L}_{(p)} := \frac{1}{2^p}\sqrt{-g}\,\delta^{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}_{\rho_1\cdots\rho_p\sigma_1\cdots\sigma_p} R_{\mu_1\nu_1}{}^{\rho_1\sigma_1}\cdots R_{\mu_p\nu_p}{}^{\rho_p\sigma_p}$$

$$\delta^{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}_{\rho_1\cdots\rho_p\sigma_1\cdots\sigma_p} := \frac{1}{p!}\delta^{\mu_1}_{[\rho_1}\cdots\delta^{\mu_p}_{\rho_p}\delta^{\nu_1}_{\sigma_1}\cdots\delta^{\nu_p}_{\sigma_p]}.$$

Up to [(n-1)/2]-Lovelock Lagrangian contributes to dynamics

Examples up to the 2nd order

Cosmological constant $\mathcal{L}_{(0)} := 1,$

Einstein-Hilbert $\mathcal{L}_{(1)} := R,$

Gauss-Bonnet $\mathcal{L}_{(2)} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$

=> non-trivial for n>4

Lovelock equations

$$\mathcal{G}_{\mu\nu} := \sum_{p=0}^{[n/2]} \alpha_{(p)} G^{(p)}_{\mu\nu} = \kappa_n^2 T_{\mu\nu},$$

$$G^{\mu(p)}_{\ \nu} := \delta^{\mu\eta_1\cdots\eta_p\zeta_1\cdots\zeta_p}_{\nu\rho_1\cdots\rho_p\sigma_1\cdots\sigma_p} R_{\eta_1\zeta_1}{}^{\rho_1\sigma_1} \cdots R_{\eta_p\zeta_p}{}^{\rho_p\sigma_p}$$

Examples up to the 2nd order

$$G^{(0)}_{\mu\nu} = -\frac{1}{2} g_{\mu\nu},$$

$$G^{(1)}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu},$$

$$G^{(2)}_{\mu\nu} = 2\left(R R_{\mu\nu} - 2R_{\mu\rho}R^{\rho}{}_{\nu} - 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} + R_{\mu}{}^{\rho\sigma\gamma}R_{\nu\rho\sigma\gamma}\right) - \frac{1}{2} g_{\mu\nu}\mathcal{L}_{(2)},$$

Result summary

- In the (spherically) symmetric spacetime, Lovelock equations may be treated in the same way as in GR
- Consequence: Techniques developed in GR may be used also in Lovelock gravity

Ansatz: symmetric spacetime

- n(>2)-dim symmetric spacetime $M^2 \times K^{n-2}$
  - M^2 is a Lorentzian
  - K^{n-2} is maximally symmetric (curvature k=1,0,-1)
  - Metric: $$g_{\mu\nu}dx^\mu dx^\nu = g_{ab}(y)dy^a dy^b + r^2(y)\gamma_{ij}(z)dz^i dz^j,$$

  M^2 K^{n-2}

- General matter field on this spacetime

$$T_{\mu\nu}dx^\mu dx^\nu = T_{ab}(y)dy^a dy^b + p(y)r^2(y)\gamma_{ij}dz^i dz^j,$$

M^2 K^{n-2}

Generalized Misner-Sharp mass

D_a: covariant derivative on M^2

- Generalized Misner-Sharp mass (HM & Nozawa '09)

$$m_{\rm L} := \frac{(n-2)V^{(k)}_{n-2}}{2\kappa_n^2}\sum_{p=0}^{[n/2]}\tilde{\alpha}_{(p)} r^{n-1-2p}[k-(Dr)^2]^p,$$

where $\tilde{\alpha}_{(p)} := \dfrac{(n-3)!\alpha_{(p)}}{(n-1-2p)!}$,

- Lovelock eq simplified as

Energy flux   Work term

$$\mathrm{d}m = A\psi_a \mathrm{d}x^a + P\mathrm{d}V.$$

$A := V^k_{n-2} r^{n-2}$ (Area of K^{n-2}), $\psi^a := T^a{}_b D^b r + P D^a r$ (Local Bondi flux), $P := -\frac{1}{2}T^a{}_a$ (Energy)

- m_L converges to ADM mass in the asymp. flat spacetime
- Under the dominant energy condition
  - **Monotonic** in the untrapped region
  - **Positive** on an untrapped hypersurface with a regular center

Vacuum solutions

- **m_L is constant in vacuum**
- Schwarzschild-Tangherlini-type solution (Zegers `00)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\gamma_{ij}dz^i dz^j.$$

$$\tilde{M} = \sum_{p=0}^{[n/2]} \tilde{\alpha}_{(p)} r^{n-1-2p}[k - f(r)]^p,$$

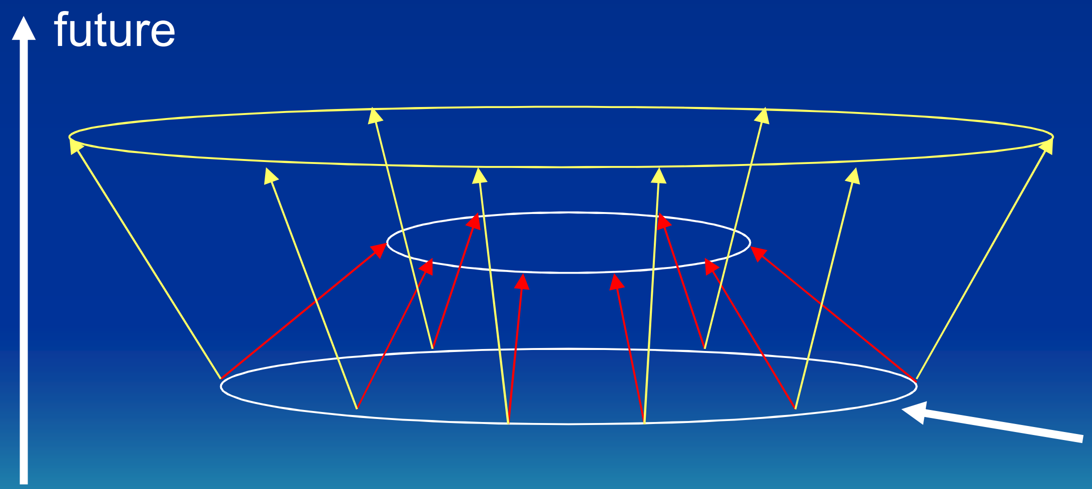
- There may be multiple max. symmetric vacua (M=0)
- Birkhoff theorem under
  - C^2 metric
  - $(Dr)^2$ is non-zero
  - The theory does not admit degenerate vacua

A quasi-local definition of a BH

- Consider a compact spatial (n-2)-surface S
- Area expansions θ_+, θ_- along two independent null rays orthogonal to S

$$\theta_+ := (n-2)r^{-1}r_{,v},$$
$$\theta_- := (n-2)r^{-1}r_{,u}.$$

Yellow: ``outgoing” null rays orthogonal to S
Red: ``ingoing” null rays orthogonal to S



Untrapped surface

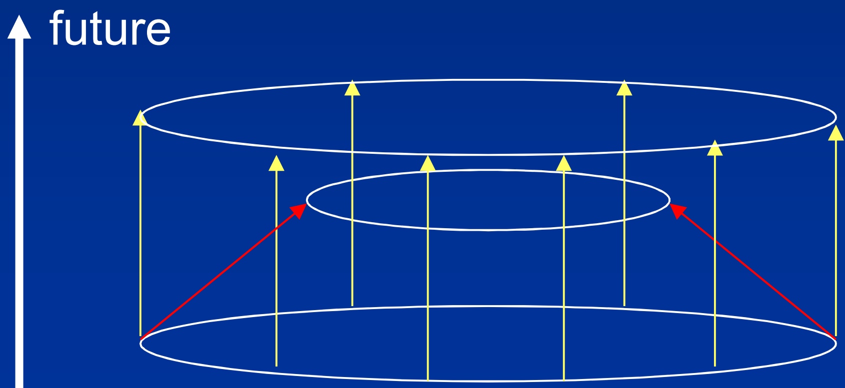
$\theta_+ > 0,$ $\theta_- < 0;$

A compact spatial (n-2)-surface S

FOTH as a dynamical BH horizon

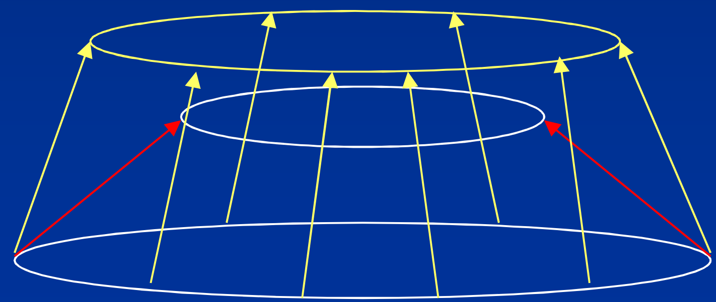
- **Future outer** trapping horizon (Hayward '94)

$$\underbrace{\theta_+ = 0,}_{\text{marginality}} \underbrace{\theta_- < 0,}_{\text{future}} \underbrace{\partial_- \theta_+ < 0}_{\text{outer}}$$



Future marginal surface

$\theta_+ = 0,$ $\theta_- < 0;$



Future trapped surface

$\theta_+ < 0;$ $\theta_- < 0;$

Einstein-Gauss-Bonnet gravity: Two branches of solutions

- Let us focus on EGB gravity (2nd-order Lovelock)
- Definition of g-Misner-Sharp mass gives

$$\frac{2}{(n-2)^2} r^2 e^f \theta_+\theta_- = -k - \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1+\frac{8\kappa_n^2\tilde{\alpha} m}{(n-2)V^k_{n-2} r^{n-1}} + 4\tilde{\alpha}\tilde{\Lambda}}\right)$$

- Two **<u>branches</u>** of solutions
  - **GR branch (upper sign): GR limit exits**

$$\frac{2}{(n-2)^2} r^2 e^f \theta_+\theta_- = -k + \frac{2\kappa_n^2 m}{(n-2)V^k_{n-2} r^{n-3}} + \tilde{\Lambda} r^2.$$

  - **non-GR branch (lower sign): no GR limit**

Pathological behavior in non-GR branch

- For a radial null vector k,

$$\pm R_{\mu\nu}k^{\mu}k^{\nu}\sqrt{1+\frac{8\kappa_n^2\tilde{\alpha}m}{(n-2)V^k_{n-2}r^{n-1}}+4\tilde{\alpha}\tilde{\Lambda}}=\kappa_n^2 T_{\mu\nu}k^{\mu}k^{\nu}$$

- Raychaudhuri equation

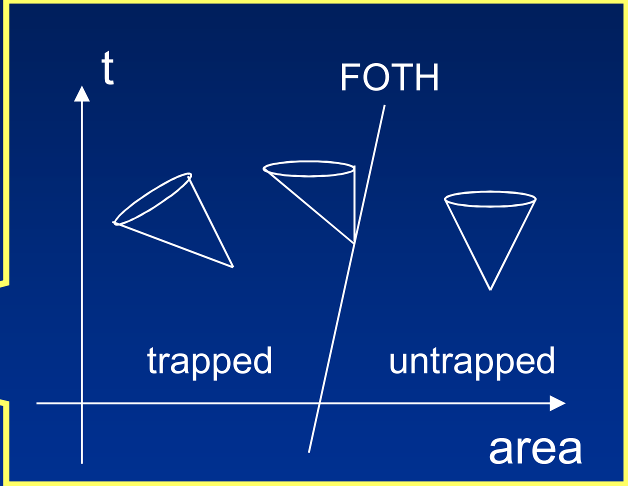
$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda}=-\frac{1}{n-2}\theta^2-R_{\mu\nu}k^{\mu}k^{\nu}$$

in GR → $T_{\mu\nu}k^{\mu}k^{\nu}$ >0(NEC)

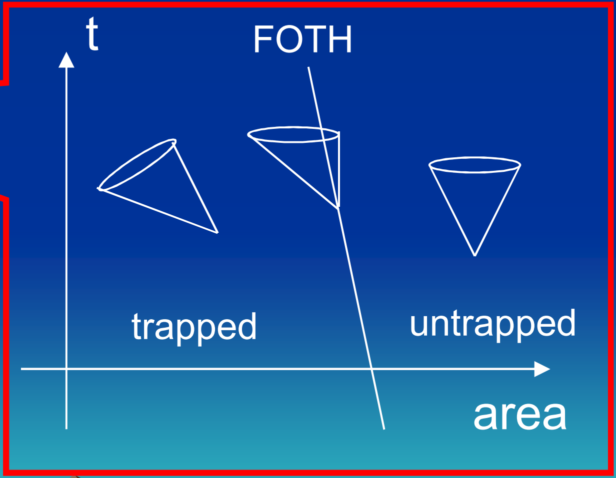
  – In GR, NEC means the null convergence condition ($R_{ab}k^a k^b>0$)
- Two branches have opposite properties
  – **GR branch**: well-behaving under NEC ($R_{ab}k^a k^b>0$)
  – **Non-GR branch**: ill-behaving under NEC ($R_{ab}k^a k^b<0$)

Signature and area laws

- Under null energy condition, a future outer trapping horizon is
  - Non-timelike in GR branch
    - **The area is non-decreasing**



  - Non-spacelike in non-GR branch
    - **The area is non-increasing**



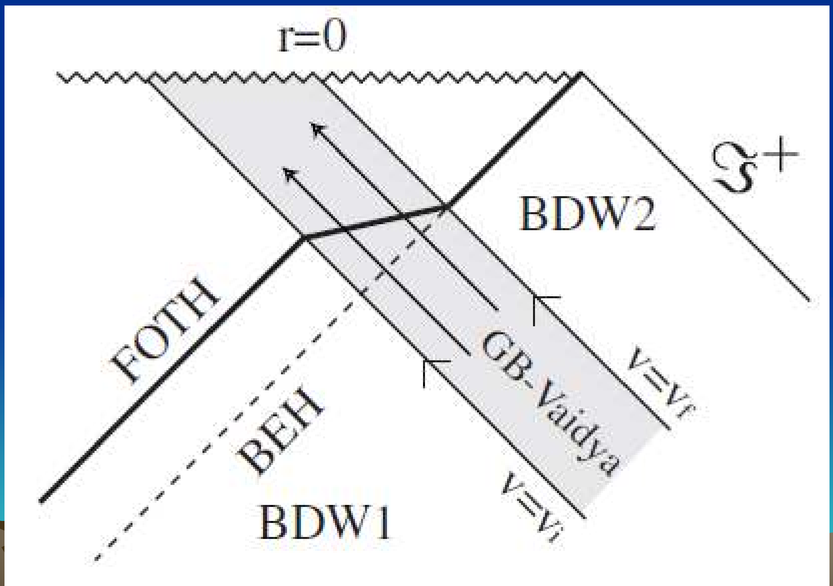
Entropy law

- The Iyer-Wald-Kodama dynamical entropy of a future outer trapping horizon is **<u>non-decreasing</u>** along its generator **in BOTH branches**
  - Under null energy condition

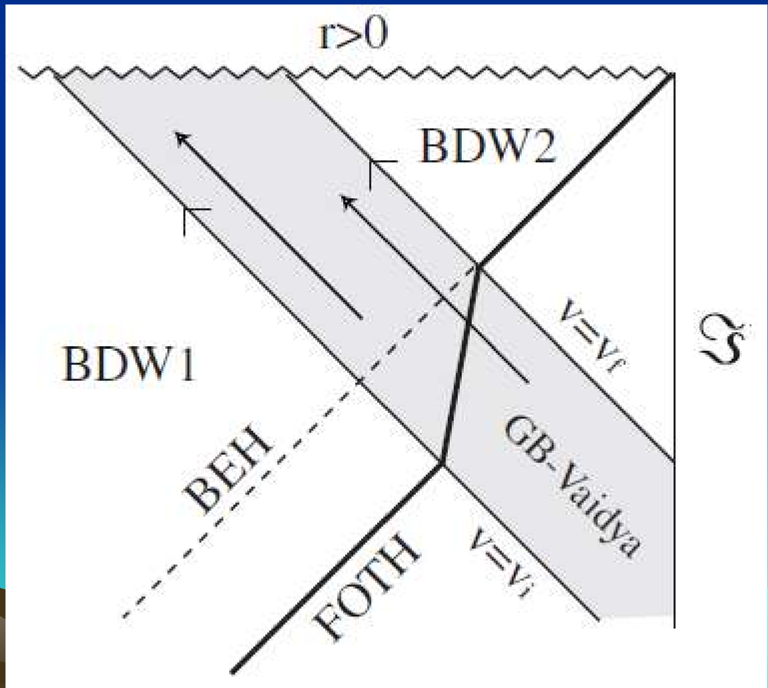
$$S_{\rm TH} = \frac{V^k_{n-2} r_{\rm h}^{n-2}}{4G_n}\left[1 + \frac{2(n-2)(n-3)\alpha k}{r_{\rm h}^2}\right]$$

Deviation from Bekenstein-Hawking formula

GR branch



Non-GR branch



Summary

- Generalization of the Misner-Sharp quasi-local mass
  - The system can be treated in a similar manner
- Dynamical BH defined by a future outer trapping horizon
  - Einstein-Gauss-Bonnet gravity
  - GR branch: area & entropy increase
  - Non-GR branch: area decreases but entropy increases
  - Lovelock with non-negative coupling constants
    - Area & entropy increase
    - Area & entropy law with general coupling constants remain open

Application for Gravitational Collapse

- Spherical gravitational collapse of a dust fluid
  - In GR, n=4, Naked Singularity formation from analytic initial data (Christodoulou `84), n>5, BH formation (Goswami-Joshi `07)
  - In EGB, NS for n<9 and BH for n>9 (GR-branch, HM `06)
  - In full Lovelock, NS is generic (Ohashi-Shiromizu-Jhingan `11)
  - Non-negative coupling constants assumed
- Scalar field collapse in Lovelock is still incomplete
  - In GR with n=4, BH formation is generic (Christodoulou)
  - In EGB?, Lovelock?

FIN