Statefinder diagnosis and dark energy models

By

Mohammad Malekjani

Bu Ali Sina University, Hamedan, Iran

Recent astronomical data obtained by SNe Ia , WMAP , SDSS [3] and X-ray [4] experiments suggest that our universe expands under an accelerated expansion.

In the framework of standard cosmology, a dark energy component with negative pressure is responsible for this acceleration.

A major puzzle of cosmology is the nature of dark energy and therefore many theoretical models have been proposed to interpret the behavior of dark energy.

The earliest and simplest model for dark energy is the Einstein's cosmological constant, In which the equation of state is time independent. This model suffers from theoretical problems.

Theoritical problems of Cosmological constant:

Fine-tuning: why the vacuum energy density is so small.

Cosmic coincidence: why the vacuum energy and dark matter are nearly equal at the present time.

Besides the Einstein's cosmological constant, Nowadays, there are many dynamical Models , in which the equation of state is time dependent, to interpret the cosmic Acceleration.

These models are classified in to two categories:

(i) Scalar fields including: quintessence , phantom , quintom , K-essence , tachyon , dilaton and so forth.

(ii) Interacting dark energy models such as holographic, agegraphic And chaplygin gas model.

Statefinder diagnosis:
Since the various dark energy models have been proposed to interpret the puzzle Of dark energy, we need an diagnostic tool to discriminate between them.

The Hubble parameter $H = \dot{a}$/a (first time derivative of scale factor) and the deceleration parameter $q = -\ddot{a}H^2$/a (second time derivate of scale factor) Can not discriminate the dark energy models. Since for all these models, we have $H > 0$ (expansion) and $q < 0$ (accelerated expansion).

Therefore, we need a higher order of time derivative of scale factor to remove this Degeneracy.
Sahni and Alam (2003) by using the third time derivative of scale factor, introduced the statefinder pair {s,r} in order to remove the degeneracy of H and q at the present time.
The statefinder pair is given by

$$r = \frac{\dddot{a}}{aH^3}, s = \frac{r-1}{3(q-1/2)}$$

The statefinder parameters are the geometrical parameters, since they only depends On the scale factor.

Holographic dark energy (HDE)

A holographic DE model is based on holographic principle . According to the holographic principle, the numbers of degrees of freedom in bounded system should be finite and corresponds to the area of its boundary. A holographic DE with apparent or particle horizon can not obtain the accelerated expansion of the universe, but with event horizon can interpret the cosmic acceleration.

Agegraphic dark energy (ADE)

The agegraphic model has been proposed based on the uncertainty relation of quantum mechanics together with general relativity . Recently, this model Has been extended to new agegraphic model to explain the matter-dominated era.

Chaplygin gas and Polytropic gas models

The chaplygin gas is a phenomenological model and has a dual role in the history
Of the universe. It can be as a EoS of matter at the early time and therefore can
interpret the decelerated expansion of the early universe. Also, it behaves as a dark energy
Later, and can indicate the accelerated expansion of the current universe.

This model suffer from the singularity (at a characteristic scale factor
Energy density tends to infinity.)

The advantage of these models is that they can cross the phantom divide without a need
Of interaction between dark matter and dark energy.

The dynamical dark energy models that have been investigated by statefinder diagnostic tool are:
The quintessence DE model (Sahni, et. Al., 2003, Alam, et. Al., 2003),
The interacting quintessence models [Zimdahl & Pavon 2004, Zhang, X, 2005],
The holographic dark energy models [Zhang, X. 2005, Zhang, J, et al. 2007],
The holographic dark energy model in non-flat universe [Setare, M.R., et al. 2007],
The phantom model [Chang, B.R., et al. 2007],
The tachyon [Shao, Y., Gui, Y., 2007],
The generalized chaplygin gas model [Malekjani, M., et al. 2011],
The interacting new agegraphic DE model in flat and non-flat universe [Zhang, L. et al. 2010. & Khodam-Mohammadi, A., Malekjani, M., 2011],.
The agegraphic dark energy model with and without interaction in flat and non-flat universe [Wei, H., Cai, R.G., 2007 & Malekjani, M., et al. 2010]
The polytropic gas model (Malekjani. M., Khodam-Mohammadi , A., 2012)
The Ghost DE model (Malekjani, M., kodam-Mohammadi, A., 2012)
are analyzed through the statefinder diagnostic tool.

Here I focus on the polytropic gas DE model.
In stellar astrophysics, the polytropic gas model can explain the equation of state of degenerate white dwarfs, neutron stars and also the equation of state of main sequence stars . The idea of dark energy with polytropic gas equation of state has been investigated by U. Mukhopadhyay and S. Ray in cosmology .

The equation of state (EoS) of polytropic gas is given by

$$p_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}},$$

In the framework of FRW cosmology and using the continuity equation

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_\Lambda)$$

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q,$$

We have

$$\rho_\Lambda = \left(\frac{1}{Ba^{\frac{3(1+\alpha)}{n}} - \tilde{K}} \right)^n,$$

And therefore

$$w_\Lambda = -1 - \frac{a^{\frac{3(1+\alpha)}{n}}}{c - a^{\frac{3(1+\alpha)}{n}}} - \alpha$$

It is clear to see that at the early time in and in the absence of interaction $(\alpha = 0)$, this model mimics the \Lambda CDM model $w_\Lambda \to -1$.

The deceleration parameter in this model is

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}$$

$$q = -1 + \frac{3}{2}\left[\Omega_\Lambda \frac{c(1+\alpha)}{a^{\frac{3(1+\alpha)}{n}} - c} + 1 + \frac{\Omega_k}{3}\right]$$

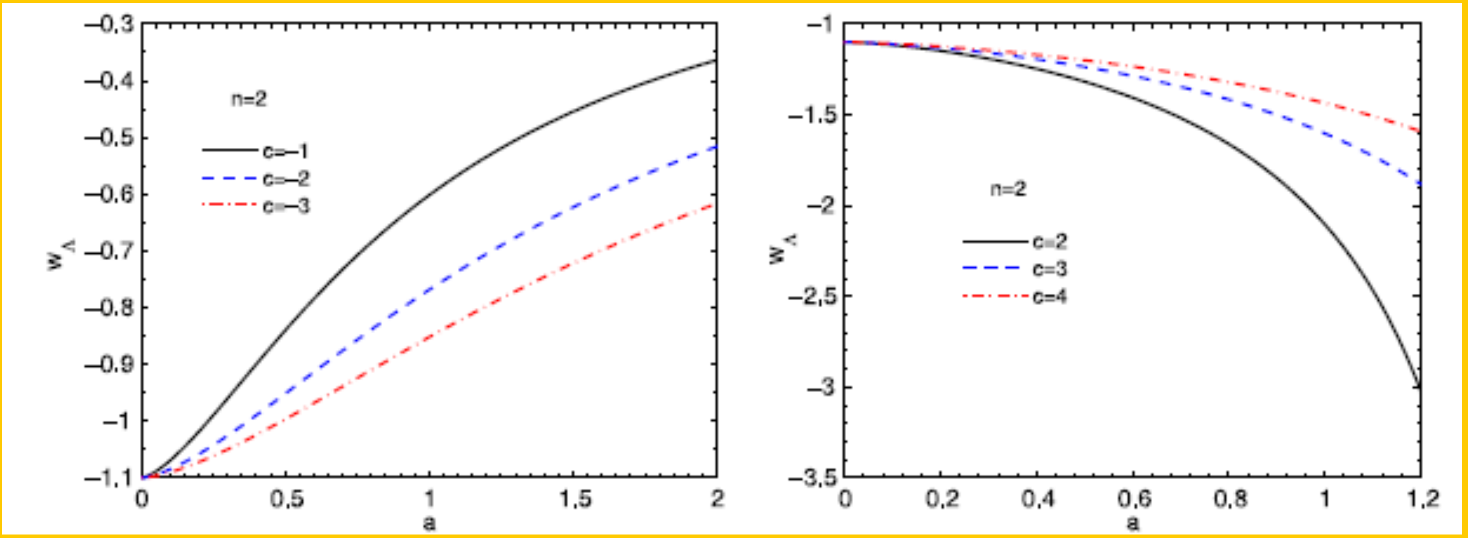


Fig. 1 The EoS parameter w_Λ of interacting polytropic gas model as a function of cosmic scale factor a for different model parameters c and n. The interaction parameter is chosen as $\alpha = 0.1$

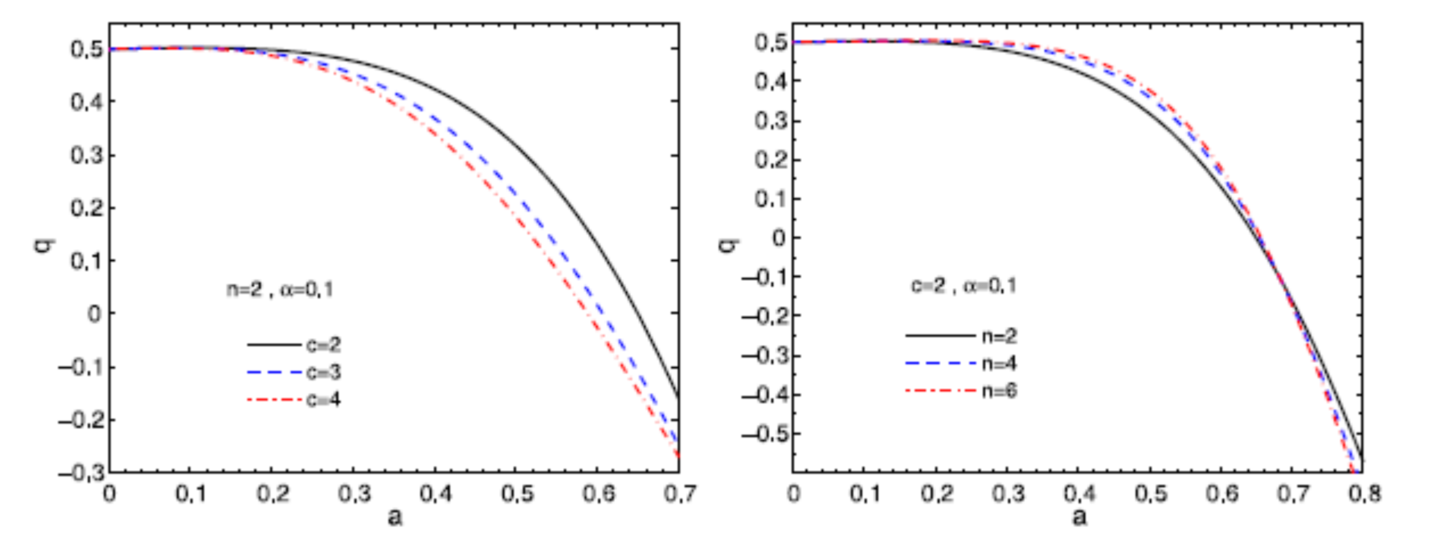


Fig. 3 The evolution of deceleration parameter q versus of scale factor a for interacting polytropic gas model. In *upper left panel*, the parameter n is fixed and the parameter c is varied. In *upper right panel*, we fix c and vary n. In *lower panel*, by fixing the parameters c and n, we vary the interaction parameter α

Statefinder diagnosis for polytropic gas model

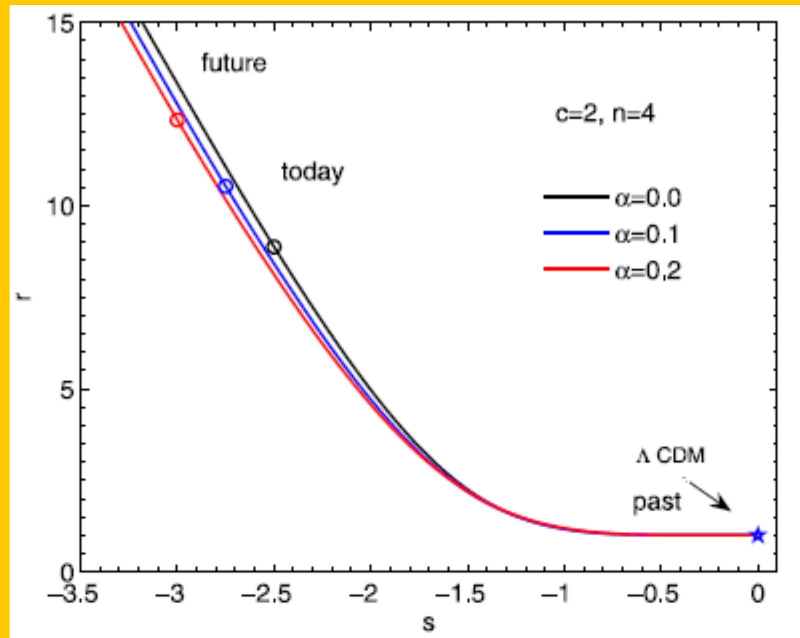
here I consider a flat FRW cosmology with present density parameters

$\Omega_m^0 = 0.3$ $\Omega_\Lambda^0 = 0.7$

$$r = \frac{\dddot{a}}{aH^3}, s = \frac{r-1}{3(q-1/2)}$$

$$r = 1 + \frac{3}{2}\Omega_\Lambda(1+\alpha)\left[3(1+\alpha)(\alpha+w_\Lambda)(1+\alpha+w_\Lambda) - w'_\Lambda\right]$$

$$s = \frac{2}{3}\frac{3\alpha(\alpha+1)^2 + 3\alpha w_\Lambda(2\alpha+w_\Lambda+3) + 3w_\Lambda(1+w_\Lambda) - w'_\Lambda}{\alpha+w_\Lambda}$$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today
α=0.0
α=0.1
α=0,2
Λ CDM
past
r
s
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10
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0
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
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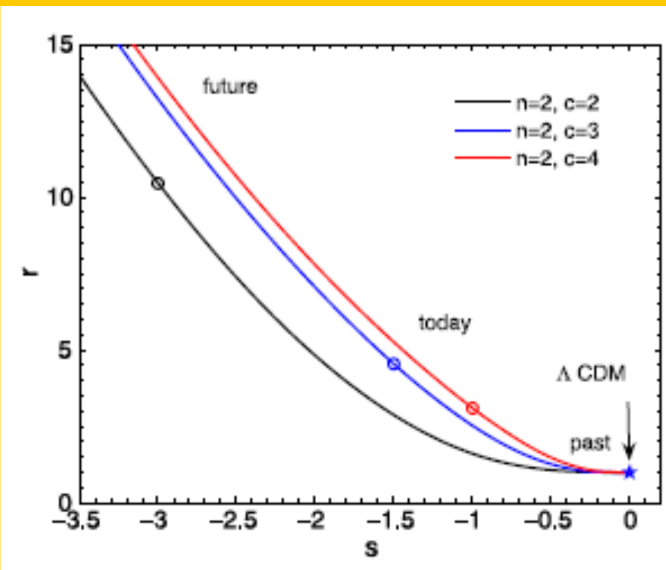
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n=2, c=3
n=2, c=4
today
Λ CDM
past
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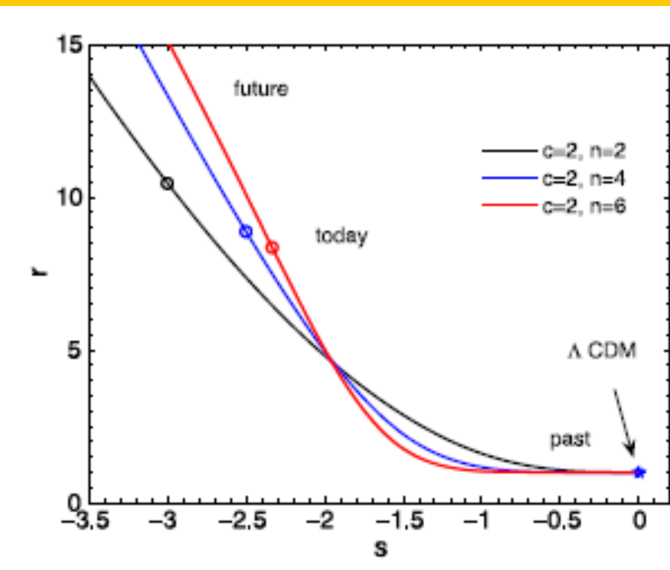
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c=2, n=4
c=2, n=6
today
Λ CDM
past
r
s
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−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

Conclusion

The ploytropic gas model can achieve the phantom regime without a need for interaction.

This model also behaves as a quintessence model for negative value of parameter “c”.

In this model the decelerated expansion (at the early time) and accelerated expansion at the late time can be interpreted.

By means of statefinder, this model is diagnosed for different values of integration Parameter as well as different values of the model parameters.