Parity Violating Hydrodynamics in 2+1 Dimensions

René Meyer

Department of Physics, University of Crete, Heraklion, Greece

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K. Jensen, M. Kaminski, P. Kovtun, R.M., A. Ritz, A. Yarom,
JHEP 1205 (2012) 102 (arXiv:1112.4498 [hep-th])
arXiv:1203.3556 [hep-th]
(see also [S. Minwalla etal. 1203.3544])

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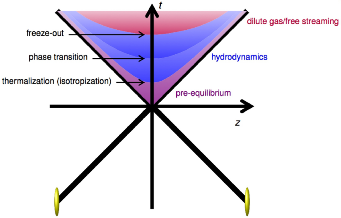

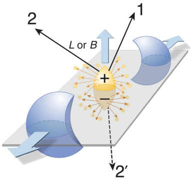

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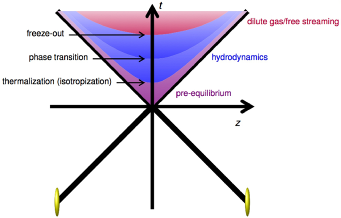

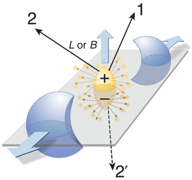

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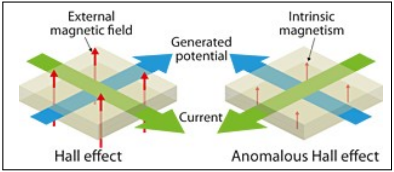

[Haldane PRL 61 1988]

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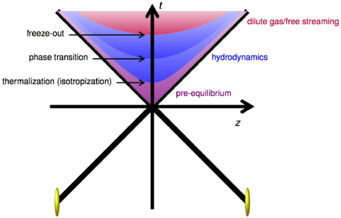

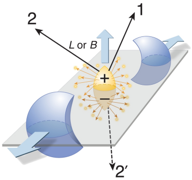

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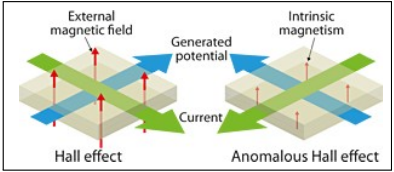

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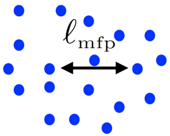
- AdS/CFT $\Rightarrow$ Response of Black Hole Horizons & QNMs

Hydrodynamics

- Intrinsic Scale ℓ_{mfp} in interacting systems



[Pics by Amos Yarom]

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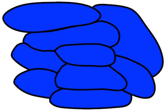
  $\rightarrow$ Continuum description



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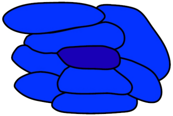

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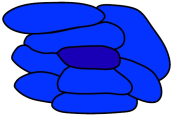

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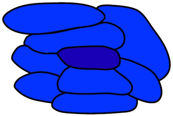

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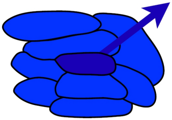

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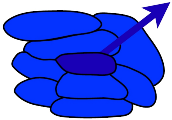

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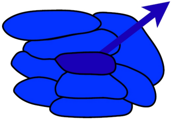

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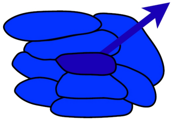

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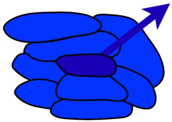

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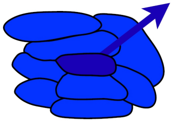

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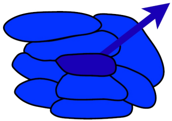
  $d\epsilon = Tds + \mu d\rho$ 1st Law
  $\epsilon + P = sT + \mu\rho$ Euler



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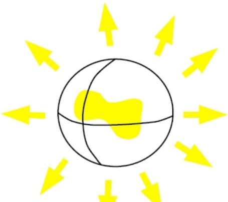
- Out of Equilibrium $\rightarrow$ Derivative Expansion
- Local form of the 2nd Law of thermodynamics:



$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \geq 0$$

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- Parity Odd: Hall Viscosity η_H , P-odd conductivity $\tilde{\sigma}$, Four thermodynamic response parameters $\tilde{\chi}_E, \tilde{\chi}_T, \tilde{\chi}_B, \tilde{\chi}_\Omega$

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\tilde{\chi}_\Omega &= \frac{\partial P}{\partial \epsilon}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu \frac{\partial \mathcal{M}_\Omega}{\partial \mu} - 2\mathcal{M}_\Omega\right) + \frac{\partial P}{\partial \rho}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
\tilde{\chi}_E &= \frac{\partial \mathcal{M}_B}{\partial \mu} - \frac{\rho}{\epsilon + P}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
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- N.B. Coupling to curved backgrounds crucial: $\# u^\mu u^\nu R_{\mu\nu}$

Entropy Current Construction II

- Local Form of Second Law of Thermodynamics

$$J_s^\mu = s u^\mu + \mathcal{O}(\partial) \quad s.t. \quad \nabla_\mu J_s^\mu \geq 0$$

- Unrestricted coefficients:

$$(\eta, \zeta, \sigma) \geq 0\,, \quad (\tilde{\sigma}, \eta_H) \in \mathbb{R}\,, \quad \chi_E = \chi_T = 0$$

- Thermodynamic Response Parameters:

  Magnetizations $\mathcal{M}_B(\mu, T)$, $\mathcal{M}_\Omega(\mu, T)$

$$\begin{aligned}
\tilde{\chi}_B &= \frac{\partial P}{\partial \epsilon}\left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) + \frac{\partial P}{\partial \rho}\frac{\partial \mathcal{M}_B}{\partial \mu} \\
\tilde{\chi}_\Omega &= \frac{\partial P}{\partial \epsilon}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - 2\mathcal{M}_\Omega\right) + \frac{\partial P}{\partial \rho}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
\tilde{\chi}_E &= \frac{\partial \mathcal{M}_B}{\partial \mu} - \frac{\rho}{\epsilon + P}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
T\tilde{\chi}_T &= \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - \frac{\rho}{\epsilon + P}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - 2\mathcal{M}_\Omega\right)
\end{aligned}$$

- N.B. Coupling to curved backgrounds crucial: $\# u^\mu u^\nu R_{\mu\nu}$
- Open Point: Lack of a derivation of $\mathcal{M}_\Omega = \frac{dP}{d\Omega}$

Euclidean Generating Functional I

[1203.3556, 1203.3544]

- $\tilde{\chi}_B, \tilde{\chi}_E, \tilde{\chi}_T, \tilde{\chi}_\Omega$ are equilibrium properties

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- Generating Functional to $\mathcal{O}(\partial^1)$:

$$W_1 = \int d^3x \sqrt{-g}\,[P(T,\mu) + \mathcal{M}_B(T,\mu)B + \mathcal{M}_\Omega(T,\mu)\Omega]$$

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- Main Result:

The general (2+1)-dimensional parity violating fluid is parametrized by two magnetizations, $\mathcal{M}_B$ and $\mathcal{M}_\Omega$.

Strongly Coupled Examples

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- Hall Viscosity [Son & Saremi 2011]

$$S = S_{grav}[g, \theta] - \frac{\lambda}{4} \int d^4x \sqrt{-g}\theta(r)\epsilon^{\lambda\rho\alpha\beta} R^{\mu}{}_{\nu\alpha\beta} R^{\nu}{}_{\mu\lambda\rho}$$

Black brane Ansatz $ds^2 = 2H(r)dvdr - r^2 f(r)dv^2 + r^2 dx_m dx^m$

$$\eta_H = -\frac{\lambda}{8\pi G_N} \left. \frac{r^4 f'(r)\theta'(r)}{4H^2(r)} \right|_{r=r_H}$$

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- Anomalous Hall Conductivity [1112.4498]

$$S = S_{grav} - \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\frac{F^2}{4} - \frac{1}{64\pi^2}\int \theta(\phi)F\wedge F$$

A high temperature analytic solution [Yarom 0912.2100] reproduces, via Fluid-Gravity, the constitutive relations.

$\Rightarrow$ Anomalous Hall Conductivity: $\tilde{\sigma} + \tilde{\chi}_E = \frac{\theta(\phi(r_h))}{8\pi^2}$

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THANK YOU!