Acceleration in Weyl integrable spacetime

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NEB 15, June 20-23, 2012 Chania

Outline

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- General relativity requires a modification at cosmological distance scales.
- Less explored is the idea that the geometry of spacetime is not the so far assumed Riemannian geometry.

Weyl geometry is considered as the most natural candidate for extending the Riemannian structure.

$$\nabla_\mu g_{\alpha\beta} = -Q_\mu g_{\alpha\beta},$$

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The simplest theory that can be constructed with the constrained variational principle is obtained from the Lagrangian $L = R$. The field equations are, [JM 2004],

$$G_{(\mu\nu)} = -\nabla_{(\mu} Q_{\nu)} + Q_\mu Q_\nu + g_{\mu\nu} \left(\nabla^\alpha Q_\alpha - Q^\alpha Q_\alpha \right) =: M_{\mu\nu}.$$

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Express the tensors $G_{(\mu\nu)}$ and $M_{\mu\nu}$ in terms of the quantities formed with the Levi-Civita connection D,

$$\overset{\circ}{G}_{\mu\nu} = \frac{3}{2} \left(Q_\mu Q_\nu - \frac{1}{2} Q^2 g_{\mu\nu} \right).$$

Bianchi identities imply

$$D^\mu Q_\mu = 0.$$

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Field equations:

$$\overset{\circ}{G}_{\mu\nu} = \frac{3-4\xi}{2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\left(\partial_\alpha\phi\partial^\alpha\phi\right)g_{\mu\nu}\right) + T_{\mu\nu},$$

and

$$\overset{\circ}{\Box}\phi = \frac{1}{3-4\xi}\rho,$$

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and

$$\overset{\circ}{\Box}\phi = \frac{1}{3-4\xi}\rho,$$

$$\lambda = \frac{4\xi - 3}{2},$$

For $\lambda < 0$ the field equations are formally equivalent to the interaction of a massless scalar field coupled to a perfect fluid in general relativity.

Flat FRW models

$$p = (\gamma - 1)\rho, \quad 0 \le \gamma \le 2,$$

$$H^2 = \frac{1}{3}\rho - \frac{\lambda}{6}\dot{\phi}^2,$$

$$\dot{H} = -\frac{\gamma}{2}\rho + \frac{\lambda}{2}\dot{\phi}^2,$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{1}{2\lambda}\rho,$$

$$\dot{\rho} = -3\gamma\rho H - \frac{1}{2}\rho\dot{\phi}.$$

State vector: $(\dot{\phi}, \rho, H) \in \mathbb{R}^3$.

Introduce expansion-normalized variables

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, \quad \Omega = \frac{\rho}{3H^2}, \qquad \tau = \ln a.$$

$$x' = -3x - \sqrt{\frac{3}{2}}\frac{1}{2\lambda}\Omega + x\left(\frac{3\gamma}{2}\Omega - 3\lambda x^2\right),$$
$$\Omega' = \Omega\left(-3\gamma - \sqrt{\frac{3}{2}}x + 3\gamma\Omega - 6\lambda x^2\right),$$

The evolution of the Hubble function

$$H' = -H\left(\frac{3\gamma}{2}\Omega - 3\lambda x^2\right),$$

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Constraint:

$$\Omega = 1 + \lambda x^2.$$

One-dimensional dynamical system:

$$x' = -\sqrt{\frac{3}{2}}\frac{1}{2\lambda} + 3\left(\frac{\gamma}{2} - 1\right)x - \frac{1}{2}\sqrt{\frac{3}{2}}x^2 + 3\left(\frac{\gamma}{2} - 1\right)\lambda x^3.$$

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The only real equilibrium of this equation is

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The decceleration parameter $q = -\ddot{a}a/\dot{a}^2$ at the equilibrium is given by

$$q_* = \frac{1 + 2\lambda\,(\gamma - 2)\,(3\gamma - 2)}{4\lambda\,(\gamma - 2)}.$$

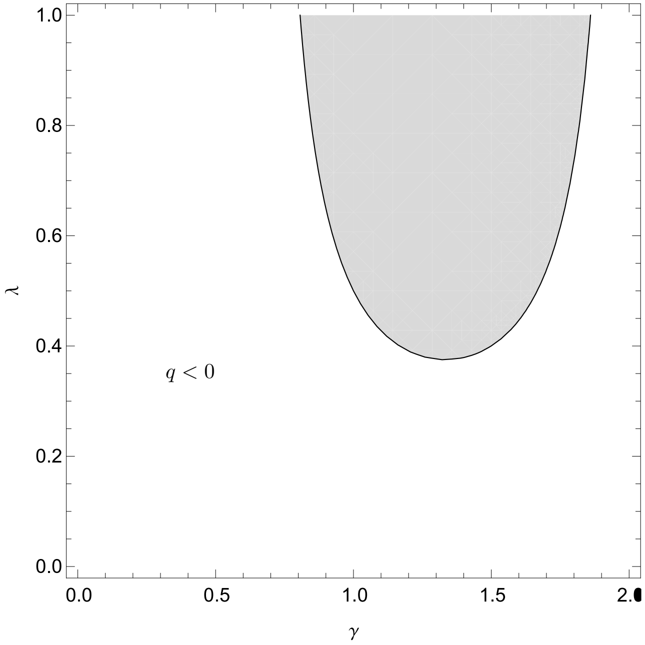

Figure :