Zero point energy in cosmology

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in collaboration with
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1104.3797 (Phys. Lett. B 2012) and 1111.5575 (PRD 2012)

Outline

- Clarifying the (non-covariant) cut-off renormalization in FRW background
- The equation of state of vacuum fluctuations
- Effective action and ultra-light fields (observable large scale quantum effects)

Cut-off regularization in Minkowski

$$\rho_{bare}(\Lambda_c) = \frac{\Lambda_c^4}{16\pi^2} \qquad p_{bare}(\Lambda_c) = \frac{\Lambda_c^4}{48\pi^2}$$

$$p_{bare}(\Lambda_c) = \frac{1}{3}\rho_{bare}(\Lambda_c)$$

- Holds for bare quantities (not physical)
- Normal since we broke Lorentz invariance
- Choose non-Lorentz invariant counter-terms to recover symmetry

$$p_{ren} = -\rho_{ren}$$

Cut-off regularization in FRW

$$\rho_{bare}(\Lambda_c) = \frac{\Lambda_c^4}{16\pi^2} + \frac{H^2(t)\Lambda_c^2}{16\pi^2} + O(H^4(t)\log(\Lambda_c))$$

$$p_{bare}(\Lambda_c) = \frac{\Lambda_c^4}{48\pi^2} + C(t)\frac{H^2(t)\Lambda_c^2}{16\pi^2} + O(H^4(t)\log(\Lambda_c))$$

$$C_{dS} = -\frac{1}{3} \qquad C_{RD} = 1 \qquad C_{MD} = \frac{2}{3} \qquad \rightarrow \qquad C(t) = w_{tot}(t) + \frac{2}{3}$$

$$p^{(4)}_{bare}(\Lambda_c) = \frac{1}{3}\rho^{(4)}_{bare}(\Lambda_c) \qquad p^{(2)}_{bare}(\Lambda_c) = \left(w_{tot}(t) + \frac{2}{3}\right)\rho^{(2)}_{bare}(\Lambda_c)$$

- Again, this holds at the bare level (not physical)

- Here, it is general covariance that we broke

- Result: bare quantities do not satisfy energy momentum conservation

$$p^{(4)}_{bare}(\Lambda_c) = \frac{1}{3}\rho^{(4)}_{bare}(\Lambda_c) \qquad p^{(2)}_{bare}(\Lambda_c) = \left(w_{tot}(t) + \frac{2}{3}\right)\rho^{(2)}_{bare}(\Lambda_c)$$

$$\dot{\rho}_{bare} + 3H\left(\rho_{bare} + p_{bare}\right) \neq 0$$

$$S^{(4)}_{count} = \int d^4x\sqrt{-g}\left(A(\Lambda_c) + B(\Lambda_c)\, g^{00}\right)$$

$$\left(T^{(4)}_{\mu\nu}\right)_{count} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{count}}{\delta g^{\mu\nu}} = A(\Lambda_c)\, g_{\mu\nu} + B(\Lambda_c)\left(g^{00} g_{\mu\nu} - 2\,\delta^0_\mu \delta^0_\nu\right)$$

$$\rho^{(4)}_{ren} = \rho^{(4)}_{bare} + \rho^{(4)}_{count} = \frac{\Lambda_c^4}{16\pi^2} - A(\Lambda_c) - B(\Lambda_c)$$

$$p^{(4)}_{ren} = p^{(4)}_{bare} + p^{(4)}_{count} = \frac{\Lambda_c^4}{48\pi^2} + A(\Lambda_c) - B(\Lambda_c)$$

We can thus choose A and B such that

$$p^{(4)}_{ren} = -\rho^{(4)}_{ren}$$

as imposed by E-M conservation (general covariance)

- The physical equation of state is thus “arbitrary”
- It would have been fixed in covariant regularizations
- We can fix it by imposing E-M conservation for the physical quantities

  → <u>same result</u> as with covariant regularizations

$$p_{ren}^{(4)} = -\rho_{ren}^{(4)} \qquad p_{ren}^{(2)} = w_{tot}(t)\rho_{ren}^{(2)} \qquad p_{ren}^{(0)} = \left(1 + 2w_{tot}(t)\right)\rho_{ren}^{(0)}$$

coincide to -1 only in de-Sitter phase!

Summary

- Non-covariant cut-off renormalization is not pathological in FRW
- Covariance is retrieved at the renormalized level

  → same result as with covariant regularization schemes
- → Renormalized quantities are scheme independent, as they should
- Equation of state is not -1 in general

Effects on cosmology?

- Setting the scale of new physics to be Planck, the naturalness argument suggests

$$\rho^{(4)}_{ren} \sim M^4_{pl} \qquad \rho^{(2)}_{ren} \sim H^2(t)\, M^2_{pl} \sim \rho_c$$

- Because of covariant EOS, they correspond to

$$\sim \sqrt{-g} \qquad \sim \sqrt{-g}\, R$$

terms in the action, respectively

→ Reabsorbed in Λ and G, respectively (CC problem)

→ Do not produce observable effects

Effects on cosmology?

- There was an unrealistic assumption:

  The zero-point energy was the only matter content

- In reality, other d.o.f. also present

  → vacuum energy <u>not</u> conserved in isolation

- This assumption is naturally relaxed in the context of <u>effective field theory</u>

Effective action point of view

- The effective action for gravity is the one where all matter has been integrated out

$$e^{-iS_{eff}[g]} = \int D\psi\, e^{-iS[g,\psi]}$$

- The VEV of the E-M tensor is just the E-M tensor of the effective action (matter sector)

$$\langle 0|T_{\mu\nu}|0\rangle = \frac{\int D\psi\, T_{\mu\nu}[g,\psi]\, e^{-iS[g,\psi]}}{\int D\psi\, e^{-iS[g,\psi]}} = -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}} S_{m,eff}[g]$$

But in the effective action point of view, we integrate out <u>only "heavy modes"</u> (compared to Hubble)

$$\omega_k = \sqrt{\frac{k^2}{a(t)^2} + m^2} \gg H(t)$$

so consider a scalar field ψ with m << H(t)

- separate high and low frequency modes $\psi = \phi + \chi$

$$e^{-iS_{m,eff}[g,\chi]} = \int D\phi\, e^{-iS_m[g,\phi+\chi]} = e^{-i\int\sqrt{-g}\,d^4x\left(L_{eff}[\chi] + f(\chi)R + O(R^2)\right)}$$

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- the χ dependence prevents absorption in G

$$\rho_{Z,ren} \sim f(\chi) H^2(t) M_{pl}^2 + \dot{f}(\chi) H M_{pl}^2$$

- early DE, tracking mechanism

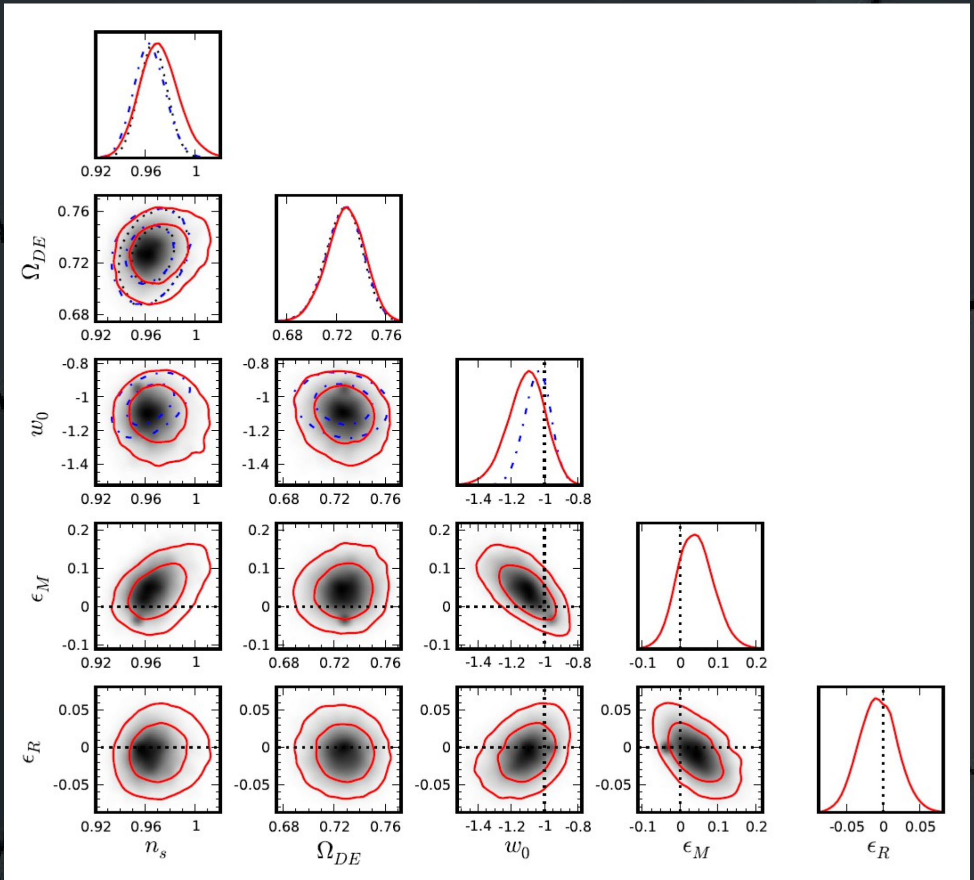
Summary

- The computation of zero-point energy amounts to integrating out the matter d.o.f. completely

- In the effective theory point of view, we integrate out only « heavy » modes

- If there is ultra-light matter
  - its long-wavelength modes survive integration
  - effective coupling to the zero point energy of the field

$$\rho_{DE}(a) = C\,a^{-3(1+w_0)} + \epsilon_R\,\rho_R(a) + \left[\epsilon_M + \left(\epsilon_R - \epsilon_M\right)\log\left(a - a_{eq}\right)\right]\rho_M(a)$$



2-parameter extension of wCDM

ΛCDM compatible

$$-1.35 < w_0 < -0.903$$

$$-0.0465 < \epsilon_M < 0.140$$

$$-0.0558 < \epsilon_R < 0.0446$$

Thank you very much

Backup

Renormalization of the quadratic term

$$\rho^{(2)}_{bare}(\Lambda_c) = \frac{H^2(t)\Lambda_c^2}{16\pi^2} \qquad p^{(2)}_{bare}(\Lambda_c) = \left(w_{tot}(t) + \frac{2}{3}\right)\frac{H^2(t)\Lambda_c^2}{16\pi^2}$$

The covariant counter-term

$$S^{(2)}_{cov.count} = A(\Lambda_c)\int d^4x\sqrt{-g}\,R$$

$$\rho^{(2)}_{cov.count} = -6\,A(\Lambda_c)\,H^2(t) \qquad p^{(2)}_{cov.count} = -6\,A(\Lambda_c)\,w_{tot}(t)\,H^2(t)$$

Ok for the energy but not for all of the pressure

→ must find a term with no T_{00}

Renormalization of the quadratic term

For the non-covariant part we restrict to given FRW

$$g_{\mu\nu} = \left(-N^2(t), b^2(t)\delta_{ij}\right) \qquad \tilde{g}_{\mu\nu} = \left(-1, a^2(t)\delta_{ij}\right)$$

$$S^{(2)}_{n-cov.count} = B(\Lambda_c)\int d^4x\left[\sqrt{-\tilde{g}}\left(\tilde{R} + 2\tilde{R}_{00}\right)\right]\frac{\sum_i g_{ii}}{\sum_j \tilde{g}_{jj}} \sim \int dt\,\frac{\dot{a}^2(t)}{a(t)}b^2(t)$$

No contribution to the energy density and $\sim H^2$

Testing the model

- The total dark energy (χ sector) is well described by

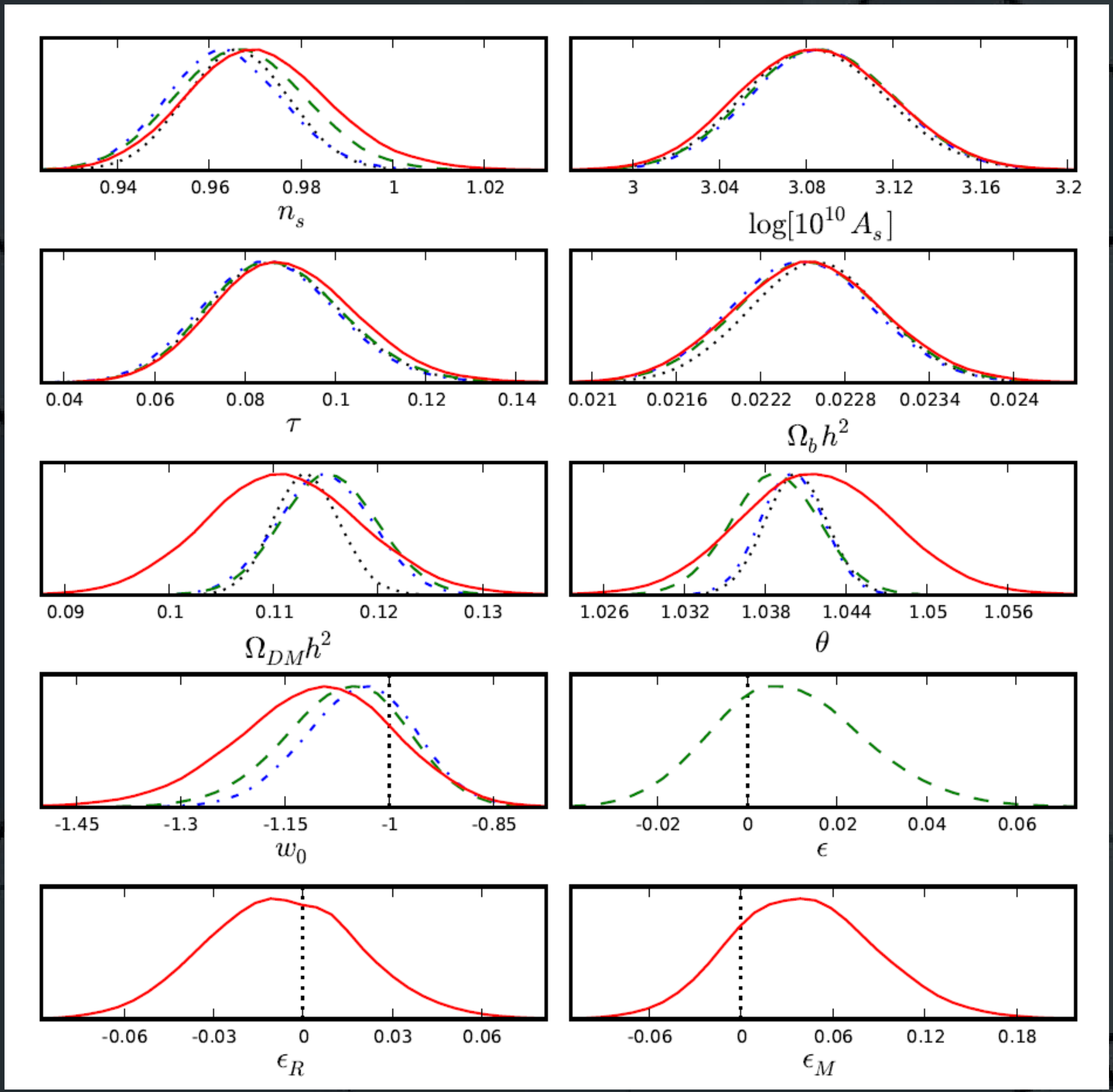
$$\rho_{DE}(a) = C\, a^{-3(1+w_0)} + \epsilon_R \rho_R(a) + \left[\epsilon_M + \left(\epsilon_R - \epsilon_M\right)\log\left(a - a_{eq}\right)\right]\rho_M(a)$$

where $w_0, \epsilon_M, \epsilon_R$ are constants

- This a two-parameter extension of wCDM

- Tested with modified CAMB and CosmoMC
  - Sne Ia, BAO, CMB, BBN, HST

CAMB / CosmoMC results



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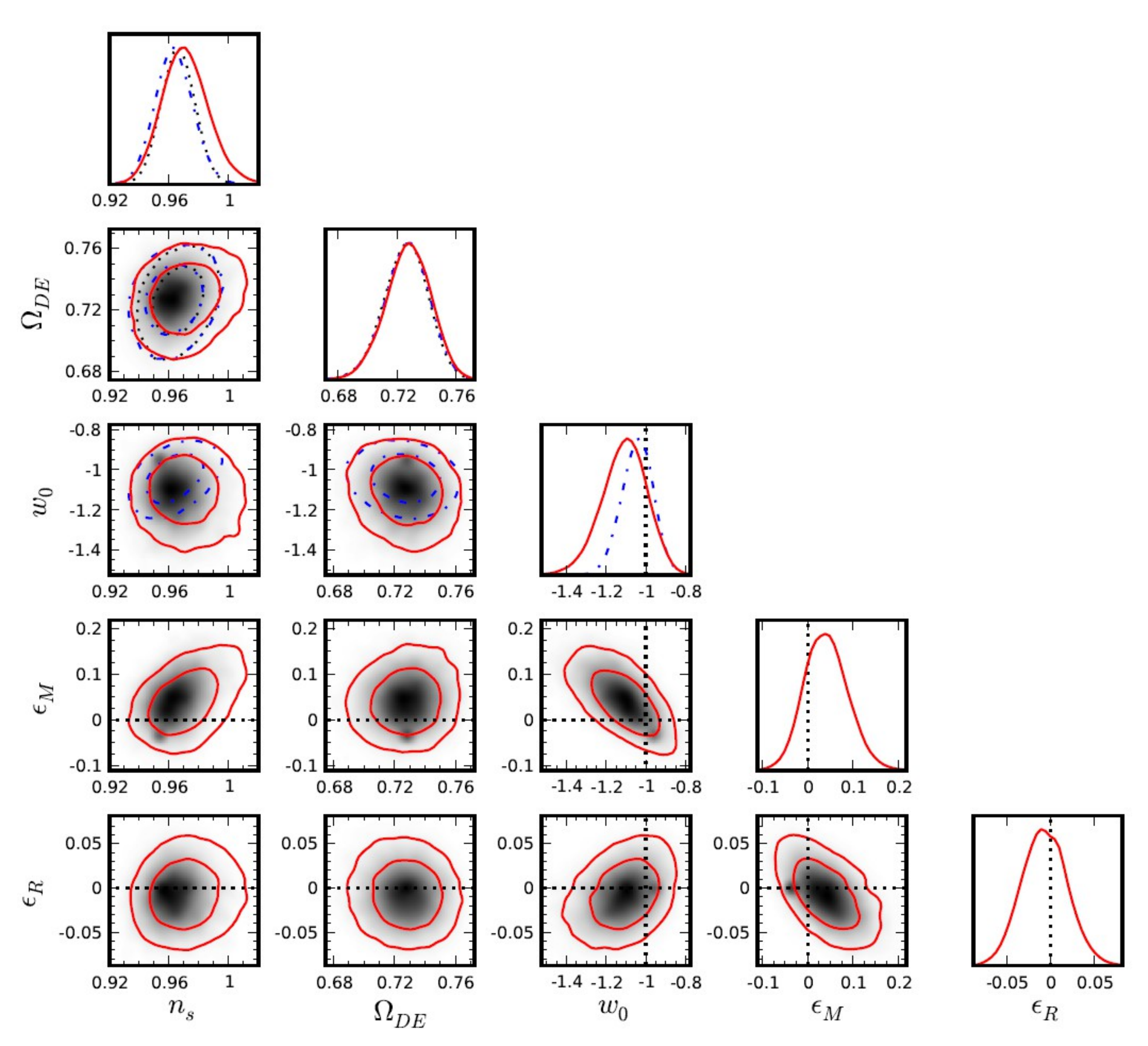

CAMB / CosmoMC results