THE EVOLUTION OF THE F-MODE INSTABILITY

Andrea Passamonti

University of Tübingen

In collaboration with E. Gaertig and K. Kokkotas

20 June 2012, Chania (Greece)

Motivation

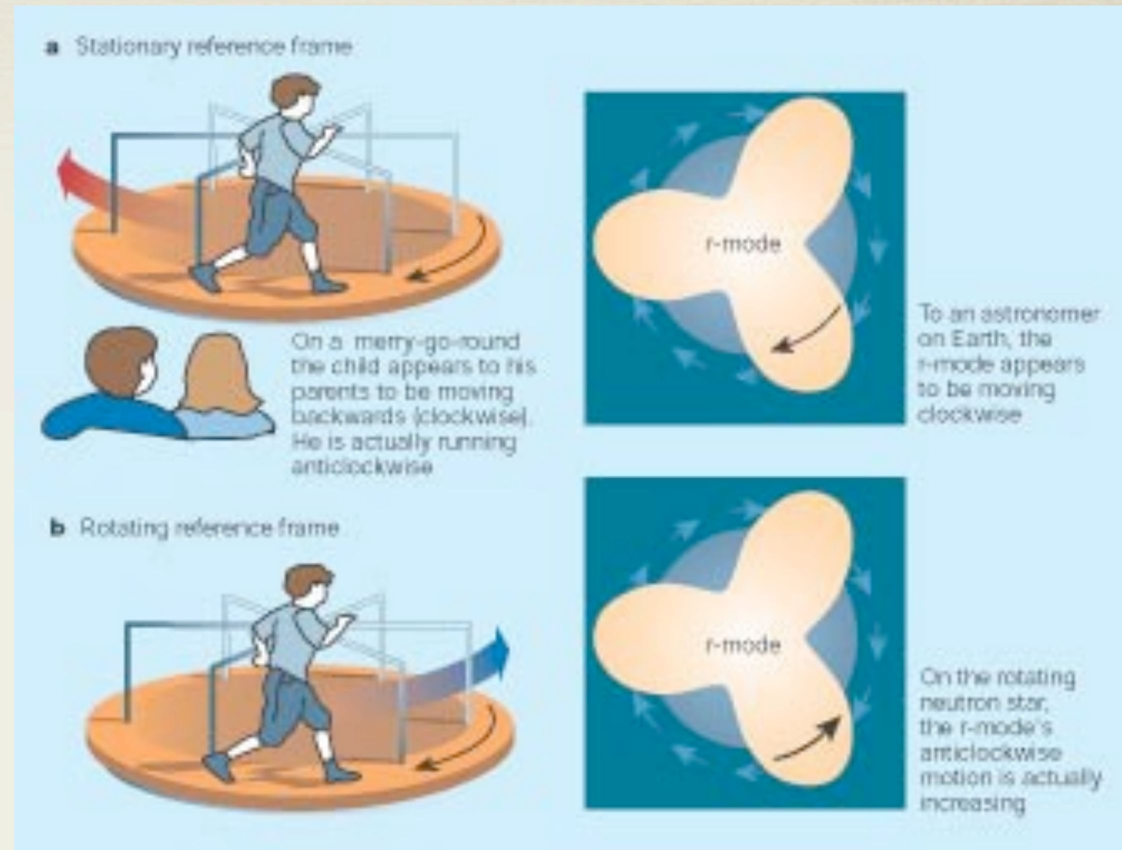
- GW driven f-mode instability of relativistic stars
- Time evolution of the instability
- GW signal of the f-mode and its detection prospectives

CFS instability

- Rotating NS are prone to CFS gravitational-wave instability
- Radiation drives a mode unstable if

$$\omega_r \left(\omega_r - m\Omega\right) \leq 0 \quad \Longrightarrow \tau_{\rm gw} \leq 0$$

and $$\delta\rho \sim e^{-t/\tau_{\rm gw}}$$



- Viscous mechanisms limit the gravitational-wave instability

$$\delta\rho \sim e^{i\omega t - t/\tau} \qquad \text{where} \qquad \frac{1}{\tau} = \frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm b}} + \frac{1}{\tau_{\rm s}} + \dots \quad \text{and} \qquad \frac{1}{\tau} = \frac{\dot{E}}{2E}$$

Instability condition: $$\frac{1}{\tau} \leq 0 \qquad \text{where} \qquad \tau = \tau\left(\Omega, T\right)$$

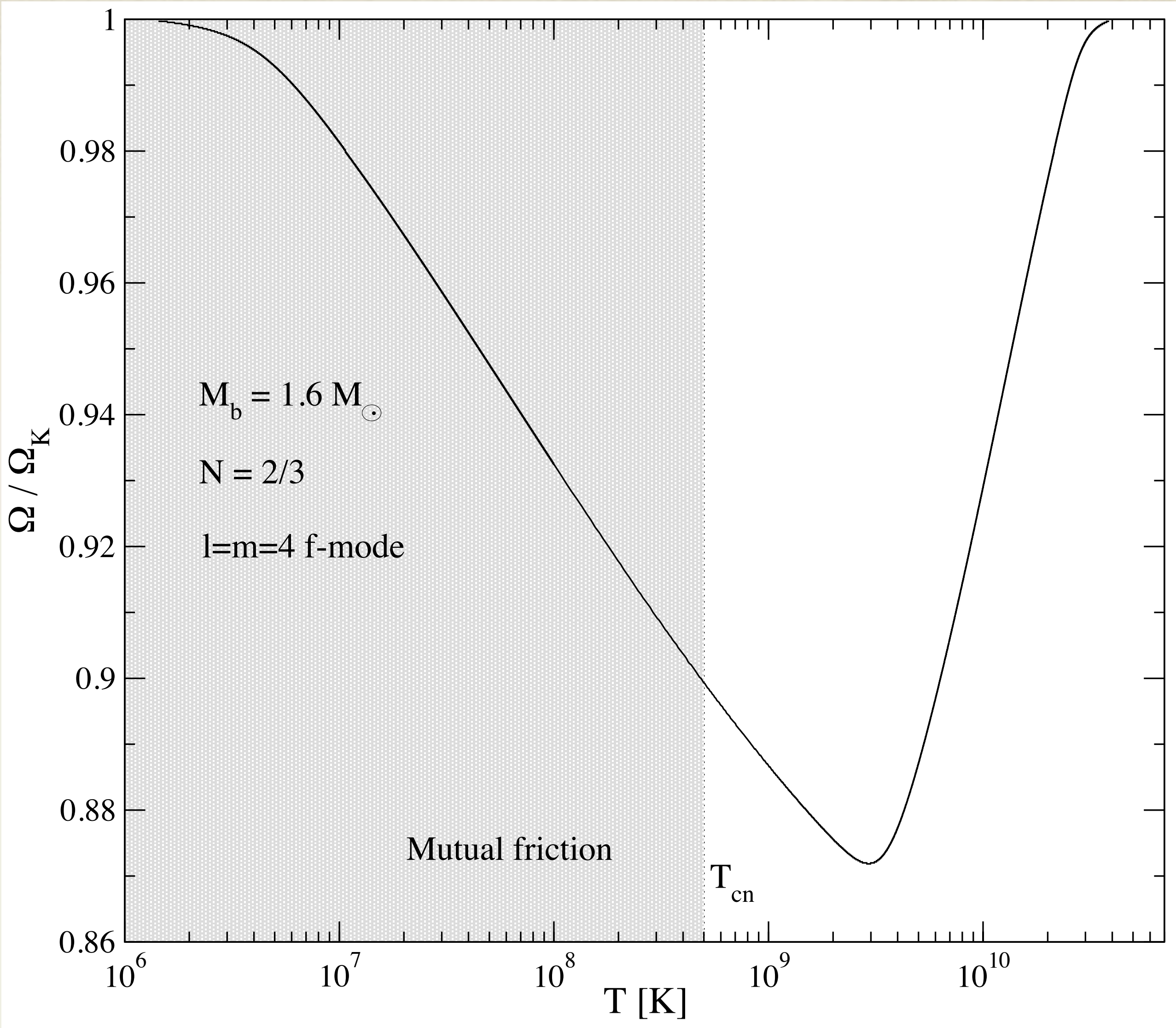
CFS instability

- Rotating NS are prone to CFS gravitational-wave instability
- Radiation drives a mode unstable if

$$\omega_r\left(\omega_r - m\Omega\right) \leq 0 \quad \Longrightarrow \tau_{\rm gw} \leq 0$$

and $$\delta\rho \sim e^{-t/\tau_{\rm gw}}$$

- Viscous mechanisms limit the gravitational-wave instability



$$\delta\rho \sim e^{i\omega t - t/\tau} \qquad \text{where} \qquad \frac{1}{\tau} = \frac{1}{\tau_{\rm gw}} + \frac{1}{\tau_{\rm b}} + \frac{1}{\tau_{\rm s}} + \ldots \quad \text{and} \qquad \frac{1}{\tau} = \frac{\dot{E}}{2E}$$

Instability condition: $$\frac{1}{\tau} \leq 0 \qquad \text{where} \qquad \tau = \tau\left(\Omega, T\right)$$

Equations

Method

- We calculate the mode-frequency and eigenfunctions from time simulations.
- With the energy volume integrals we determine the damping/growth times.
- We study the instability evolution with a set of evolution equations

Mode Frequency

- Evolution of the relativistic perturbation equations in Cowling approximation

$\delta\left(\nabla_\nu T^{\mu\nu}\right) = 0$ where $\delta g_{\mu\nu} = 0$

- Standard model

$N = 1$

$M = 1.4 M_\odot$

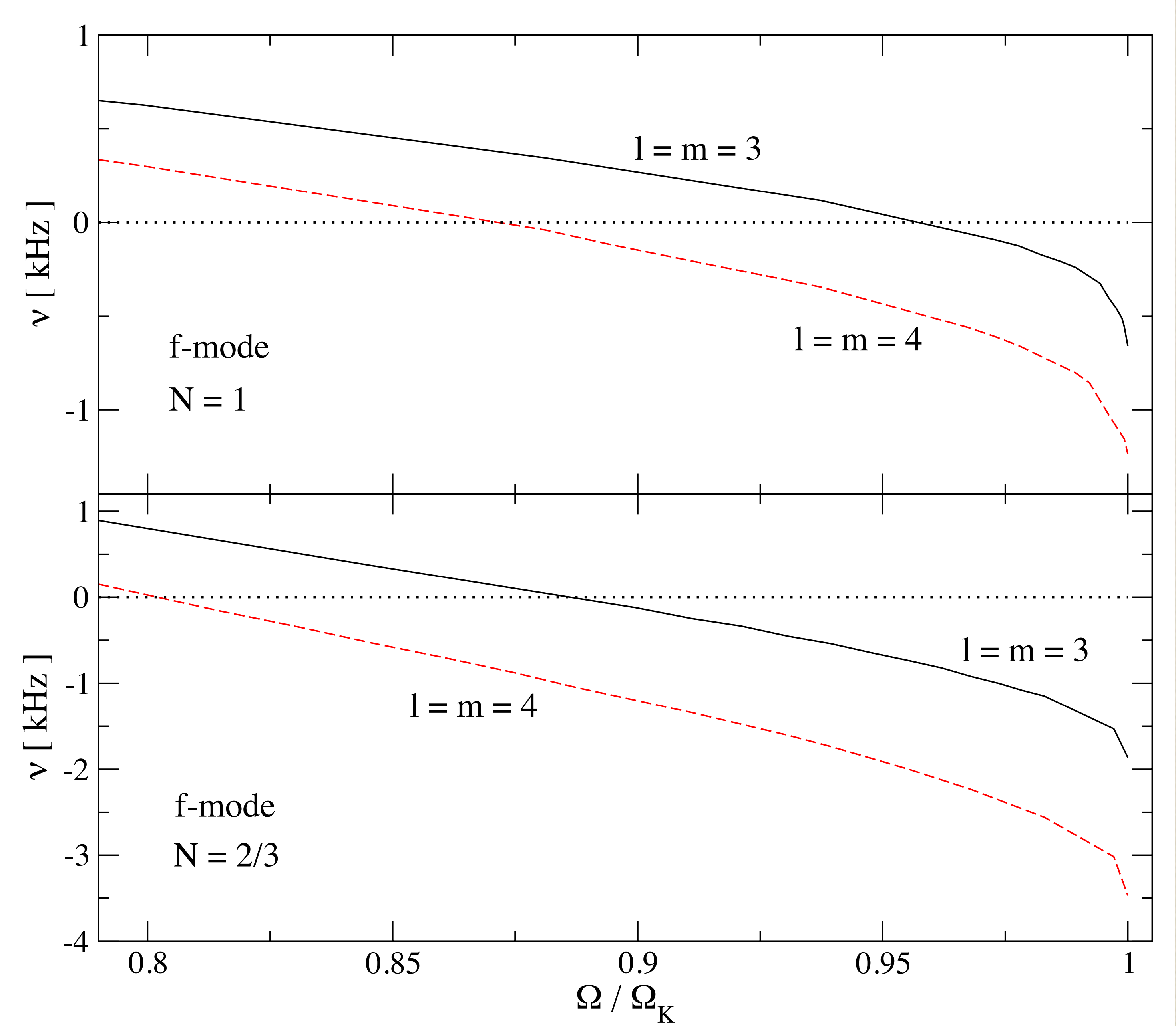
$\Omega_K = 4.229$ kHz

- Supramassive model

$N = 2/3$

$M = 1.6 M_\odot$

$\Omega_K = 11.206$ kHz



Mode Frequency

- Evolution of the relativistic perturbation equations in Cowling approximation

$\delta\left(\nabla_\nu T^{\mu\nu}\right) = 0$ where $\delta g_{\mu\nu} = 0$

- Standard model

$N = 1$

$M = 1.4 M_\odot$

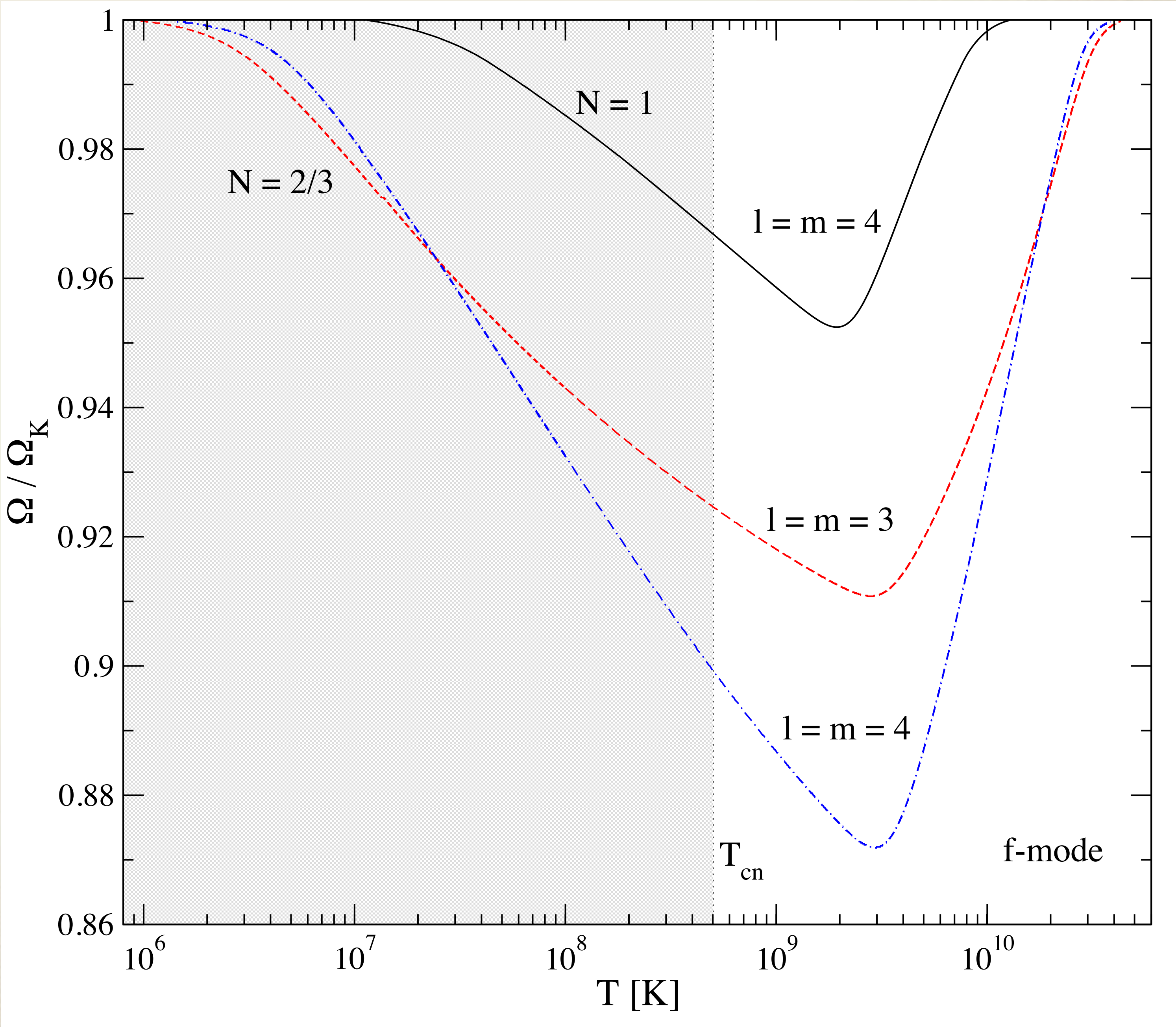
$\Omega_K = 4.229 \text{ kHz}$

- Supramassive model

$N = 2/3$

$M = 1.6 M_\odot$

$\Omega_K = 11.206 \text{ kHz}$



Instability Evolution

- Basic equations

$$\frac{dE}{dt} = -\frac{2E}{\tau} \qquad E = \alpha \tilde{E}(\Omega)$$

$$\frac{dJ}{dt} = \frac{dJ_{gw}}{dt} + \frac{dJ_{mag}}{dt} \qquad J = J_s + \alpha \tilde{J}_c(\Omega)$$

$$C_v \frac{dT}{dt} = -L_\nu + H_s \qquad H_s = \frac{2E}{\tau_s}$$

- Amplitude Normalization

$$E = \alpha E_{\rm rot} \qquad \alpha = 1 \Longrightarrow E \simeq 10^{-2} M_\odot c^2$$

Note: $\delta\rho \sim \alpha^{1/2}$

- Mode growth

$$\frac{d\alpha}{dt} = -\frac{2\alpha}{\tau_{gw}} - \frac{2\alpha}{\tau_v}\frac{1+\alpha Q}{D} + \frac{2P}{D}\frac{\alpha}{\tau_{mag}},$$

$$\frac{d\Omega}{dt} = \frac{2F}{D}\left(\frac{\alpha}{\tau_v} - \frac{1}{\tau_{mag}}\right),$$

- Non-linear saturation

$$\frac{d\alpha}{dt} = 0$$

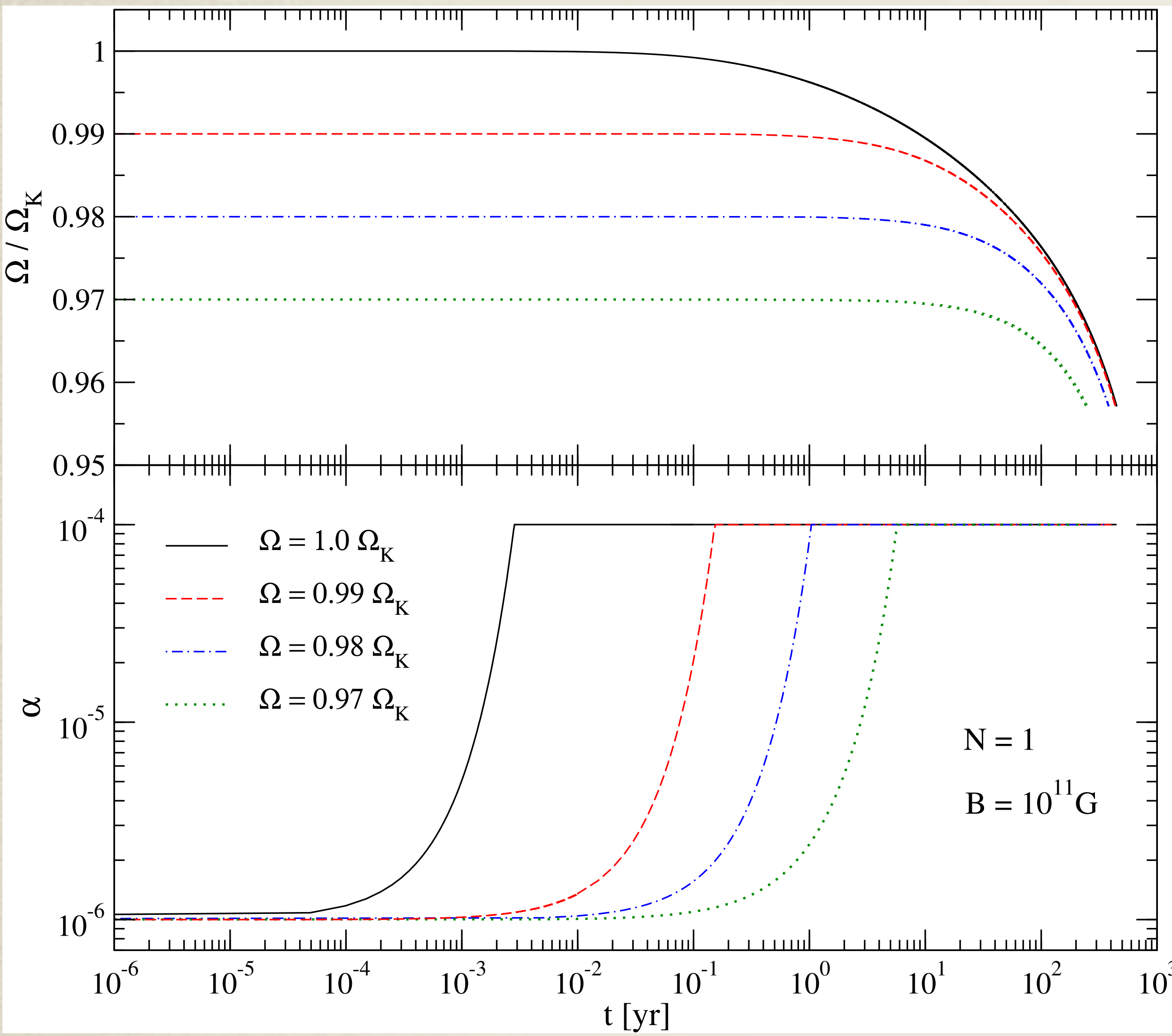
$$\frac{d\Omega}{dt} = -\frac{2F}{1+\alpha Q}\left(\frac{\alpha}{\tau_{gw}} + \frac{1}{\tau_{mag}}\right)$$

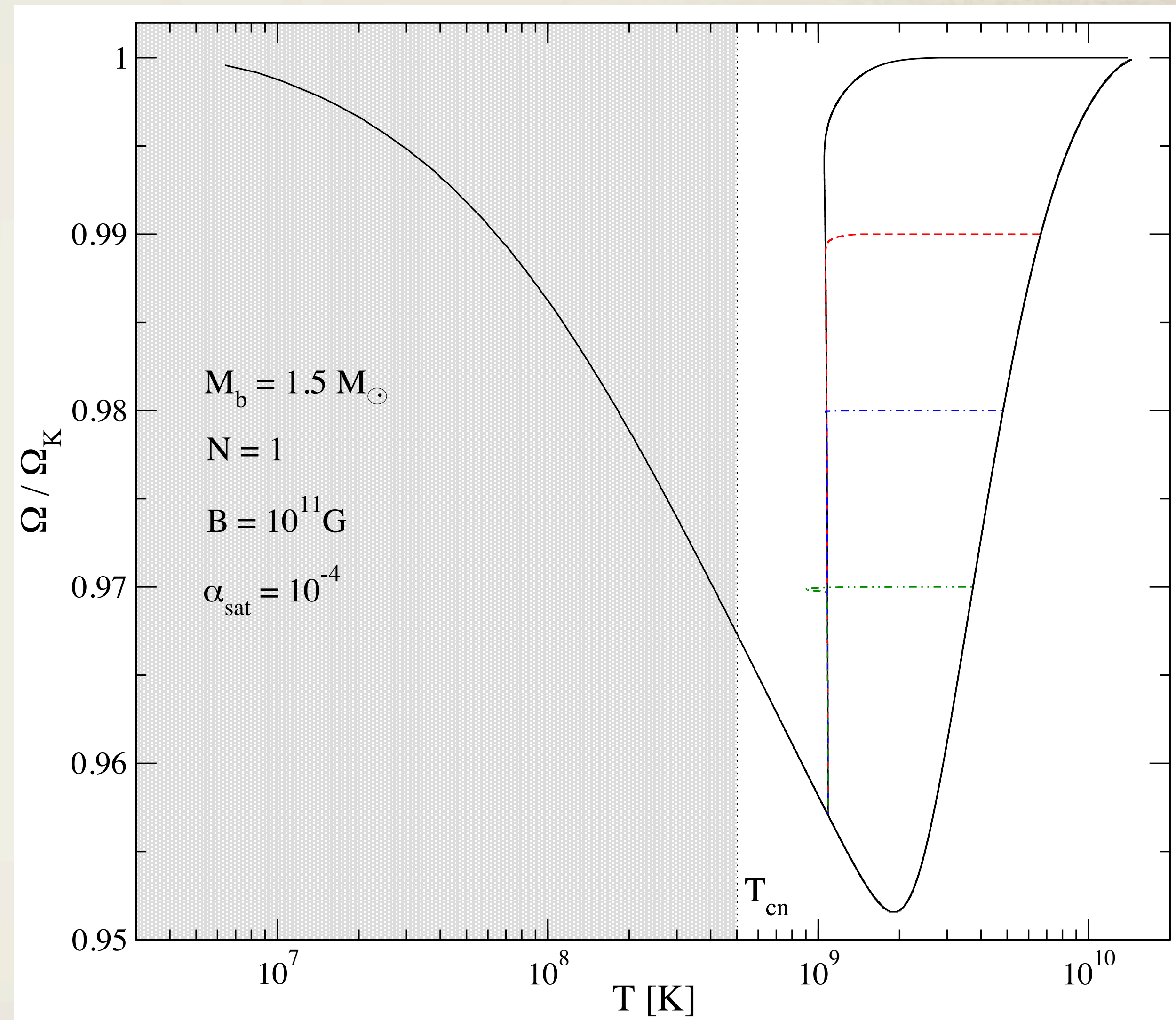
Results

l=m=4 f-mode Evolution

N = 1 polytrope

Instability trajectory





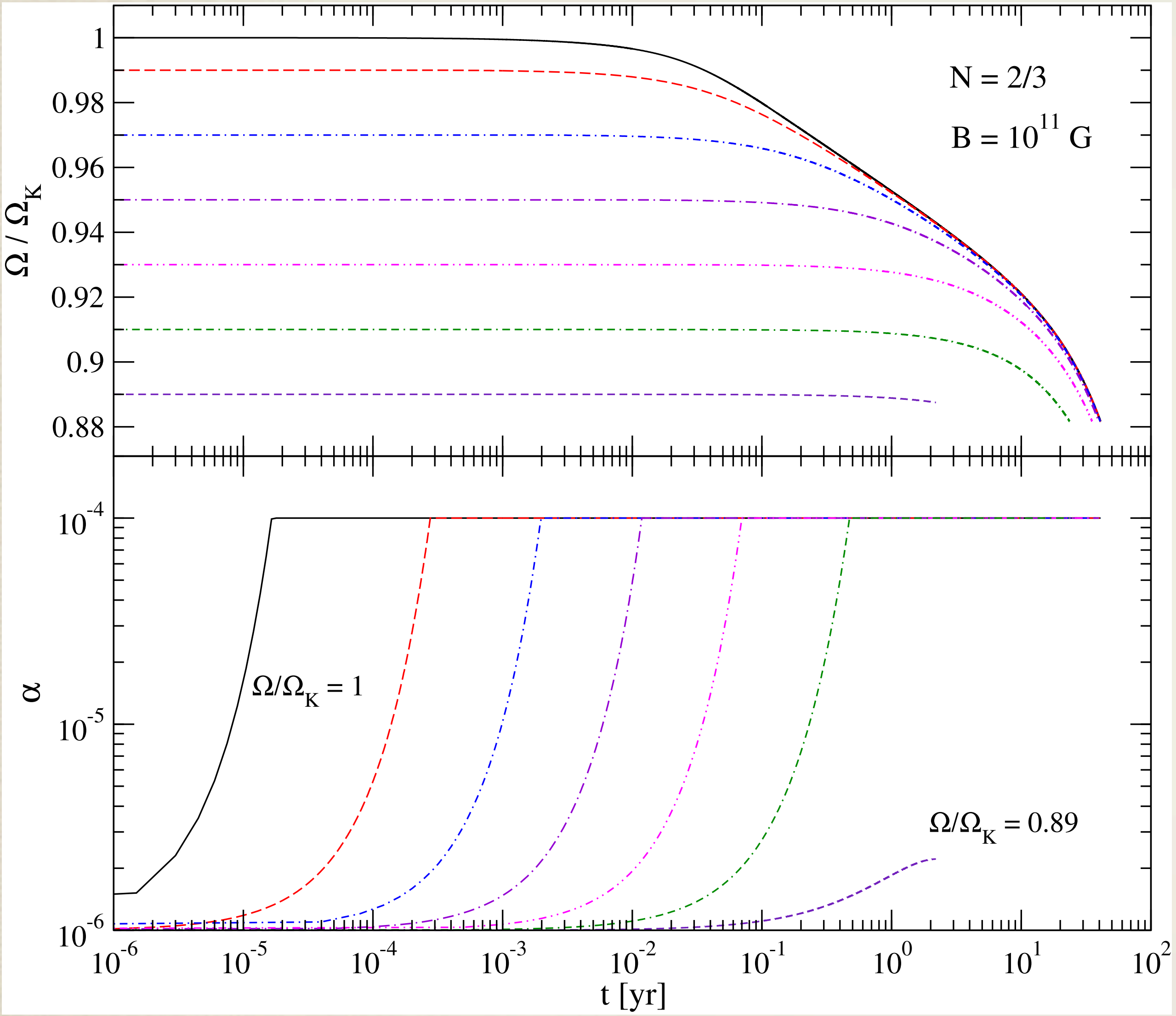
- Amplitude: $\alpha_0 = 10^{-6}$ and $\alpha_{sat} = 10^{-4} \Longrightarrow E \approx 10^{-6} M_\odot c^2$

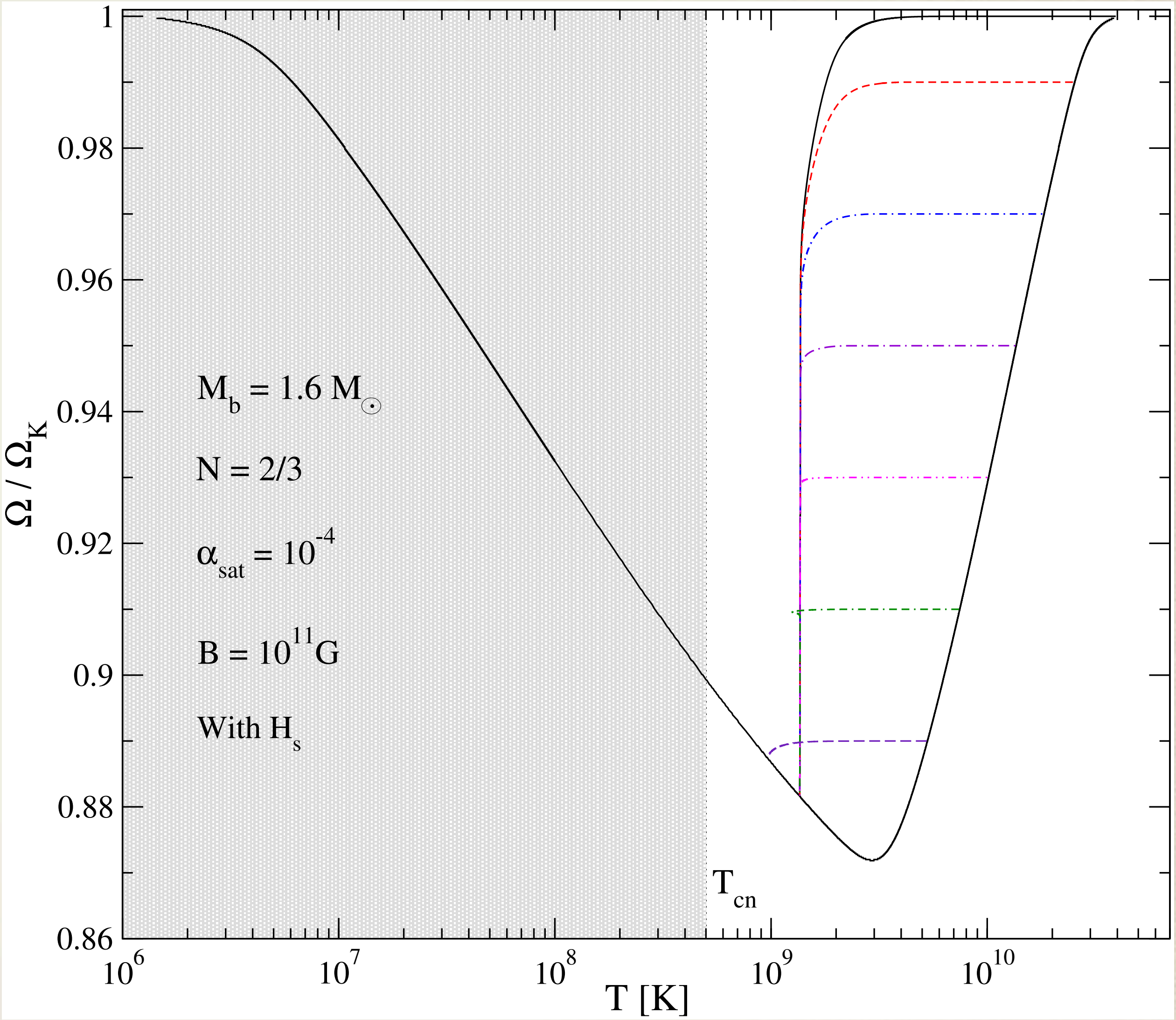
- At $\Omega = \Omega_K$ the growth time is $\tau_{gw} \sim 10^4\mathrm{s}$

l=m=4 f-mode Evolution

N = 2/3 polytrope

Trajectory



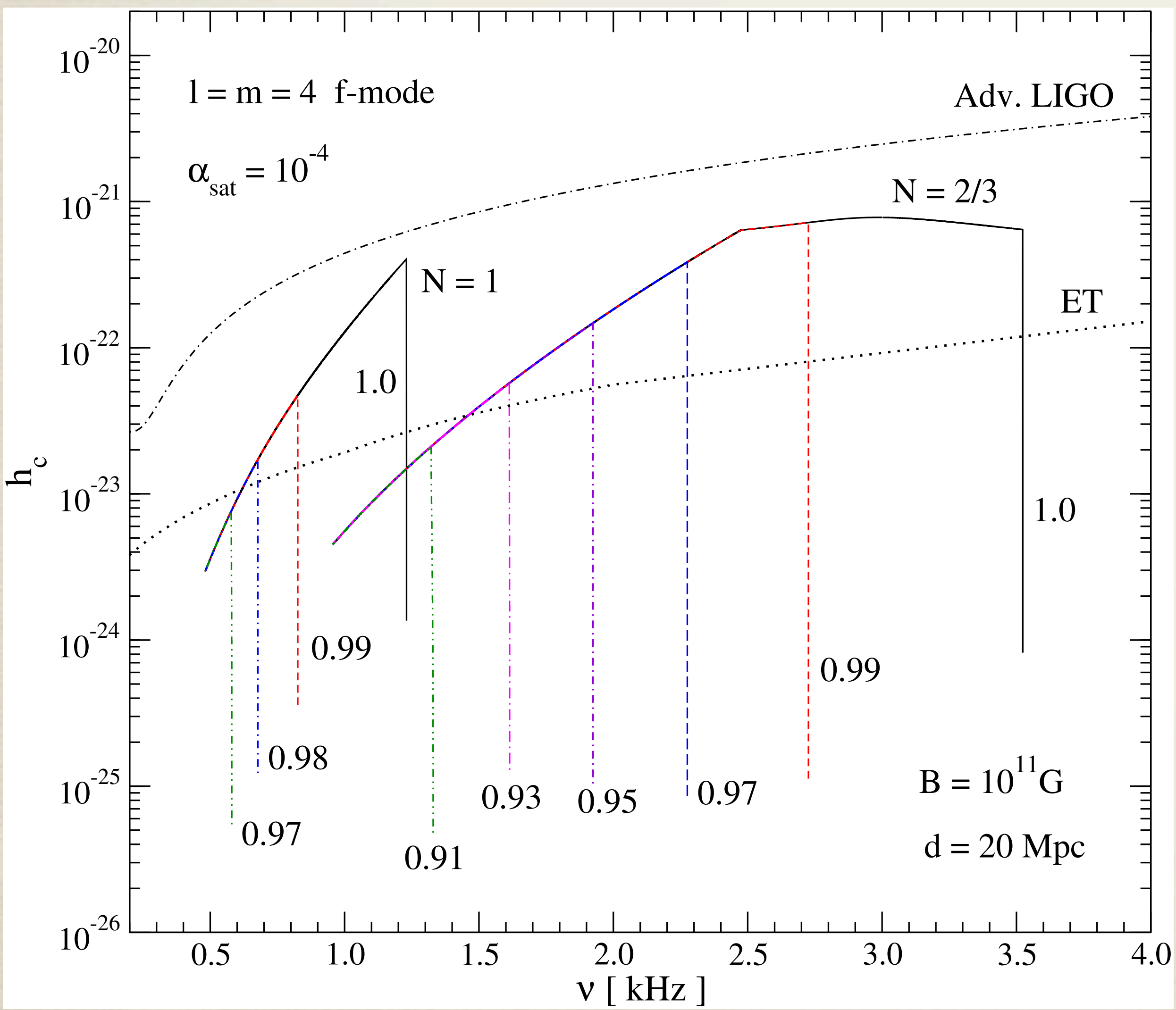


- At $\Omega = \Omega_K$ the growth time is $\tau_{gw} \sim 10^2 s$

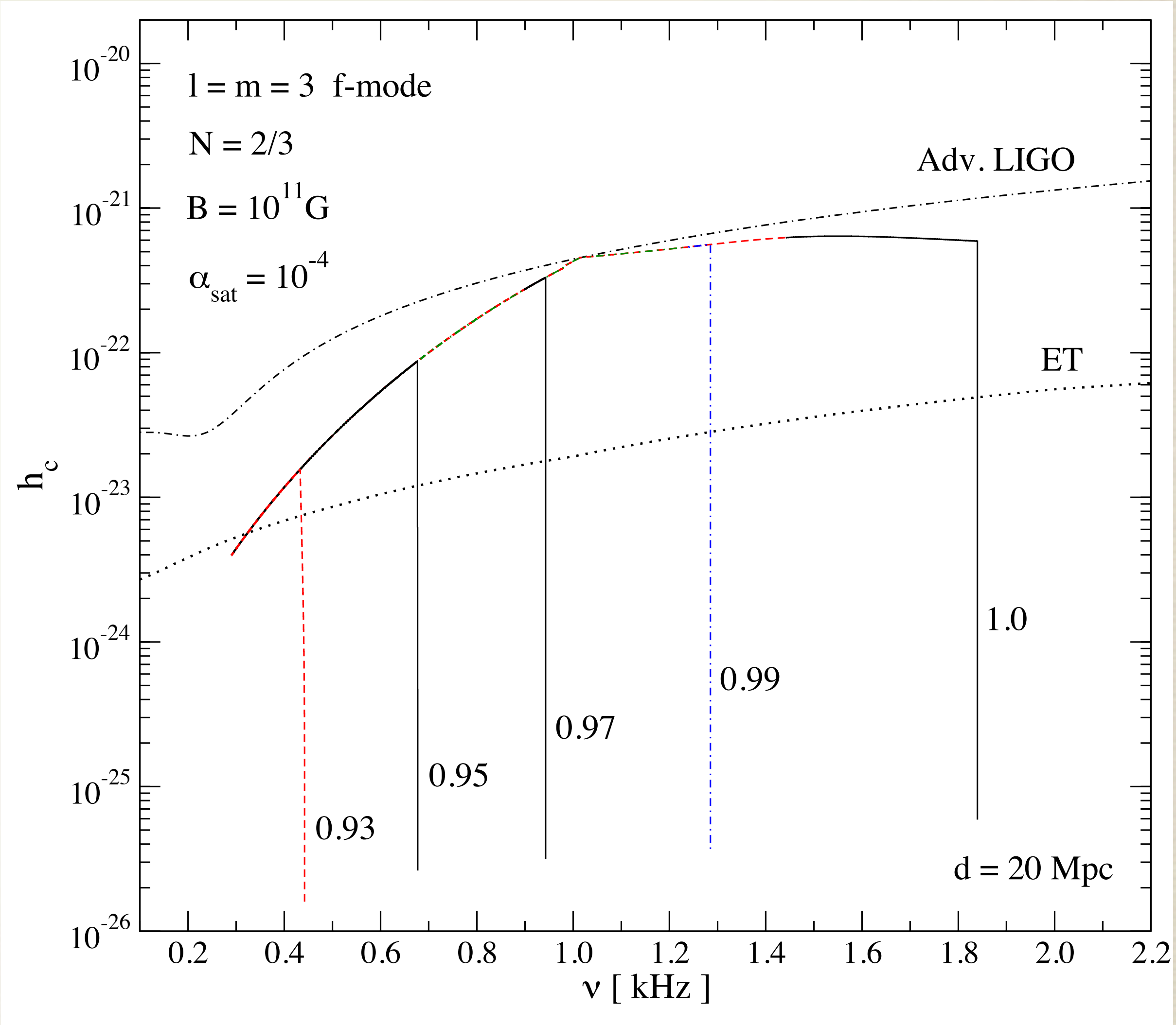
GW signal

- Characteristic strain $$h_c = h\sqrt{\nu^2 \left| \frac{dt}{d\nu} \right|}$$ $$t_{obs} \leq 1\text{yr}$$

l=m=4 f-mode



l=m=3 f-mode



Magnetic Torque

N = 1 polytrope

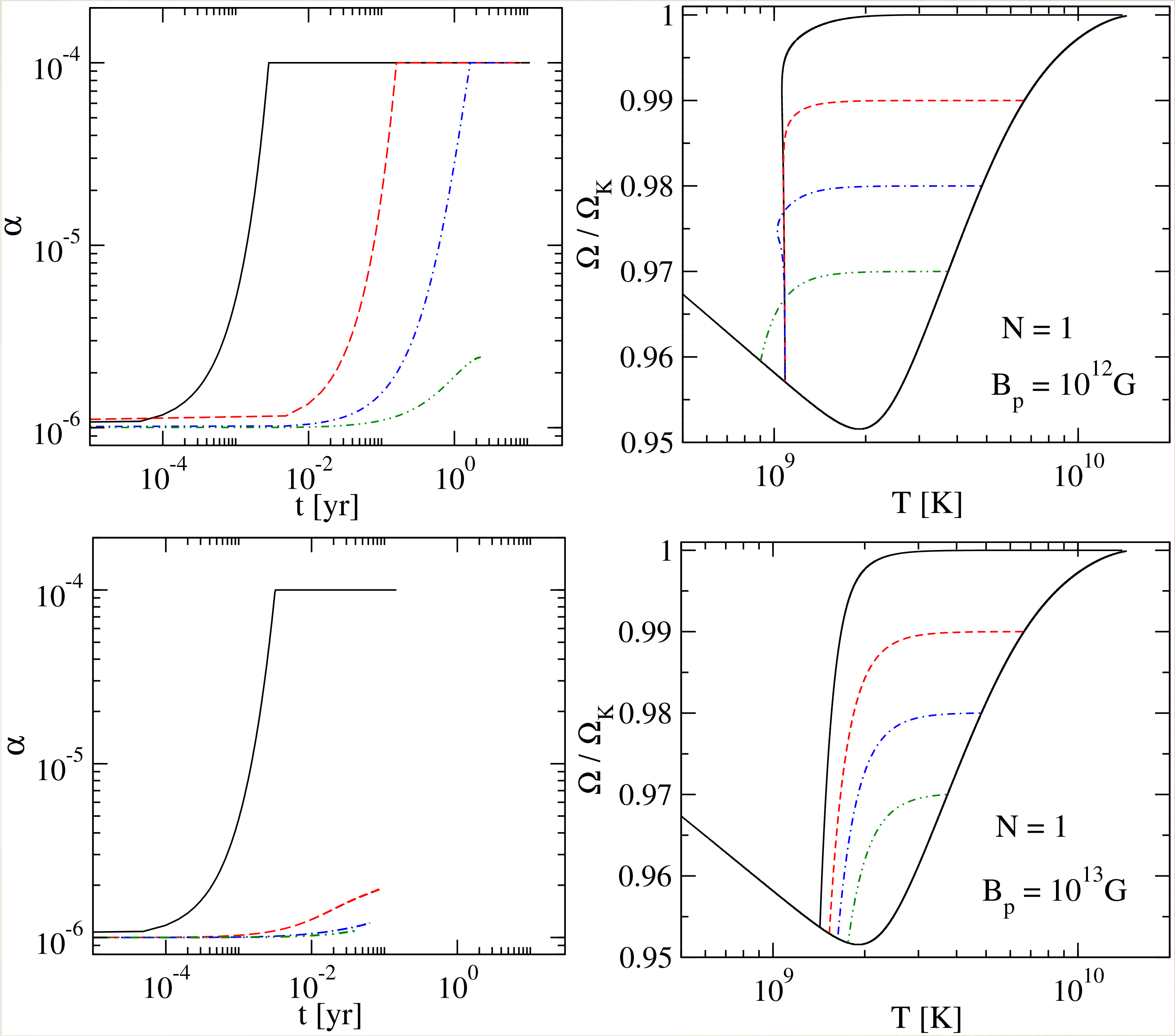
- Magnetic field accelerates the transition through the instability window

- Dipole formula

$$\frac{dJ_{mag}}{dt} \sim B_p^2 R^6 \Omega^3$$

- Mag. Torque becomes dominant for

$$B_\mathrm{p} \geq 10^{12}\mathrm{G}$$



Magnetic Torque

- Magnetic field accelerates the transition through the instability window

- Dipole formula

$$\frac{dJ_{mag}}{dt} \sim B_p^2 R^6 \Omega^3$$

- Mag. Torque becomes dominant for

$$B_{\mathrm{p}} \geq 10^{12}\mathrm{G}$$

Magnetic Torque

N = 2/3 polytrope

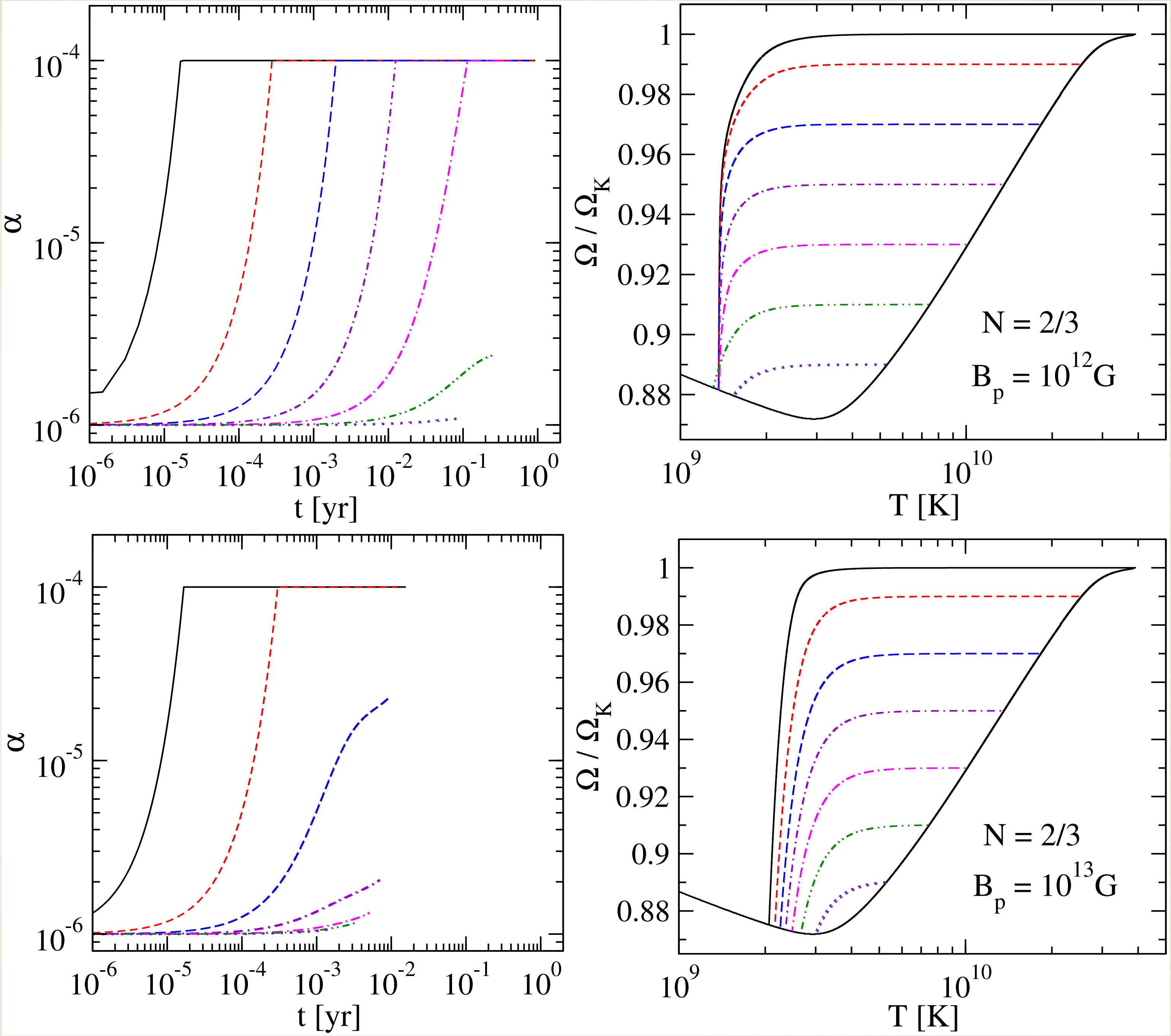
- Magnetic field accelerates the transition through the instability window

- Dipole formula

$$\frac{dJ_{mag}}{dt} \sim B_p^2 R^6 \Omega^3$$

- Mag. Torque becomes dominant for

$$B_{\rm p} \geq 10^{12}{\rm G}$$



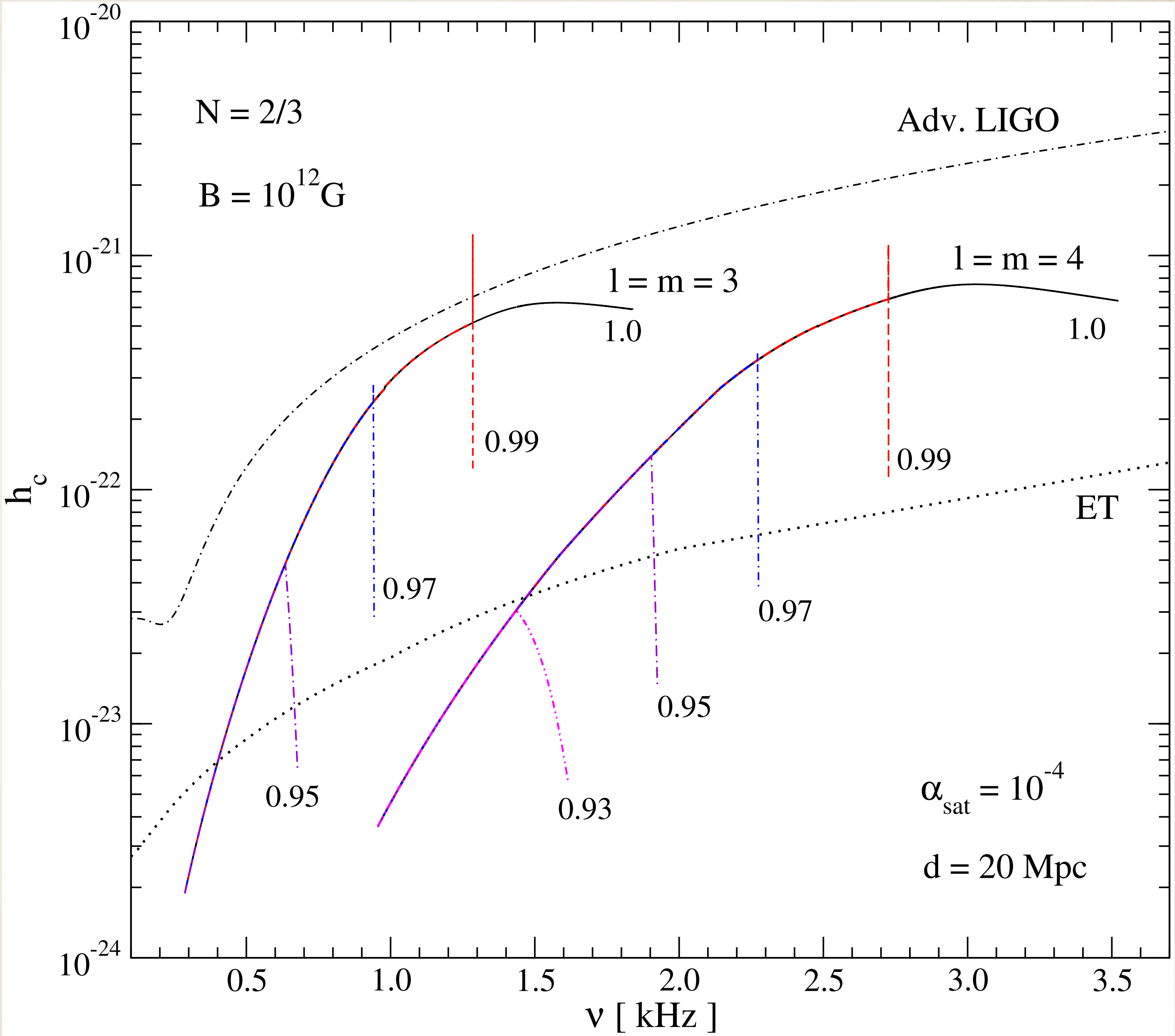
GW signal

- Characteristic strain

$t_{obs} \leq 1\mathrm{yr}$

- For the more massive model with $B_{\rm p} = 10^{12}\mathrm{G}$ the GW signal may be still detectable by ET.

- Source is 20 Mpc (Virgo Cluster)



F-mode versus R-mode

N = 2/3 model

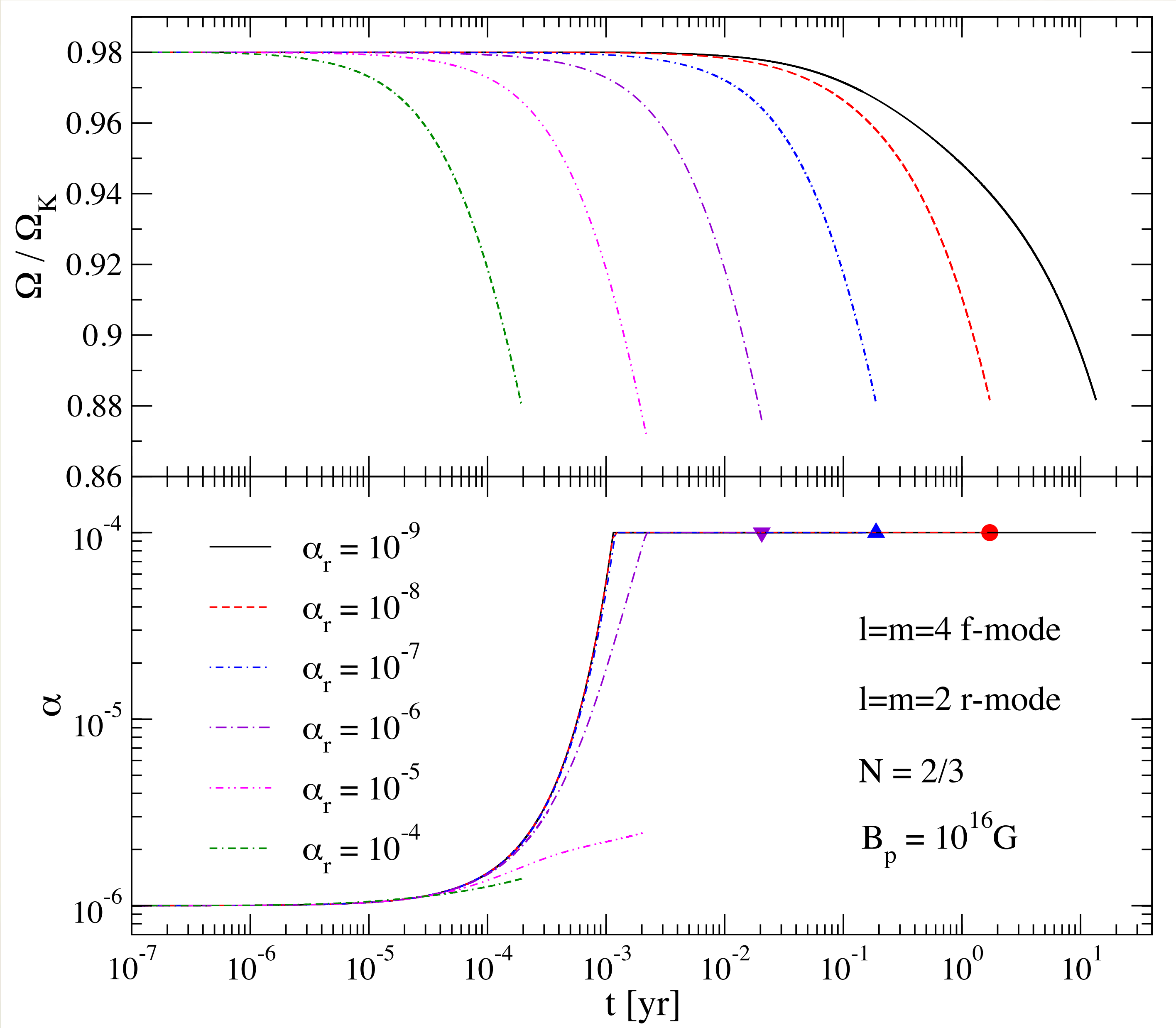
- Non-linear saturation of the f-mode

  $\alpha_{\rm f}^{sat} = 10^{-4}$

  $\Longrightarrow E \approx 10^{-6} M_{\odot} c^2$

- Non-linear mode coupling saturates r-mode

  $\alpha_{\rm r}^{sat} = 10^{-10} - 10^{-6}$



F-mode versus R-mode

N = 2/3 model

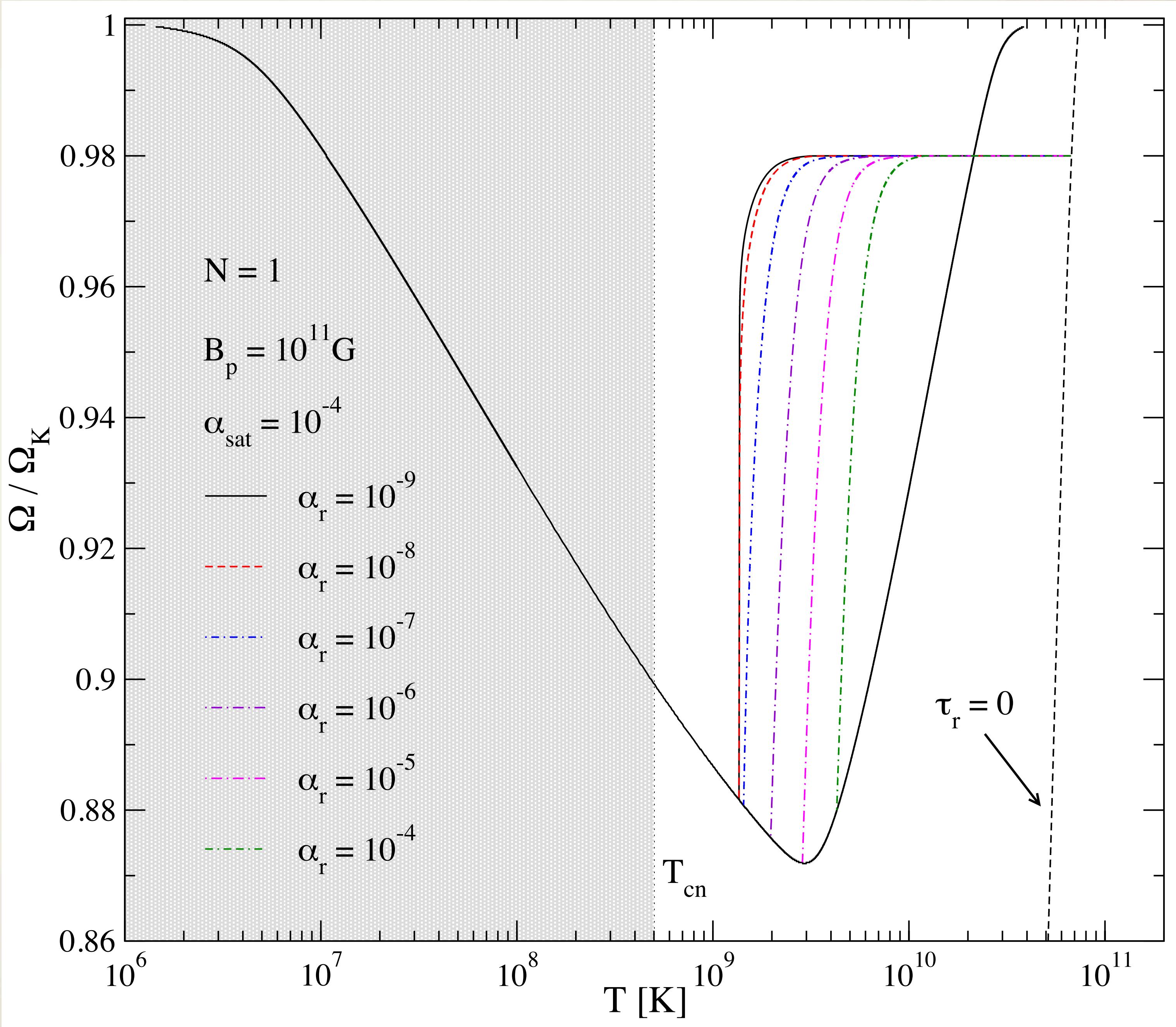
- Non-linear saturation of the f-mode

  $\alpha_{\rm f}^{sat} = 10^{-4}$

  $\Longrightarrow E \approx 10^{-6} M_{\odot} c^2$

- Non-linear mode coupling saturates r-mode

  $\alpha_{\rm r}^{sat} = 10^{-10} - 10^{-6}$



F-mode versus R-mode

N = 2/3 model

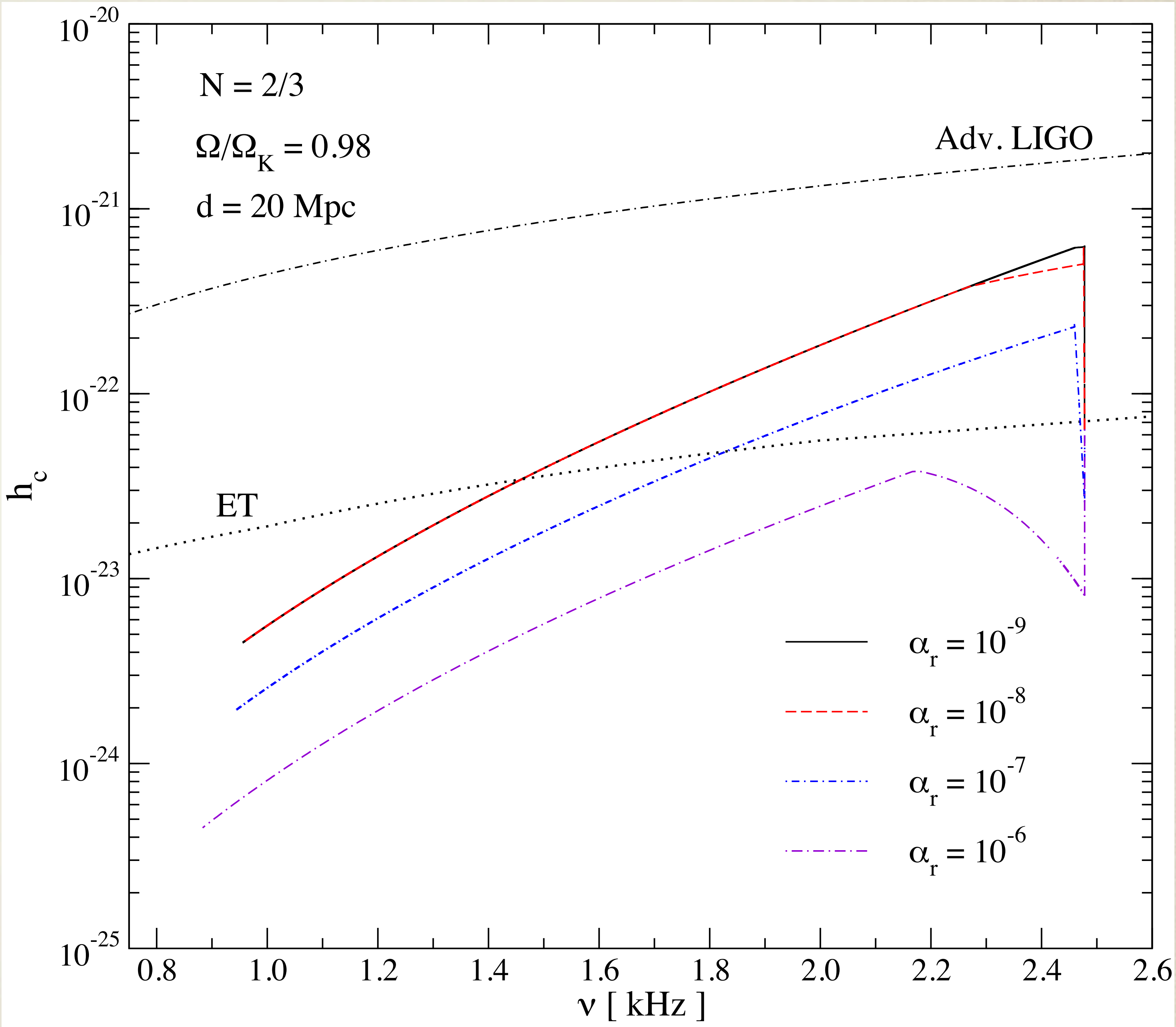
- Non-linear saturation of the f-mode

$$\alpha_{\rm f}^{sat} = 10^{-4}$$

$$\Longrightarrow E \approx 10^{-6} M_{\odot} c^2$$

- Non-linear mode coupling saturates r-mode

$$\alpha_{\rm r}^{sat} = 10^{-10} - 10^{-6}$$



Conclusions

- The GW signal of very compact objects may be detectable from Virgo Cluster by ET.
  - The magnetic torque affects the spin down when $B_{\mathrm{p}} \geq 10^{12}\mathrm{G}$
  - The r-mode may limit the f-mode instability, but we need to know the relative saturation amplitude more accurately.
- More ingredients in future work.
  - The l=m=2 f-mode may become important if we abandon the Cowling approximation.
  - Study realistic EoS and consider dUrca reactions.
  - Include the Crust and the effects of Ekman layers.

This is the End