Gravothermal Catastrophe with a Cosmological Constant

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Minos Axenides, George Georgiou and Z.R., arxiv:1206.2839

Thermodynamics of self-gravitating systems

- Non-extensivity of energy and entropy
  due to the long range character of gravitational interaction
- Non-equivalence of ensembles
  Canonical and microcanonical ensembles lead to different predictions
  Microcanonical ensemble is the proper one for gravitation
- Negative Heat Capacity (NHC) appears in the microcanonical ensemble
  Virial theorem: $2K + U = 0 \Rightarrow E = -K \Rightarrow C_V = -\frac{3}{2}Nk$
  most astrophysical systems have NHC
  NHC region is replaced by phase transition in canonical ensemble
- energy decrease⇔shrinking⇔temperature increase
  energy increase⇔expansion⇔temperature decrease
  crucial for stellar stability[1]: Gravothermal vs Thermonuclear effects
- important for Quantum Gravity

[1] H.A.Posch,W.Thirring, PRL 95, 251101 (2005)

Mean field approximation

- intermediate scale: granularity ($6N$-dim.) $\rightarrow$ continuum limit (6-dim.)
  One-particle distribution function $f(\vec{r}, \vec{p}, t)$: $dm = f(\vec{r}, \vec{p}, t) d^3\vec{r} d^3\vec{p}$
  Mean field entropy: $S = \int f \ln f d^3\vec{r} d^3\vec{v}$
- Equilibrium $f \Leftrightarrow$ extremum of entropy with constant E and M
  Stability $\Leftrightarrow$ entropy maximum
  The approximation is valid only for stable configurations

The Physical System

Self-gravitating gas of point-particles inside a spherical shell with perfectly reflecting and insulating walls

No global entropy maximum [Antonov (1962)]

Only local entropy maxima called isothermal spheres

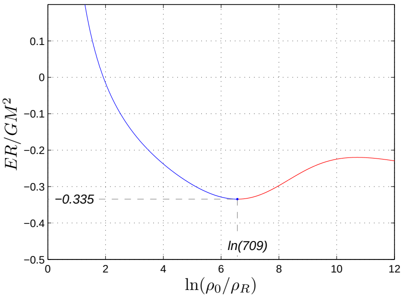
In regions of instability consider a short-distance cut-off $\rightarrow$ global entropy maxima: core-halo structure known as core collapse [Padmanabhan (1990)]

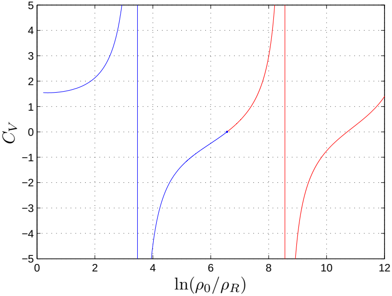
Gravothermal Catastrophe [Antonov 1962, Lynden–Bell & Wood 1968]

$f = (\beta/2\pi)^{3/2}\rho_0 e^{-\beta(\phi-\phi(0))} e^{-\beta\upsilon^2/2}$ with:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\phi\right) = 4\pi G\rho_0 e^{-\beta(\phi-\phi(0))} \quad \text{Emden equation}$$

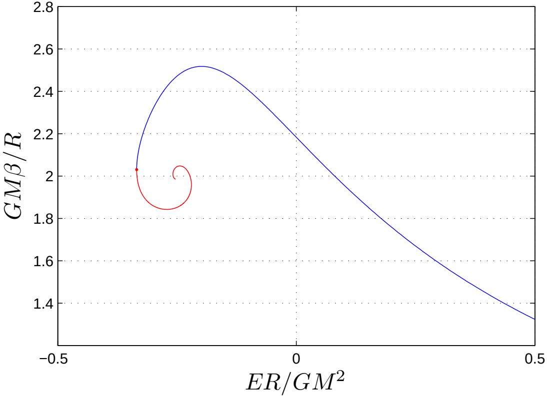
No global entropy maximum. Local entropy maxima (meta-stable states) exist only for $ER > -0.335GM^2$ and only when $\frac{\rho_0}{\rho_R} < 709$





Poincaré's theorem: stability changes at an energy extremum

The series of equilibria $\beta = \beta(E)$. The stability changes at points of infinite slope. For the specific figure, following the curve from right $(\beta \to 0)$ to left, each time we cross a transition point the equilibria become more unstable. The blue series are stable, while the red ones are unstable.



Adding a Cosmological Constant

The Emden equation becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\phi\right) = 4\pi G\rho_0 e^{-\beta(\phi-\phi(0))} - 8\pi G\rho_\Lambda \quad \text{Emden} - \Lambda \text{ equation}$$

where $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$.
Dimensionless variables: $x = r\sqrt{4\pi G\rho_0\beta}$, $y = \beta(\phi - \phi(0))$, $\lambda = \frac{2\rho_\Lambda}{\rho_0}$

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{d}{dx}y\right) = e^{-y} - \lambda \quad , y(0) = y'(0) = 0$$

Dimensionless temperature, energy and mass:

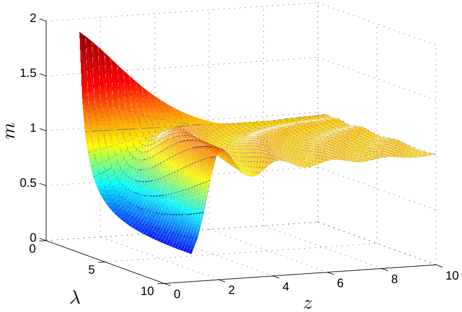
$$\bar{\beta} = \frac{GM\beta}{R} \quad , \quad Q = \frac{ER}{GM^2} \quad , \quad m = \frac{M}{2M_\Lambda}$$

For $z = x(R)$ we get $m = 3\bar{\beta}/\lambda z^2$. We want $\bar{\beta}(z)$ and $Q(z)$.
Have to solve Emden-Λ for various z keeping M, i.e. m fixed.

We constructed a computer code that solves Emden-Λ for various z keeping m constant. Then $\bar{\beta}$, Q are found by the equations:

$$\bar{\beta}(z) = zy'(z) + \frac{1}{3}\lambda z^2 \ , \ Q(z) = \frac{z^2 e^{-y(z)}}{\bar{\beta}^2} - \frac{3}{2\bar{\beta}} - \frac{\lambda}{2\bar{\beta}^2 z}\int_0^z x^4 e^{-y(x)} dx$$

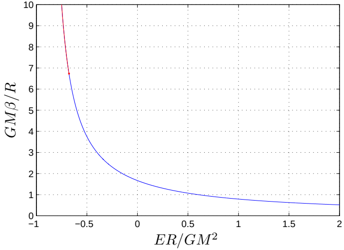
For $m = const.$ there are various series of solutions corresponding to pairs λ, z. For $m = 1$ ($M = 2M_\Lambda$) there are infinite series.
$m = \frac{3}{8\pi}\frac{M}{\rho_\Lambda R^3}$



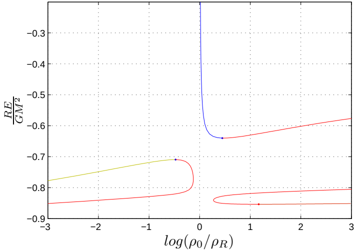
At $R_H = (\frac{3M}{8\pi\rho_\Lambda})^{\frac{1}{3}}$ there exists a homogeneous solution $\rho = 2\rho_\Lambda = const.$, that is the non-relativistic analogue of Einstein's static Universe.

The homogeneous solution suffers a transition to instability at $T_{cr} = GM/6.73R_H$. For $T \to \infty$ a collisionless gas, which is stable, while for $T \to 0$ the static unstable solution.

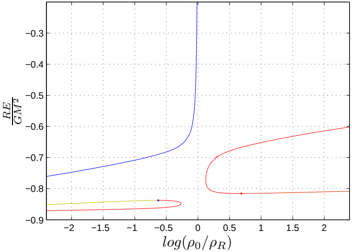
$R = R_H$

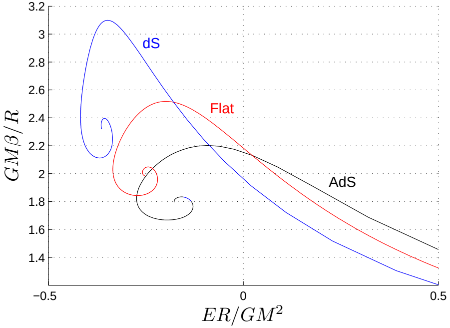


$R < R_H$



$R > R_H$





As the cosmological constant increases, there exist equilibrium states at even lower temperatures and energies.

Second Variation of Entropy

$$\delta^2 S = \int_0^R \int_0^R \delta M(r_2)\hat{K}(r_1, r_2)\delta M(r_1) dr_1 dr_2$$

where δM is a local mass perturbation and:

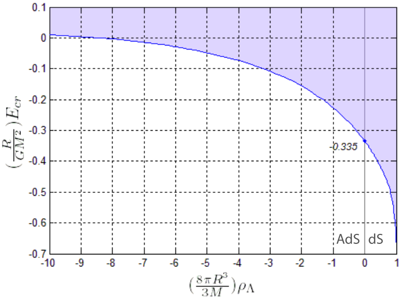
$$\hat{K}(r_1, r_2) = -\frac{\phi'(r_1)\phi'(r_2)}{3MT^2} + \frac{1}{2}\delta(r_1 - r_2)\left[\frac{G}{Tr_1^2} + \frac{d}{dr_1}\left(\frac{1}{4\pi\rho r_1^2}\frac{d}{dr_1}\right)\right]$$

The sign of $\delta^2 S$ is determined by the sign of the eigenvalues ξ of the eigenvalue problem:

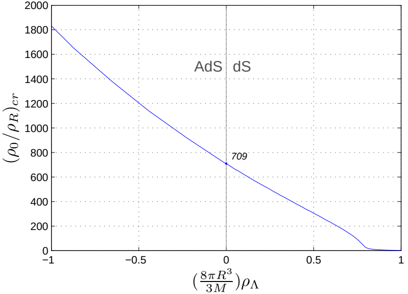
$$\int_0^R \hat{K}(r, r_1)F_\xi(r_1)dr_1 = \xi F_\xi(r) \qquad (1)$$

with $F_\xi(0) = F_\xi(R) = 0$. We developed an algorithm that can numerically determine eigenvalues and eigenstates (the perturbations) of equation (1).

For increasing cosmological constant, the critical energy decreases. In AdS, beyond some ρ_Λ, the critical energy is positive.

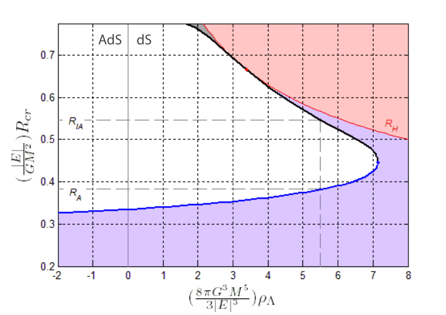


For increasing cosmological constant, the critical density contrast $(\rho_0/\rho_R)_{cr}$ decreases.

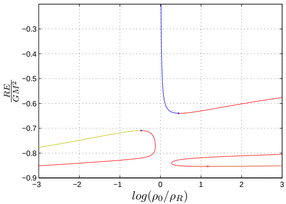


Re-entrant behaviour in dS

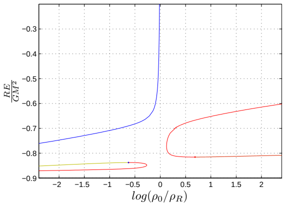
- No equilibrium for $R_A < R < R_{IA}$.
- For $R > R_{IA}$ metastable states are restored.
- R_A and R_{IA} merge at an extremal value $\bar{\rho}_\Lambda$



$R < R_H$



$R > R_H$



Results

AdS ($\Lambda < 0$) tends to destabilize the system

- as AdS becomes stronger the region of instability is enlarging
- the instability sets in at more condensed states
- beyond some Λ the instability occurs for positive energies

dS ($\Lambda > 0$) tends to stabilize the system

- as dS becomes stronger the region of no instability is diminishing
- the instability sets in at less condensed states
- Re-entrant phenomenon: a new critical radius R_{IA} appears, beyond which a series of meta-stable states is restored
- there exists a homogeneous solution for which an instability sets in at $T_{cr} = \frac{GM}{6.73R}$
- new type of configurations are found for which $\rho(r)$ is not monotonically decreasing, but any configuration is possible

Future Plans

- Perform a full General Relativistic Analysis
- Analysis on the instability region with a short-distance cut-off
- Canonical Ensemble
- Extra Dimensions
- AdS/CFT