

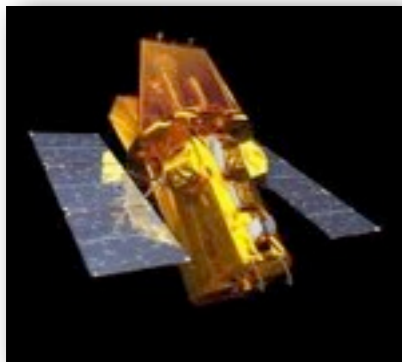
# Black Holes & Gravitational Waves: The Quest to Verify Einstein's Predictions

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Center for Relativistic Astrophysics  
School of Physics  
Georgia Tech  
visiting TAT, University of Tuebingen

# New Era in Gravity

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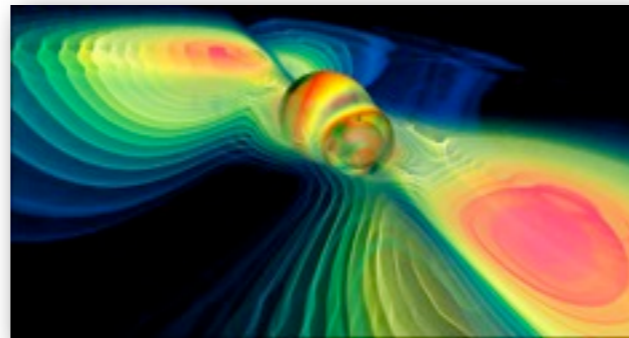
- Gravitational physics will be driven by data.
- That data is gravitational waves coupled with other messengers.
- Numerical relativity plays a fundamental role in this new era by
  - predicting and characterizing sources of gravitational radiation and
  - unveiling strong-field gravity in the universe.



SWIFT



LIGO, Livingston



Gravitational Waves from BBH  
(credit: MPI for Gravitational Physics/W.Benger-ZIB)



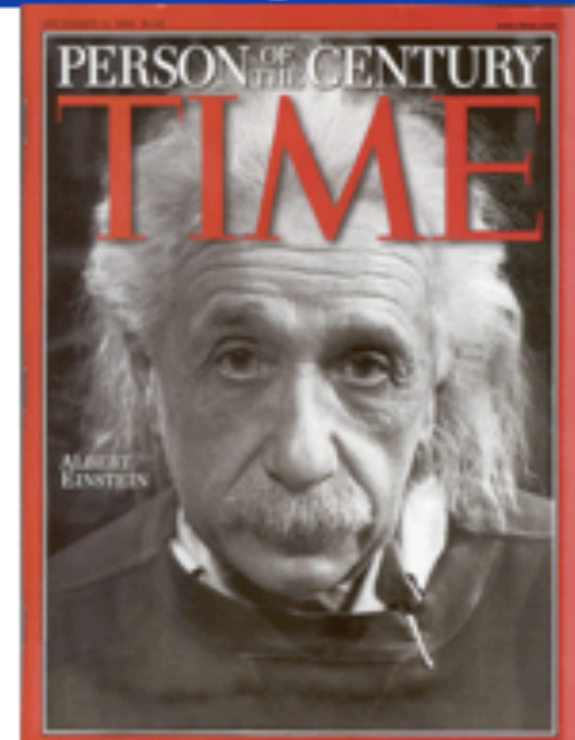
NASA

# A Brief History of Numerical Relativity

by Larry Smarr

## The Problem of the Century Posed by the Person of the Century

- 1910s-General Theory; Schwarzschild
- 1920s-Equation of Motion Posed
- 1930s-Two Body Problem Posed
- 1940s-Cauchy Problem Posed
- 1950s-Numerical Relativity Conceived
- 1960s-Geometrodynamics; First Numerical Attempts
- 1970s-Head-On Spacetime Roughed Out
- 1980s-Numerical Relativity Becomes a Field
- 1990s-Head-On Nailed; 3D Dynamics Begin
- 2000s-3D Dynamics Nailed; Grav. Wave Astronomy

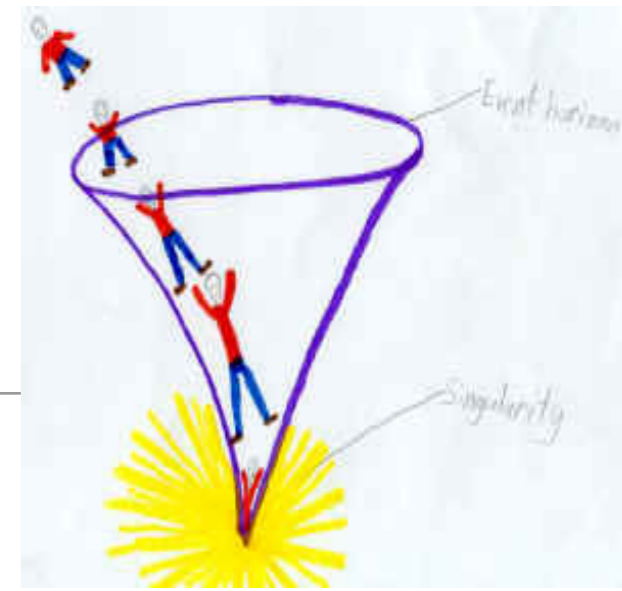


# The Two-Body Problem in General Relativity

Shapiro and Teukolsky 1985

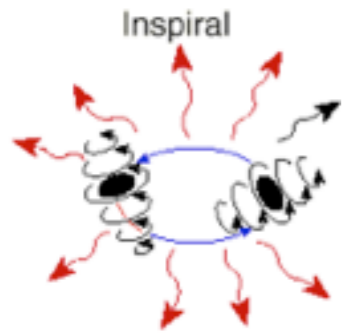
“... the Holy Grail of numerical relativity: a code that simultaneously avoids singularities, handles black holes, maintains high accuracy, and runs forever.”

- Simple? It is pure curvature, no messy neutron stars.
- Black Holes are complicated - there is a physical singularity!
- Black Holes are simple - described by mass and spin (and charge).
- Several orders of magnitude need to be resolved - mass of the black hole & wavelength of the waveform.
- It took over 20 years to successfully orbit, merge and ringdown two black holes.



# Anatomy of Binary Black Hole

BBH spacetime: equations of motion telling us how the black holes are moving and the gravitational radiation (waves) that are emitted as they interact.



Far apart: approximated with post-Newtonian models  
assume slow moving point particles

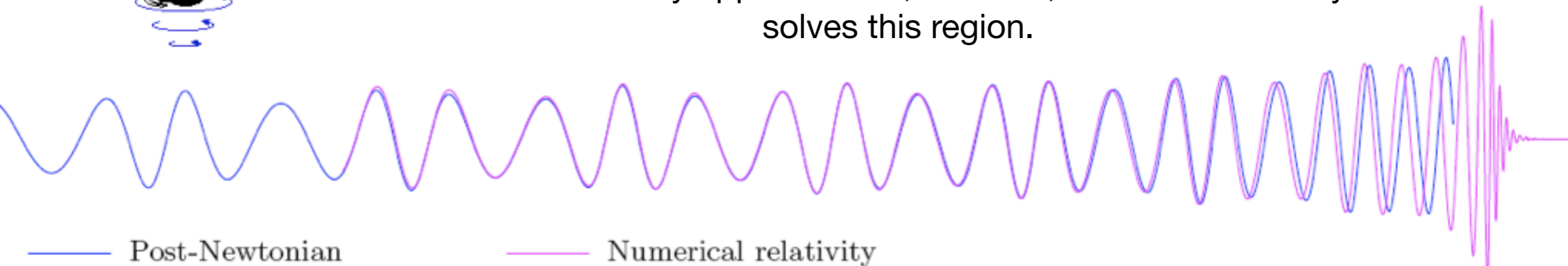


Close together: black holes move rapidly the nature of the black holes  
becomes important  
Numerical relativity solves the full Einstein equations

Ringdown



End state: single, larger Kerr black hole  
Perturbation theory approximates; however, numerical relativity also  
solves this region.

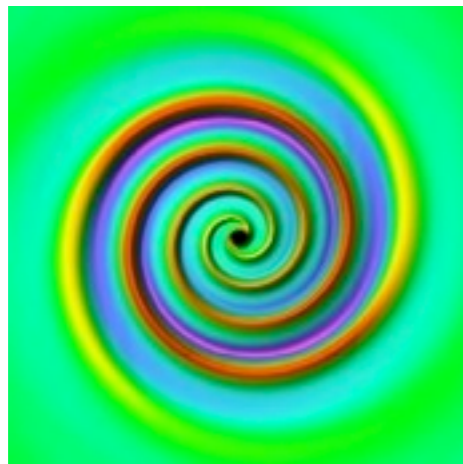


# Binary Black Hole Problem “Solved”

FIRST ORBIT by Bruegmann, Tichy and Jansen  
PRL 92:211101 (2004)

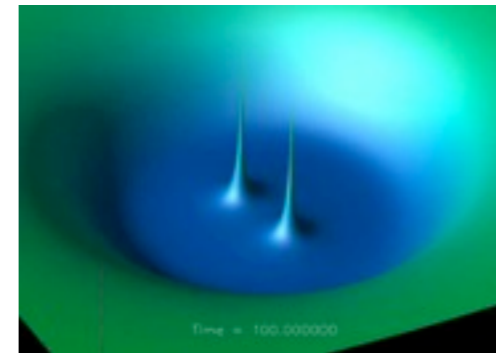
Ian Hinder

2005 Pretorius  
Binary inspiral and merger



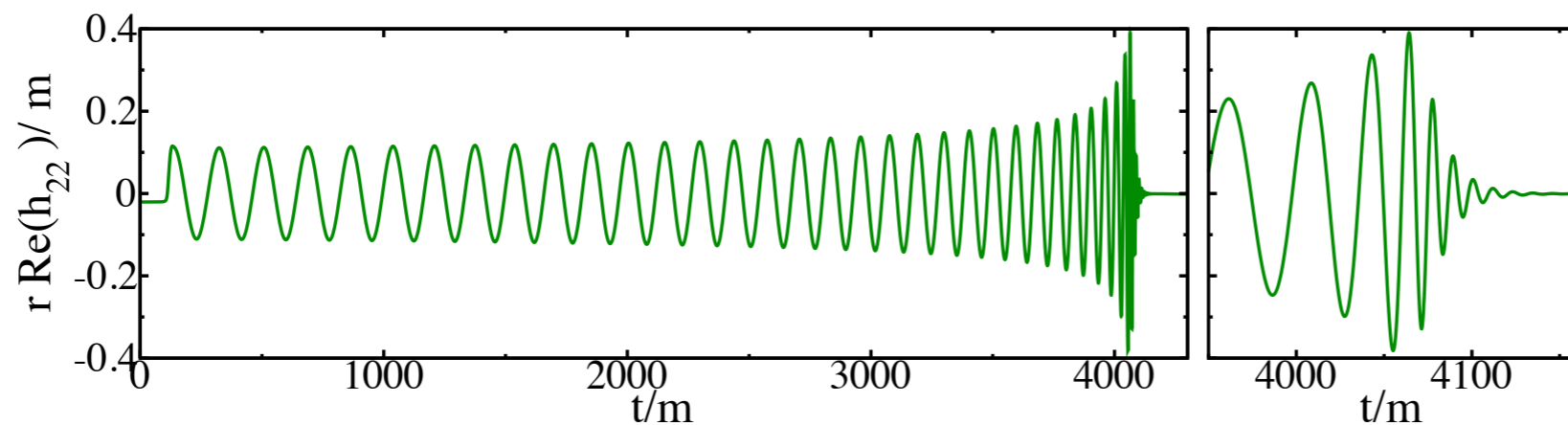
PRL 95:121101 (2005)

2006 RIT and NASA  
Moving Punctures Method



Campanelli, Lousto, Zlochower  
PRL 96:111101 (2006)

Baker, Centrella, Choi,  
Koppitz, van Meter  
PRL 96:111102 (2006)



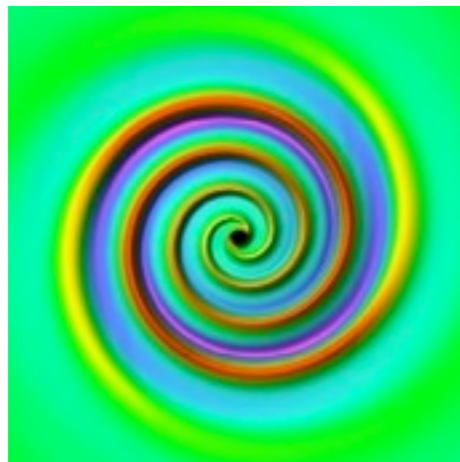
# Binary Black Hole Problem “Solved”



Ian Hinder

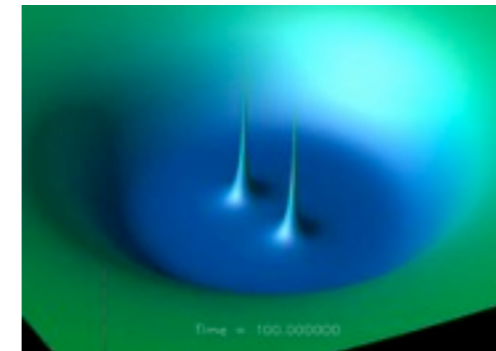
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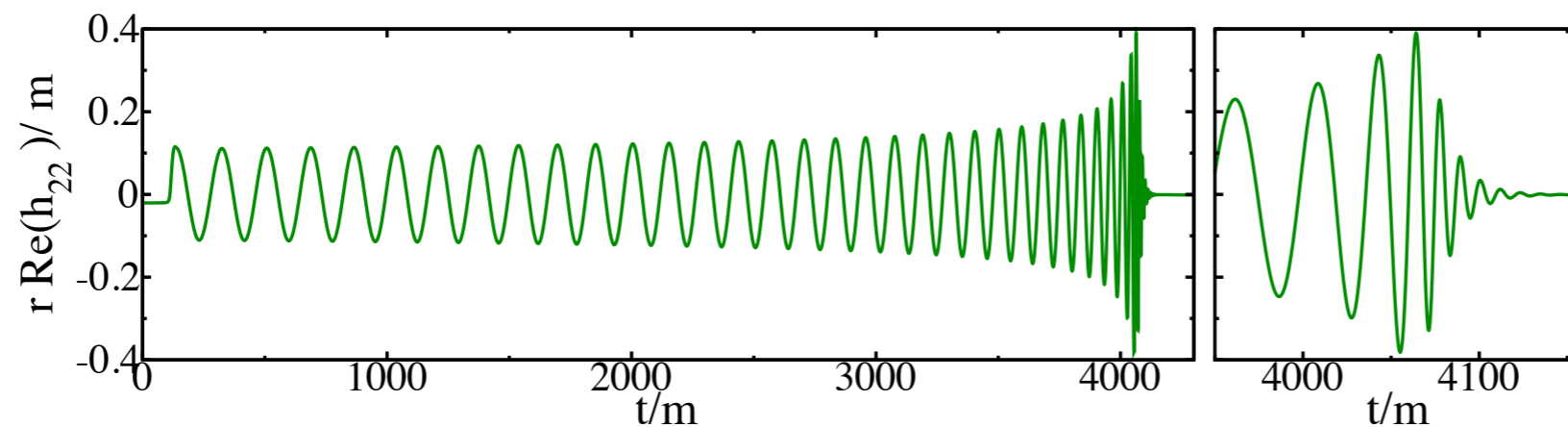
PRL 95:121101 (2005)

2006 RIT and NASA  
Moving Punctures Method



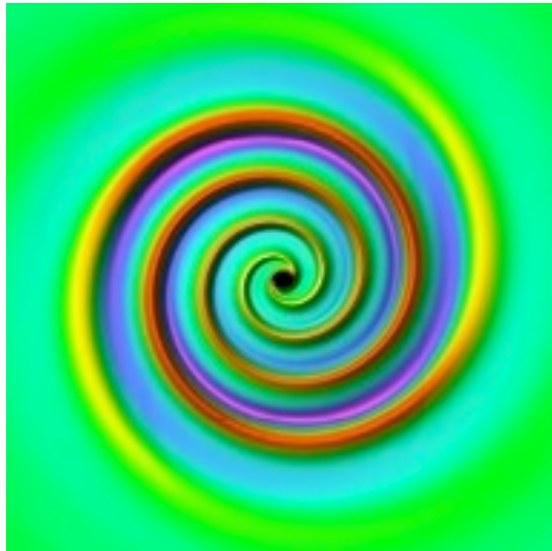
Campanelli, Lousto, Zlochower  
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Baker, Centrella, Choi,  
Koppitz, van Meter  
PRL 96:111102 (2006)

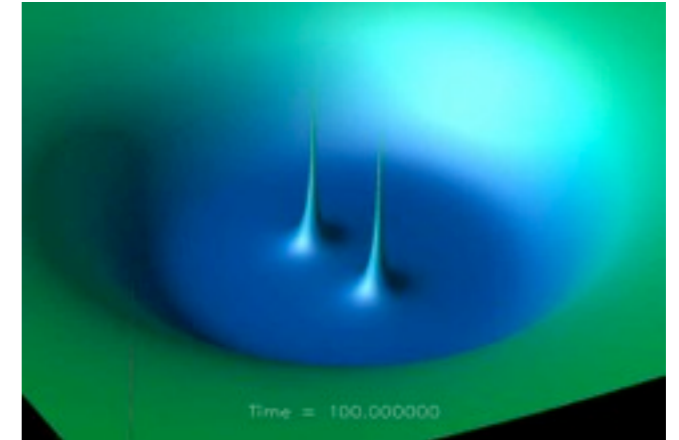


# Mathematical Infrastructure

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Two sets of equations and gauges have been successful.



## Generalized Harmonic Equations

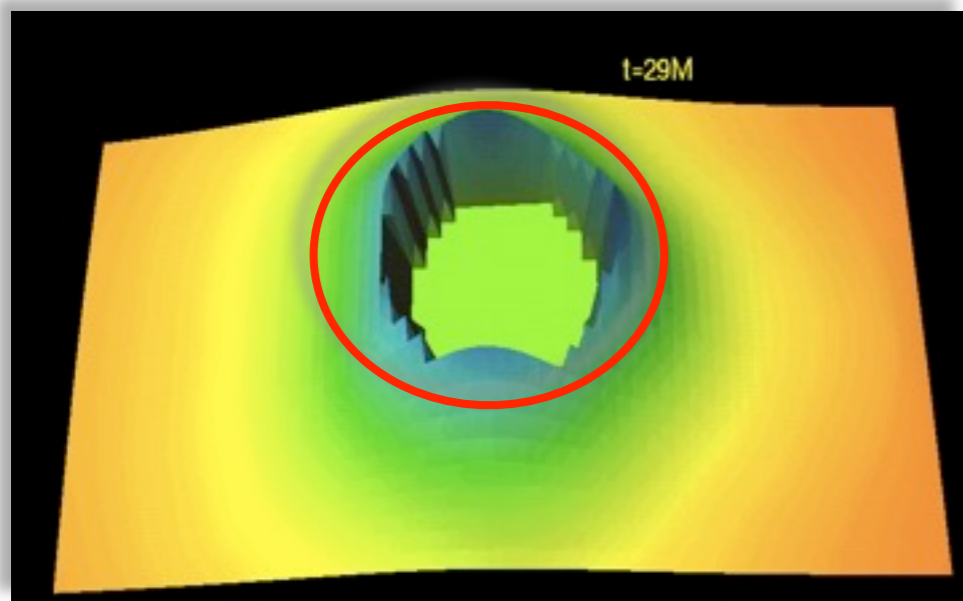
- Y. Bruhat (in Gravitation: an introduction to current research)
- Pretorius (CQG 22 (2005) 425)
- Lindblom et al (CQG 23 (2006) S447)
- Many others: Garfinkle (PRD 65 (2002) 044029), Gundlach (CQG 22 (2005) 3767), Winicour & Co., Friedrich

## BSSN Equations

- Nakamura et al (PTPS 90 (1987) 1)
- Shibata and Nakamura (PRD 52 (1995) 542)
- Baumgarte and Shapiro (PRD 59 (1999) 024007)
- Many groups use it for BBH  
Jena, AEI, LSU, RIT, NASA, GT, FAU, ...

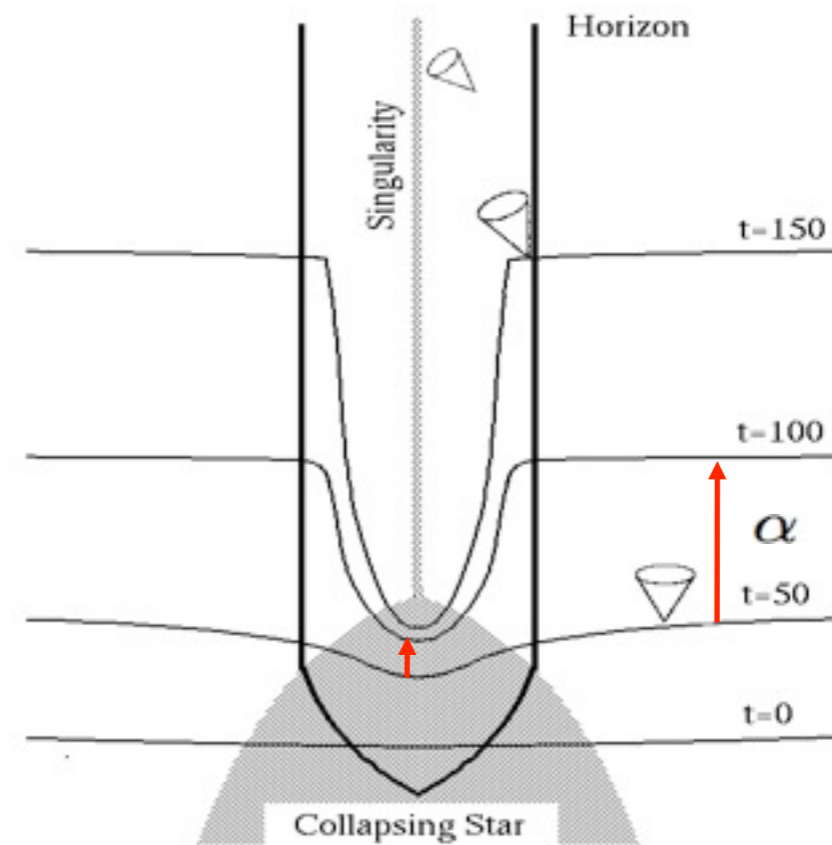


# Black Hole Singularity: Two Approaches



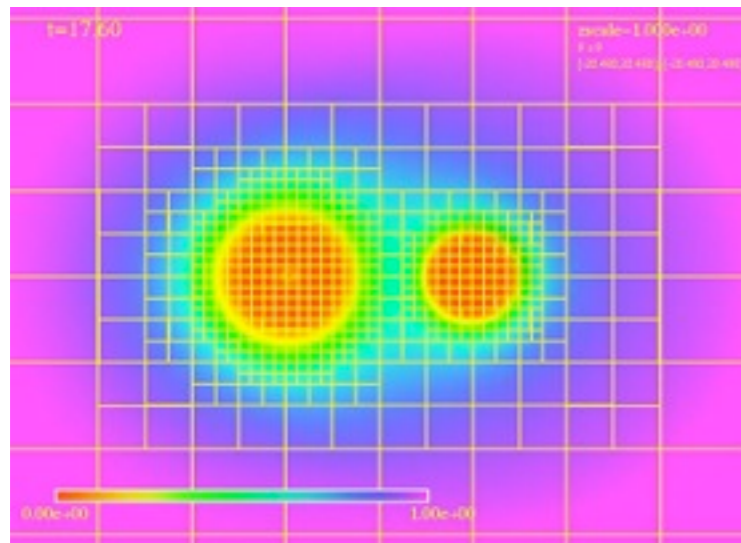
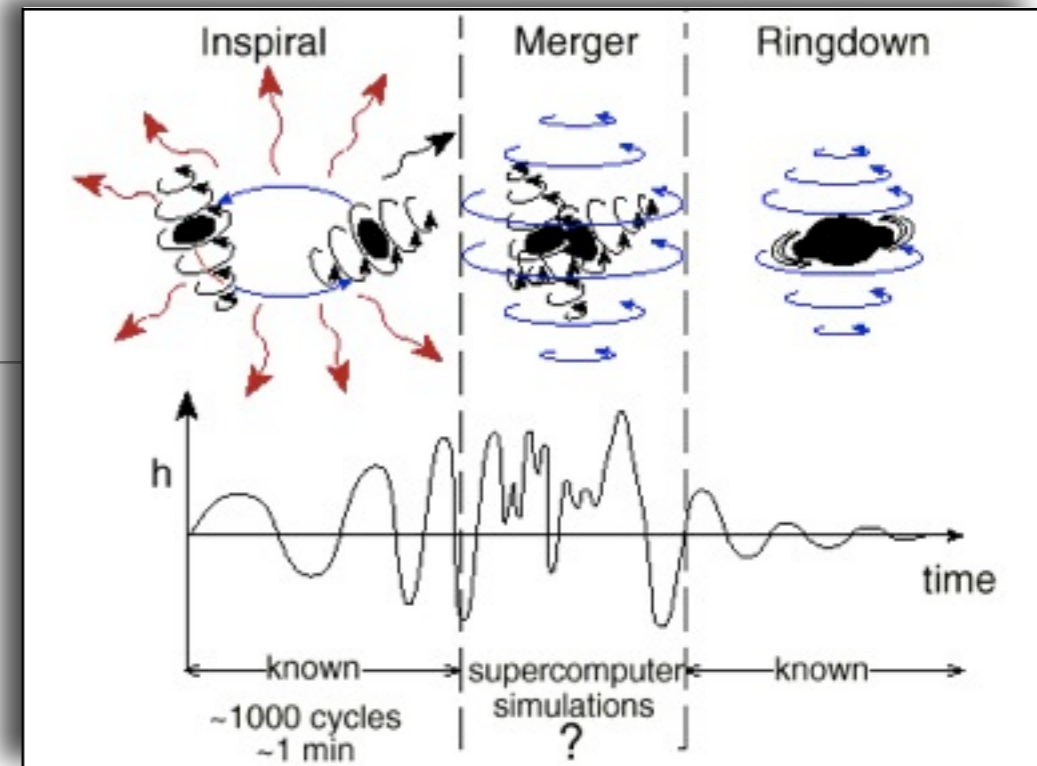
Harmonic coordinate conditions

(Pretorius CQG 22 (2005) 425, Garfinkle PRD 65 (2002) 044029, Lindblom et al CQG 23 (2006) S447)

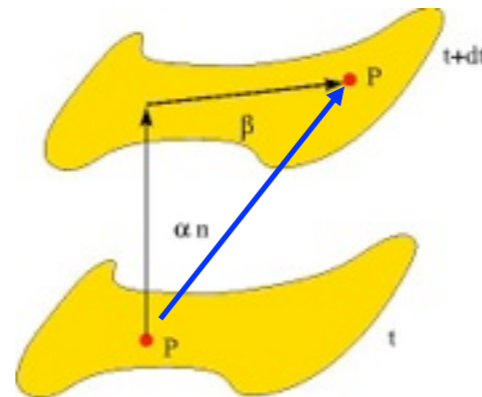


1+log lapse condition and  
Gamma-driver shift condition  
(Alcubierre et al PRD 67 (2003) 084023)

# What made the Breakthrough Possible?



Dale Choi (NASA-GSFC)



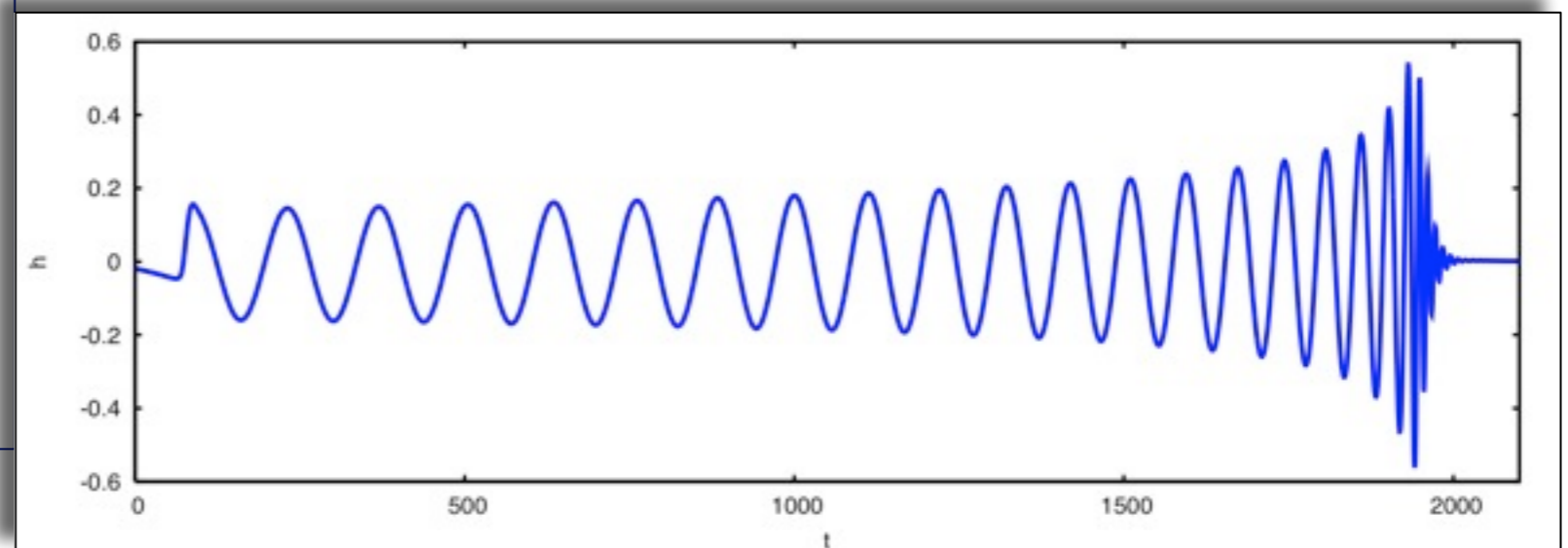
Why did it take so long?

We did not have the appropriate package of **Mathematical Tools** (e.g. Gauges, Formulations) and **Computational Infrastructure** (e.g. Adaptive Mesh Refinements, Hardware, etc.)

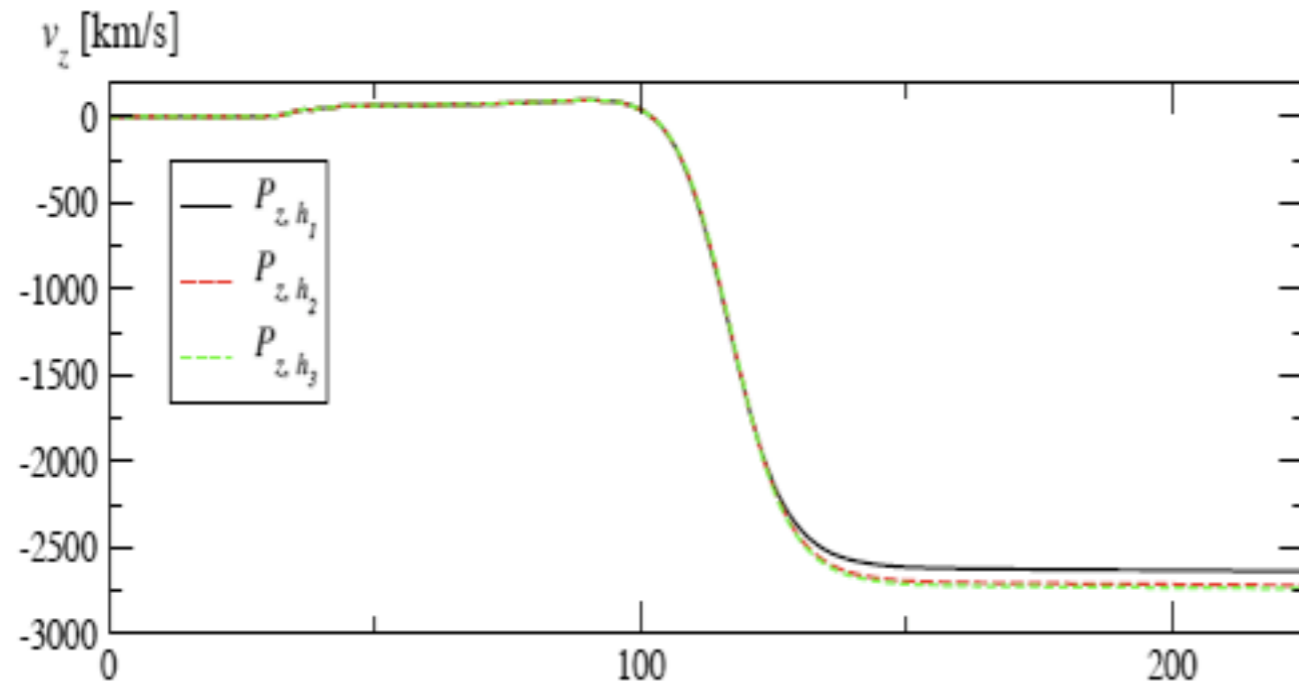
Holy Grail Obtained!

- Orbital Hang Ups
- Gravitational Recoil
- Black Hole Remnant
- Black Hole Triplets
- Extreme Orbits
- Gravitational Wave

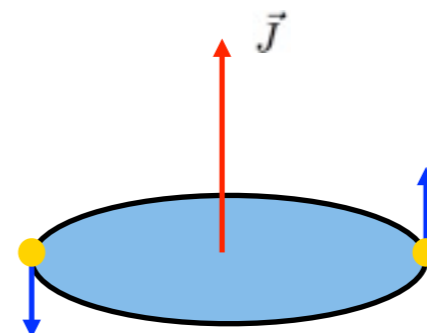
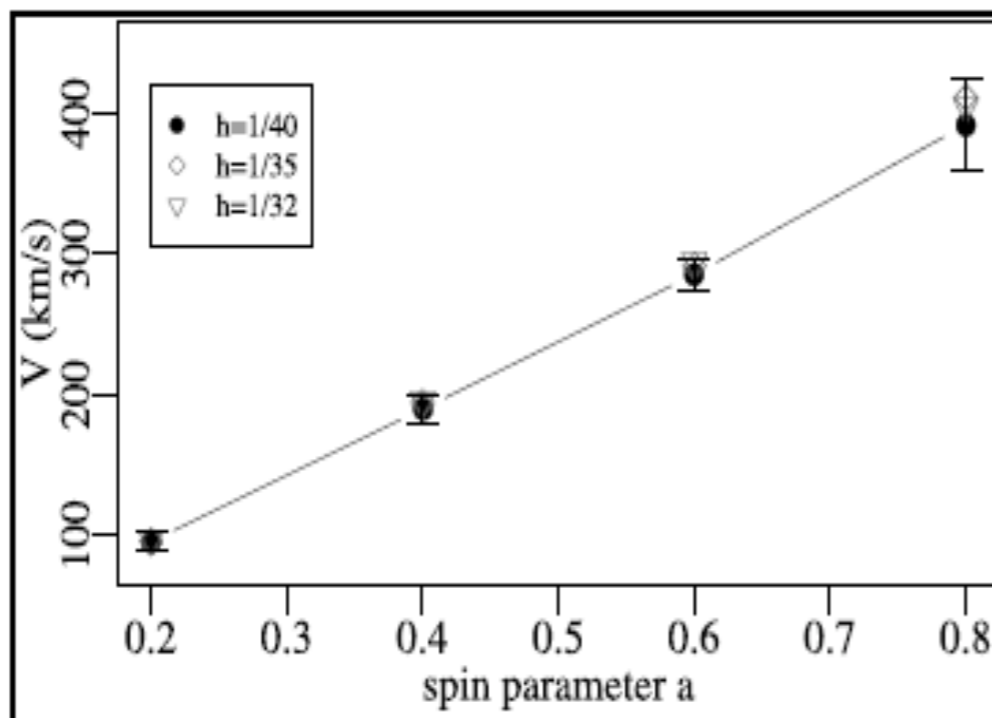
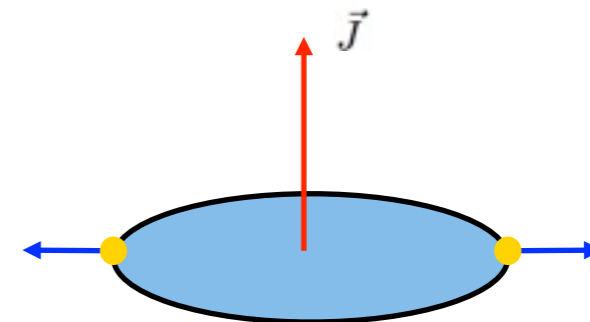
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# Gravitational Wave Recoil

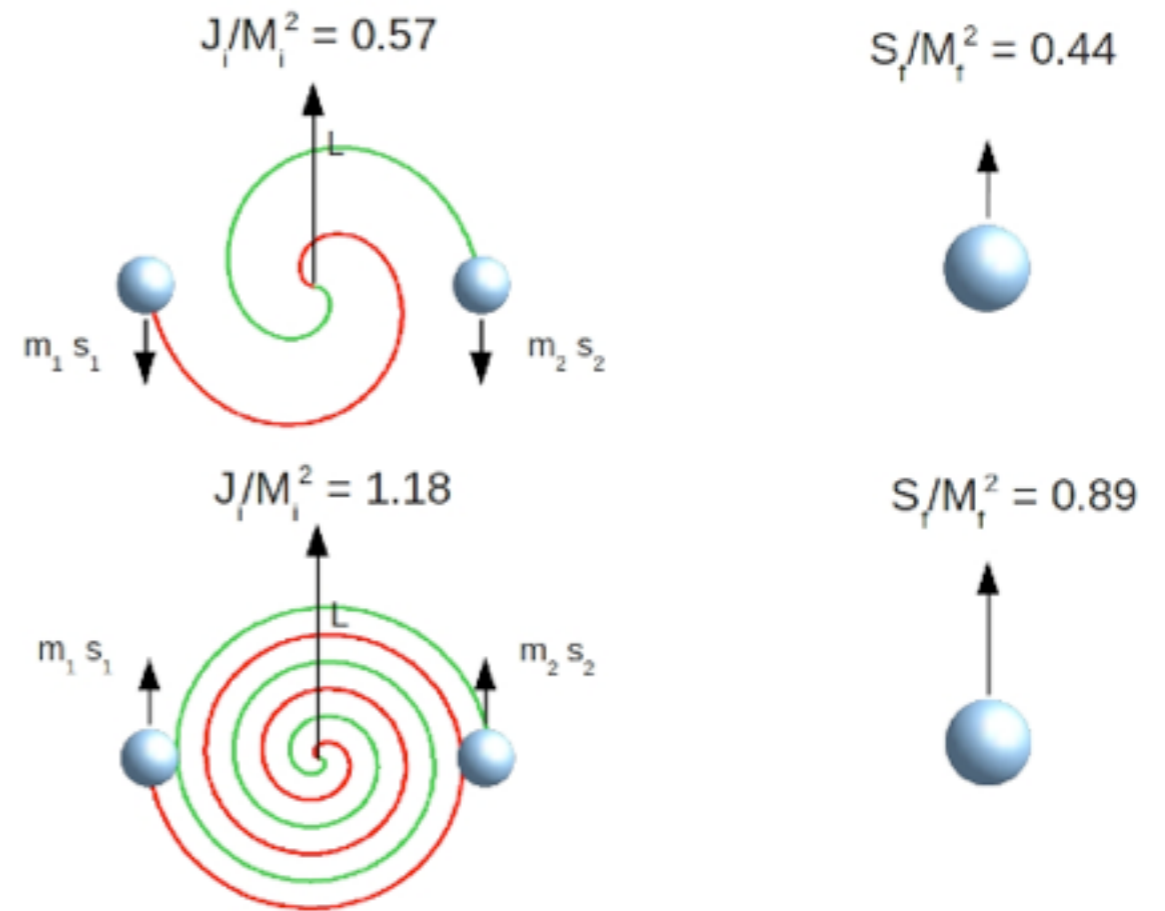


Gonzalez et al 2007  
RIT 2007 & 2008



Herrmann, Hinder,  
DS, Laguna,  
Matzner, 2007

# Spin Hangup



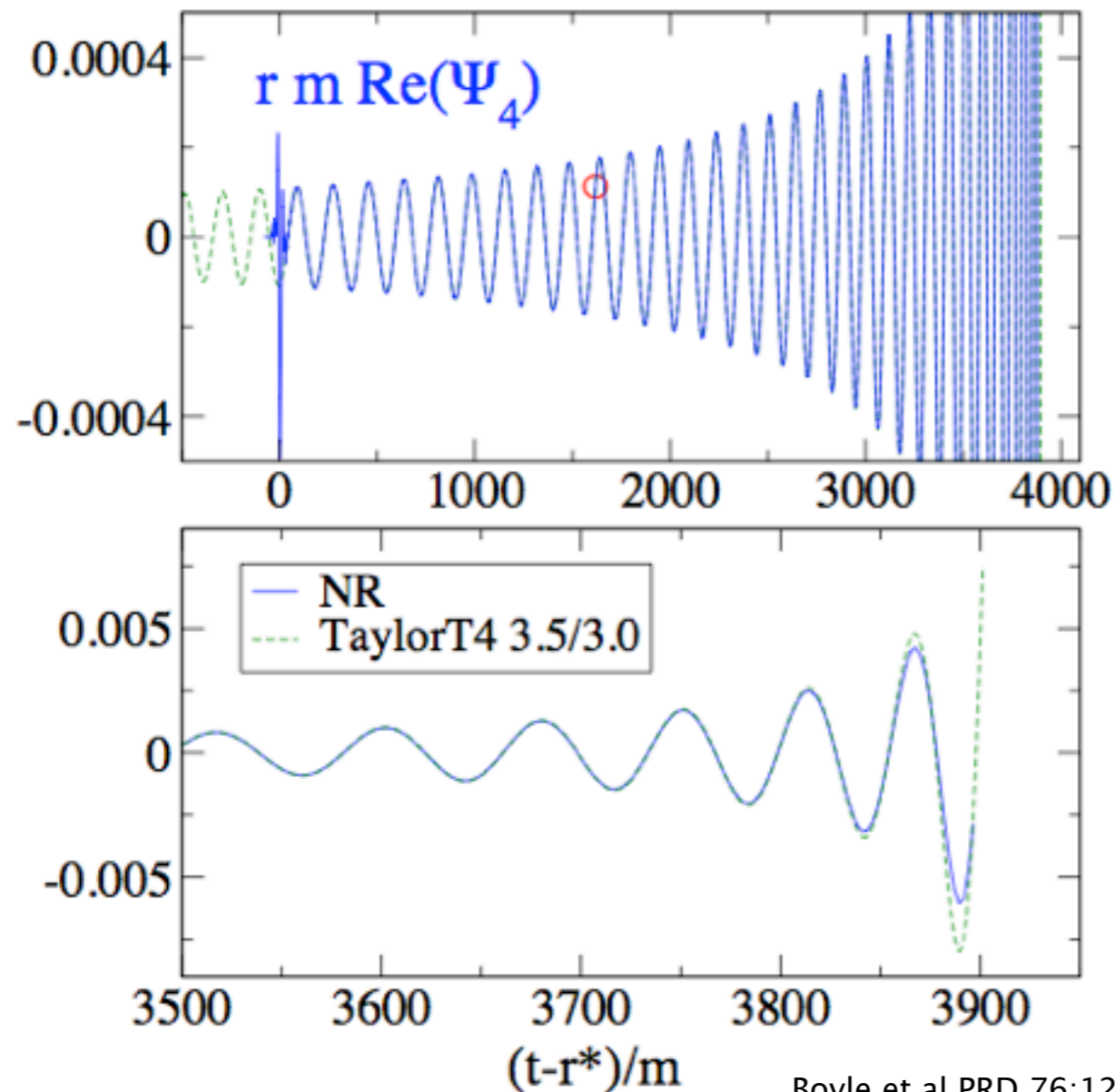
From Marronetti & Tichy 2011

- Campanelli et al 2006
- Berti & Jena 2007
- Dain et al 2008

# NR and PN Comparisons

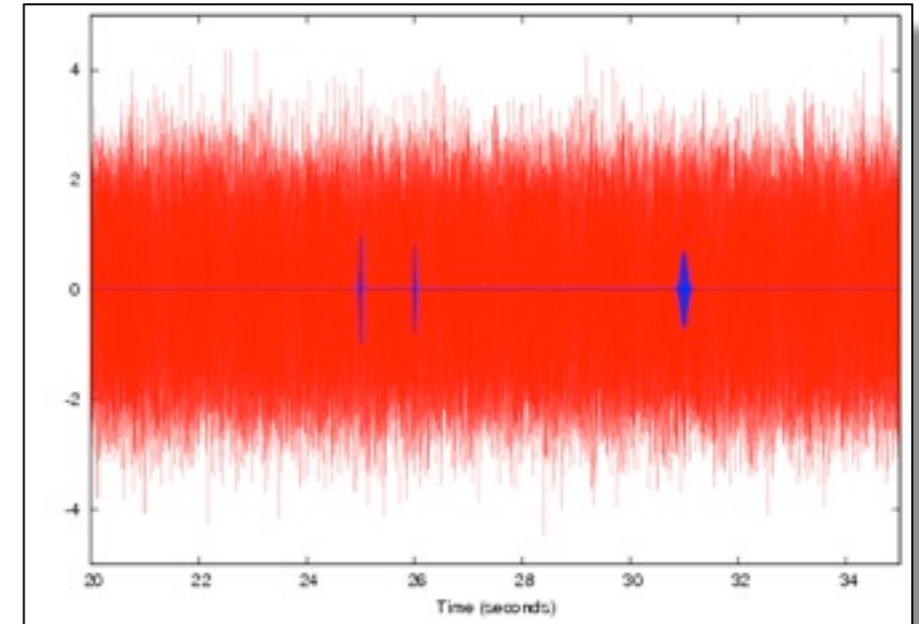
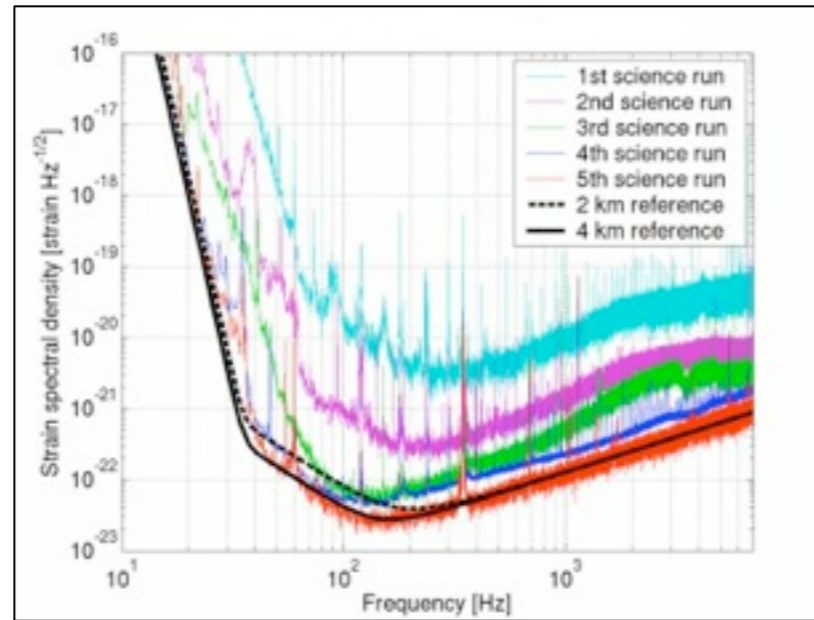


Equal-mass, non-spinning comparisons agree up to 2-3 orbits before merger.

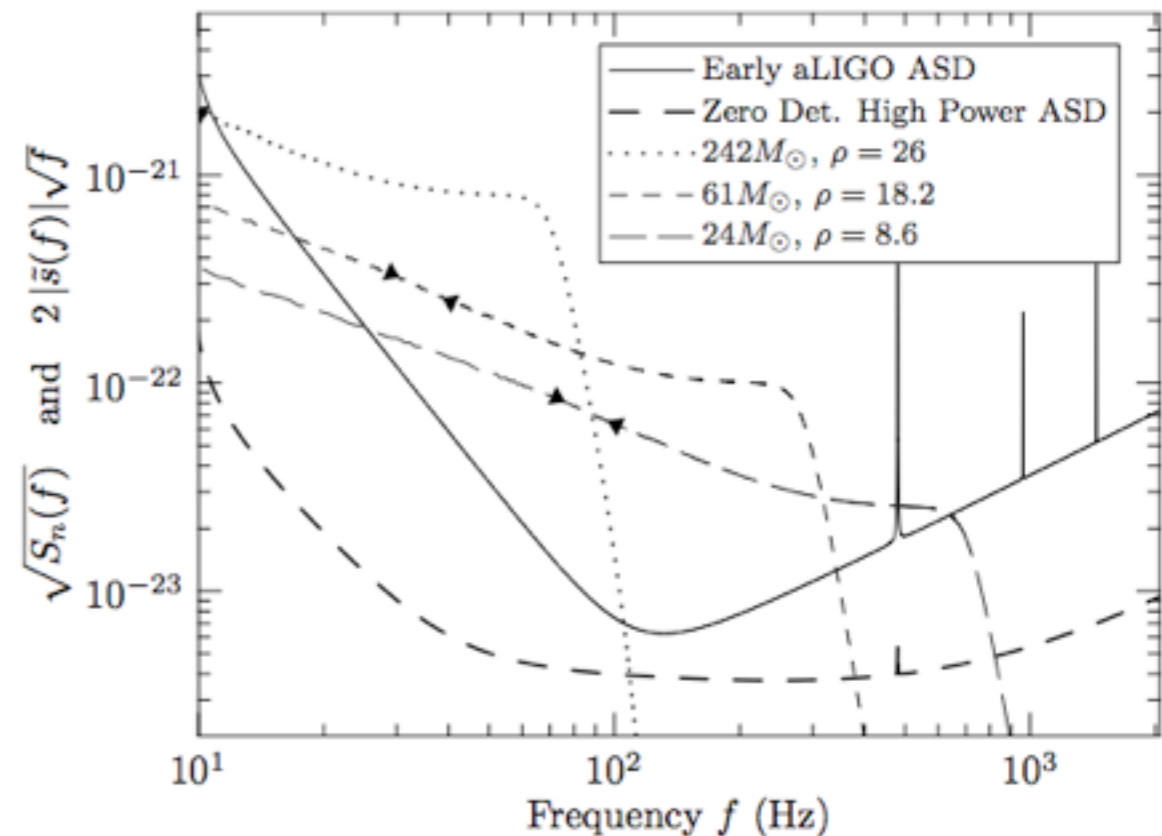
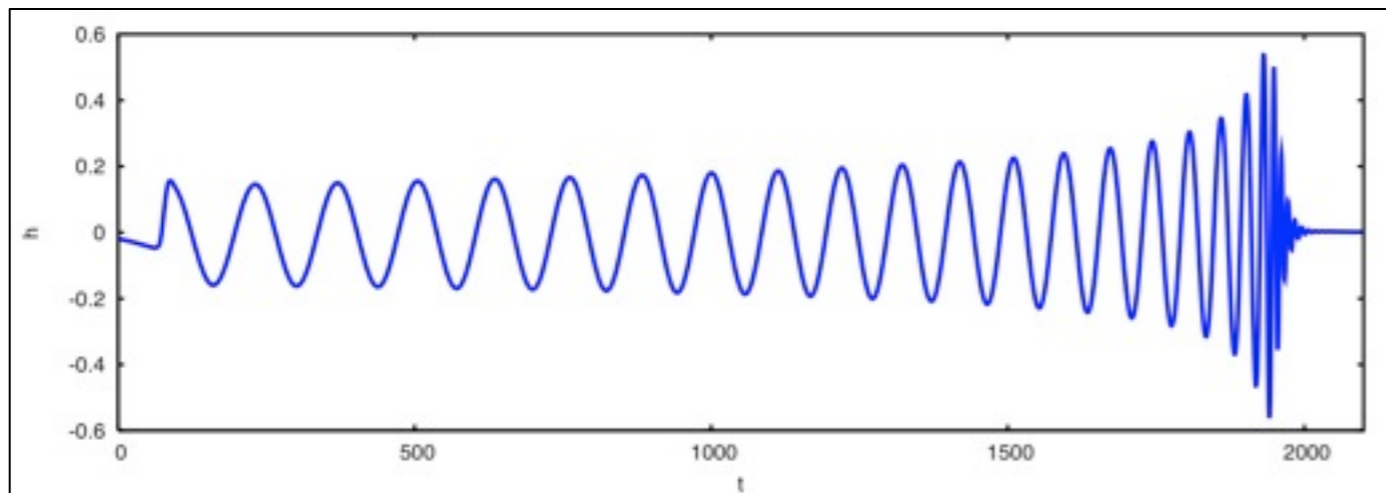


- Equal-mass, non-spinning:
  - Baker et al PRL 99, 181101 (2007)
  - Boyle et al PRD 76 (2007) - Finds  $< 0.05$  radians over 30 cycles for 3.5 PN
  - Hannam et al PRD 77 (2008)
- Non-spinning: Pan et al PRD 77 (2008)
- Equal-mass, aligned spin: Hannam et al PRD78 (2008)
- Eccentric: Hinder et al PRD 82 (2010)

# The Driving Force behind Numerical Relativity



*Numerical waveforms are essential to predict what to expect and aid detection*

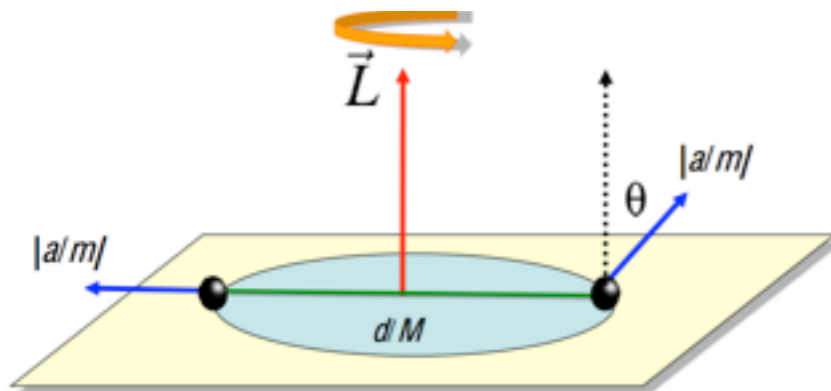


*Fortunately, the two-body problem in GR is progressing*

# Challenge: Template Banks

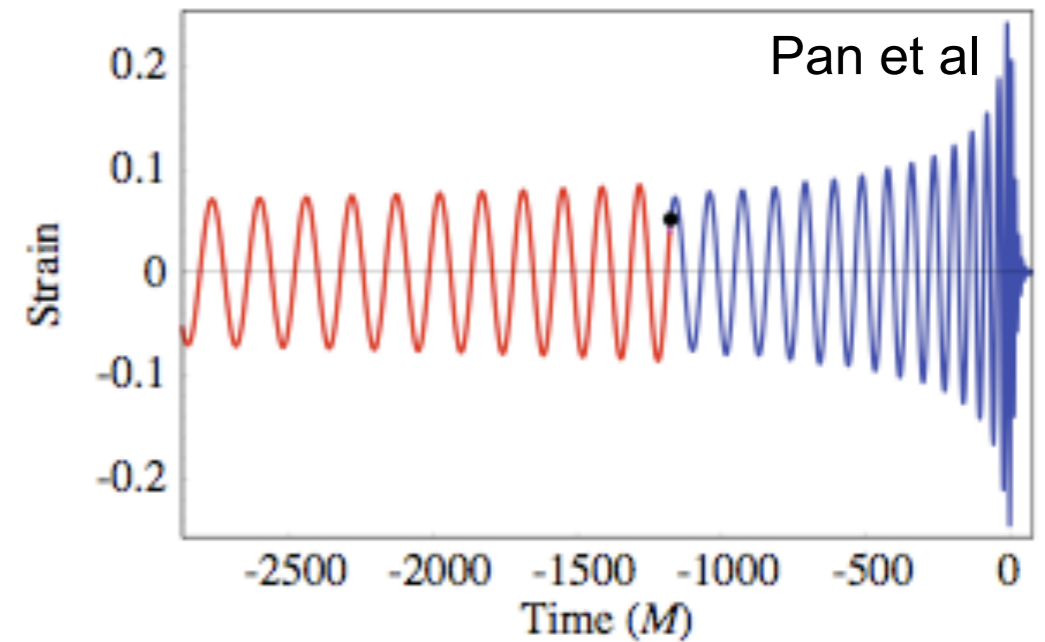
## Parameter Space:

- Masses (2)
- Spins (6)
- Eccentricity (1)
- Sky location (3)
- Time of arrival
- Phase at arrival
- Distance

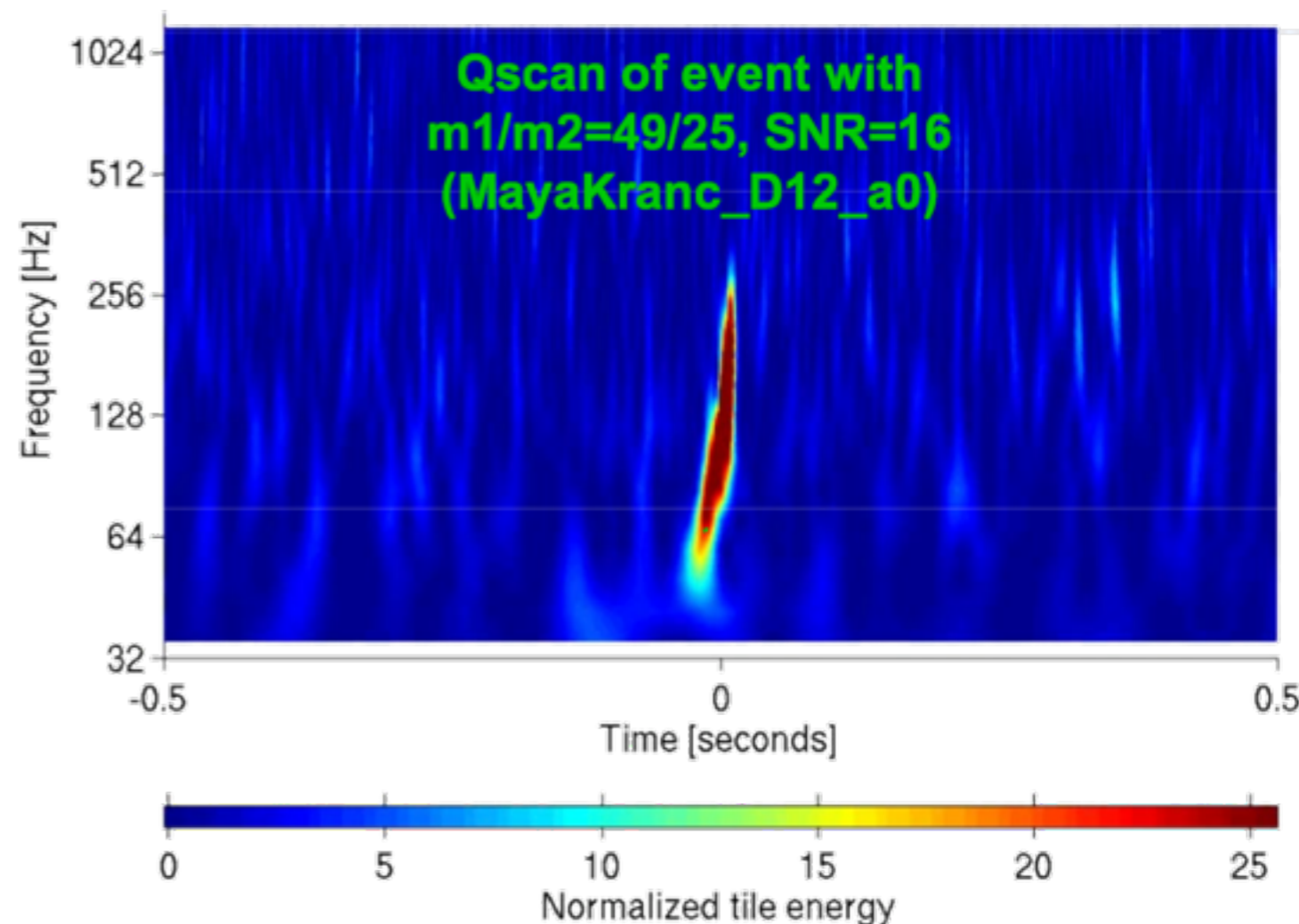


## Dirty Laundry & Needs:

- Efficiency of codes
- Limited parameter exploration



Channel 1 at 871158154.800 with Q of 5.7



# Meeting the challenge



- NINJA (Numerical Injection Analysis) Project
  - collaboration between data analysts and numerical relativists to test LIGO/VIRGO ability to detect binary black hole waveforms
  - **NINJA 1 successful built community collaboration** (Ninja Collaboration 2009 CQG 26 165008 and 114008)
  - 10 numerical relativity groups submitted 23 waveforms produced by 9 independent codes of any BBH configuration
  - 9 data analysis groups added the NR waveforms to simulated, colored Gaussian noise and analyzed the results using burst detection, inspiral detection and parameter estimation
  - However, NR waveforms were very short and the noise was Gaussian
  - **NINJA 2 using spinning, non-precessing waveforms and real data** (Ninja2 Collaboration CQG 29 (2012) 124001)
  - Majority of NINJA 2 waveforms will be made public.

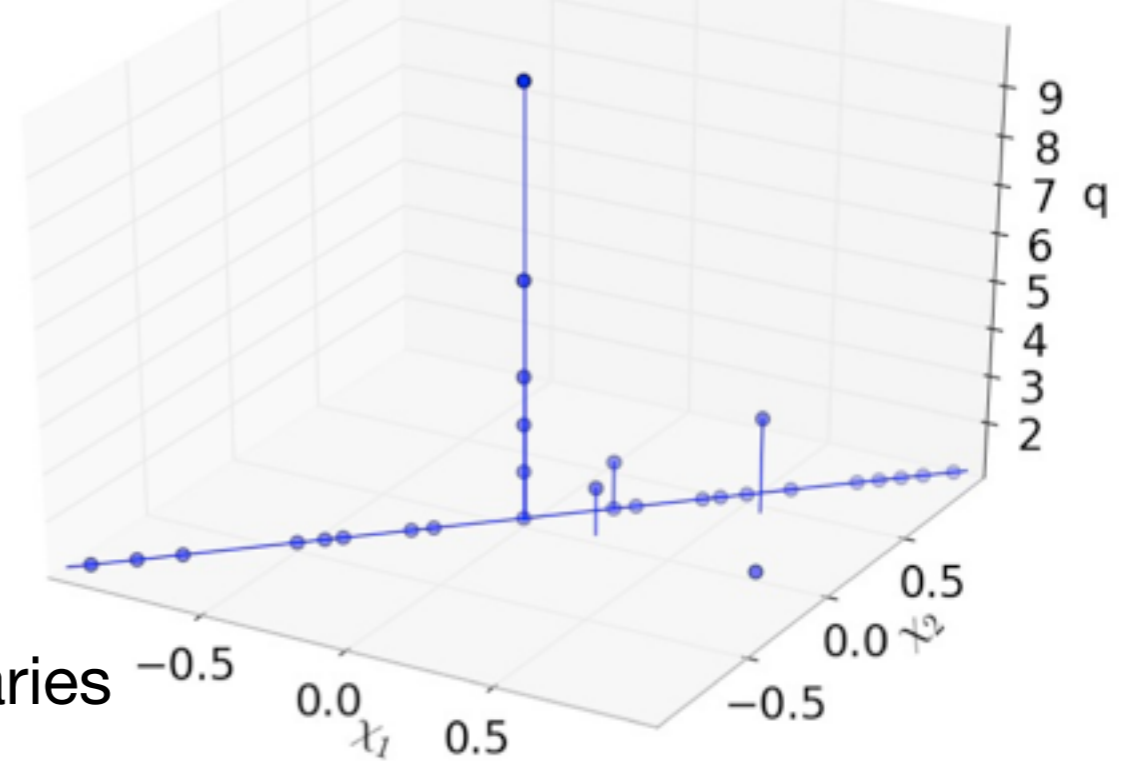




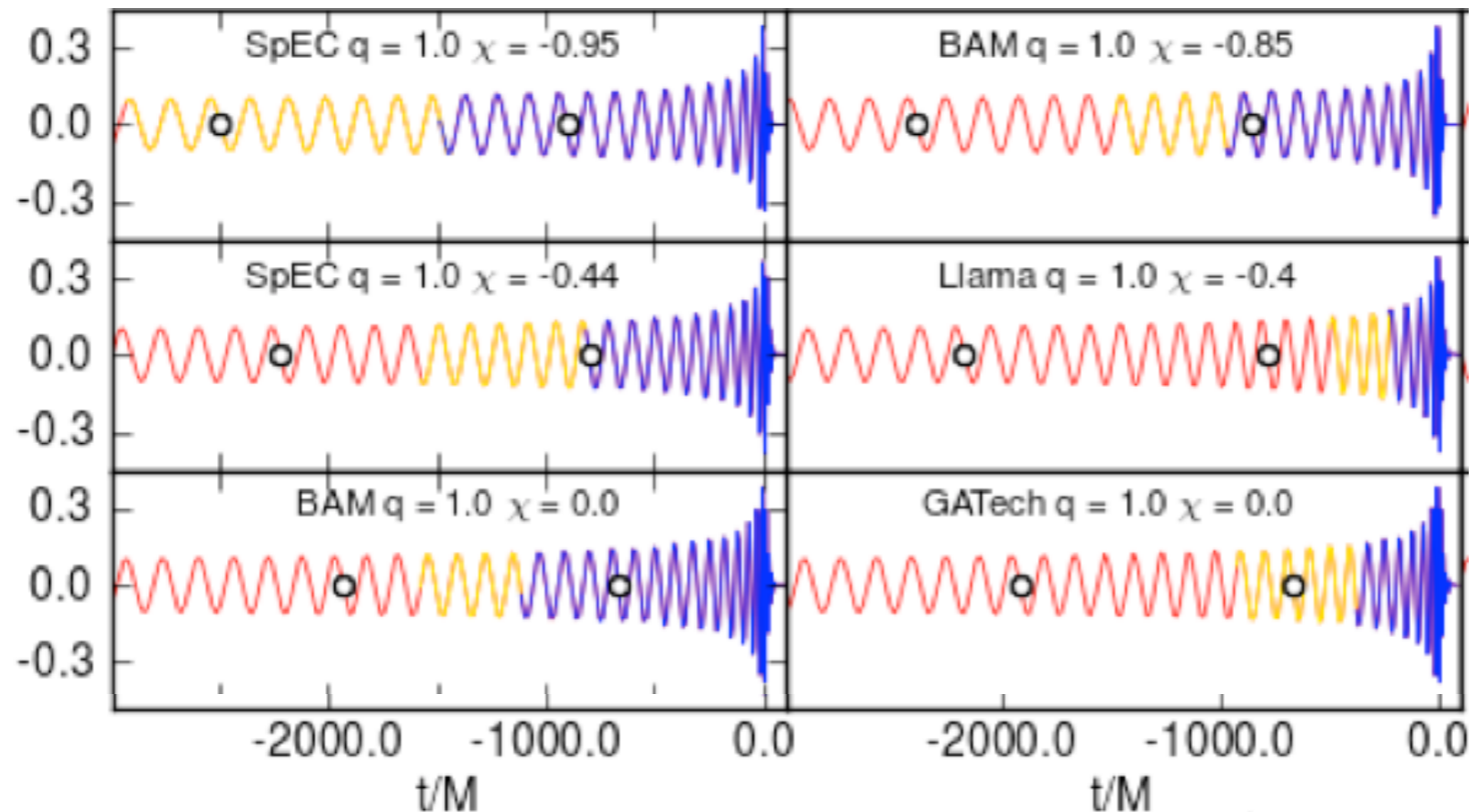
# NINJA 2

NR groups provided:

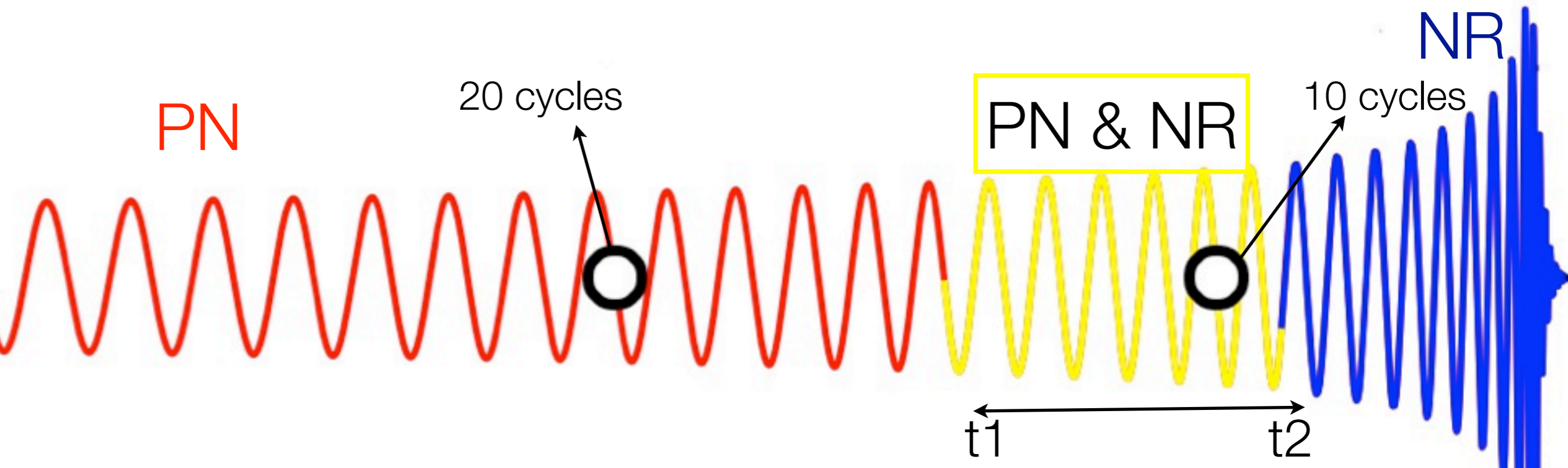
Ninja2 Collaboration CQG 29 (2012) 124001



- waveforms of non-precessing, spinning binaries
- hybrid waveforms that are created by stitching PN and NR waveforms together to produce waveforms that are relevant for the full mass range of the detectors

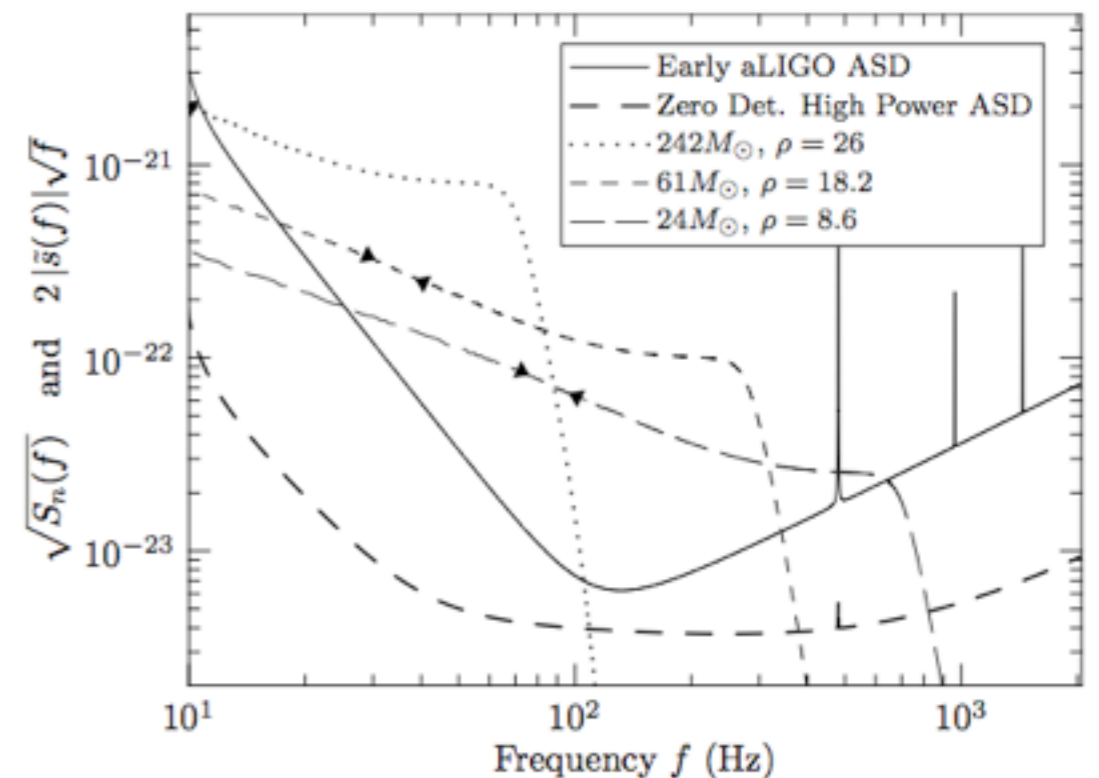


# Building Hybrid Waveforms for GW Detection



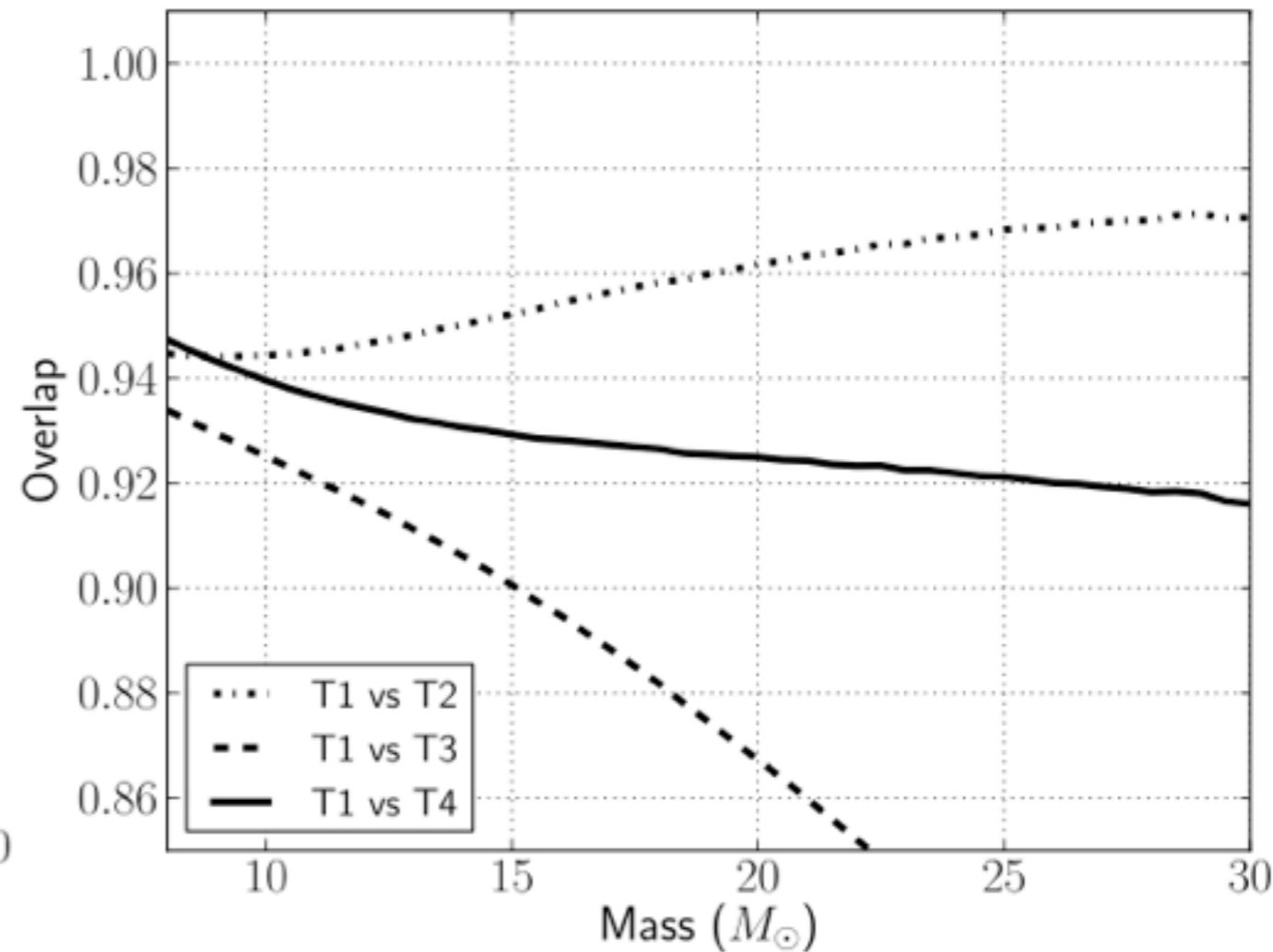
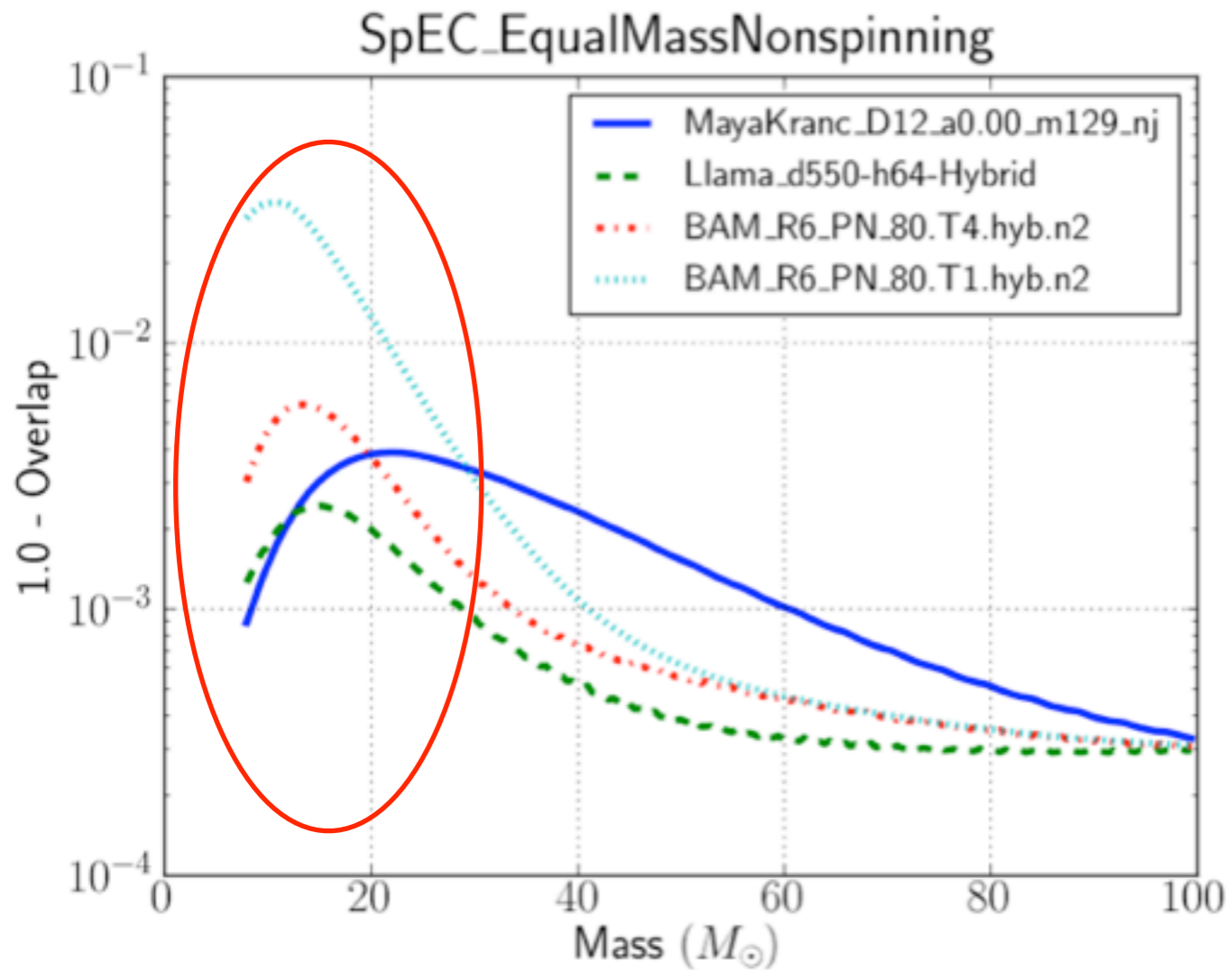
$$\delta(\vec{u}, a) = \min_{\vec{u}, a} \int_{t_2}^{t_1} |h_{PN}(t, \vec{u}) - ah_{NR}(t, \vec{u}_0)|^2 dt$$

$$\left. \begin{aligned} \vec{u} &= (t_{\text{shift}}, \phi_{\text{shift}}) \\ a &= 1 \end{aligned} \right\} \text{GT choice}$$



# Comparing the Waveforms

Ninja2 Collaboration CQG 29 (2012) 124001



$$(s_1 | s_2) = 4 \Re \int_0^{\infty} df \frac{\tilde{s}_1(f) \tilde{s}_2^*(f)}{S_n(f)}$$

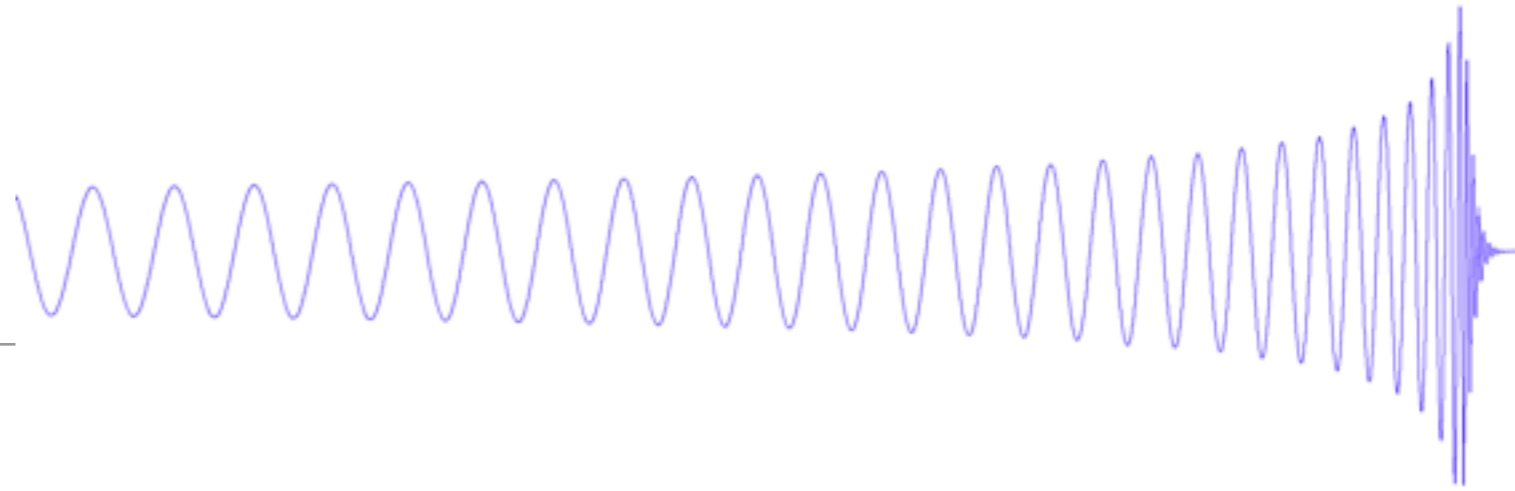
$$\langle s_1 | s_2 \rangle := \max_{\Delta t, \Delta \phi} \frac{(s_1 | s_2)}{\sqrt{(s_1 | s_1) (s_2 | s_2)}}$$

# Ramifications on GW Detection

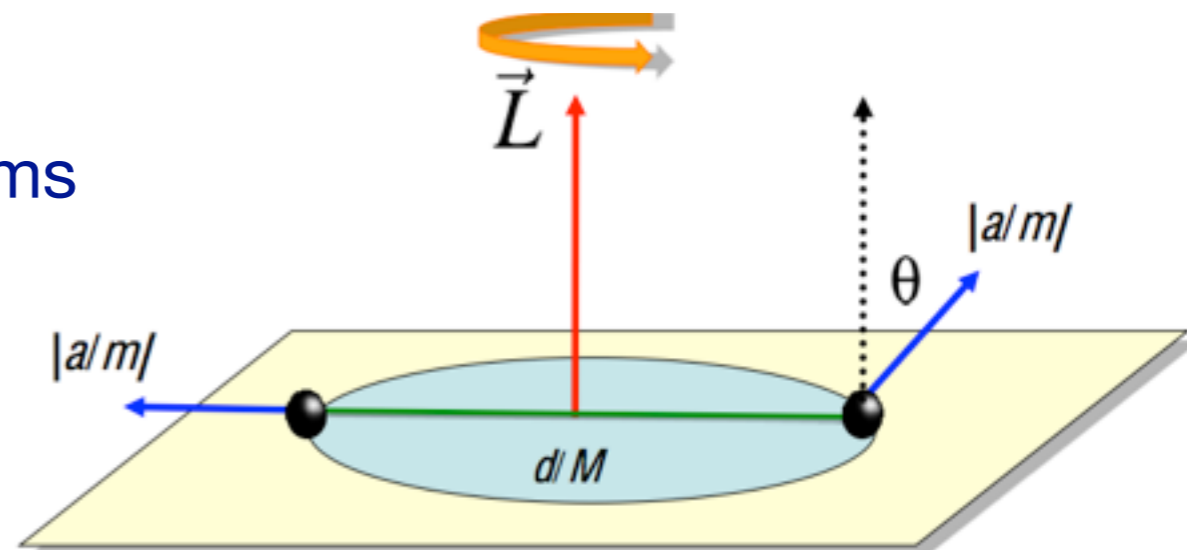
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- Work in progress for NINJA2 ...
  - testing the impact of the errors using data analysis pipelines
  - using LIGO/VIRGO data
  - testing how well existing searches do with signals that include all modes
- Expectation that these errors will not impact detection but may parameter estimation
- What can we do about reducing the errors?
  - Run longer NR simulations with black holes further apart thus reducing PN contribution
    - Ohme PRD 84:064029 (2011)
    - MacDonald et al CQG 28:134002 (2011)
  - Further orders of PN?
- Explore the parameter space more.

# Meeting the challenge



- NR-AR collaboration
  - collaboration between analytic and numerical relativists
  - science objectives
    - explore spinning, precessing parameter space
    - produce accurate analytical/phenomenological waveforms tuned to numerical relativity
    - Includes 13 million SUs on TeraGrid to perform the NR simulations dedicated to his project.
  - 9 NR groups produced about 30 waveforms
  - Currently analyzing waveforms in working groups



# Non-precessing systems in NR

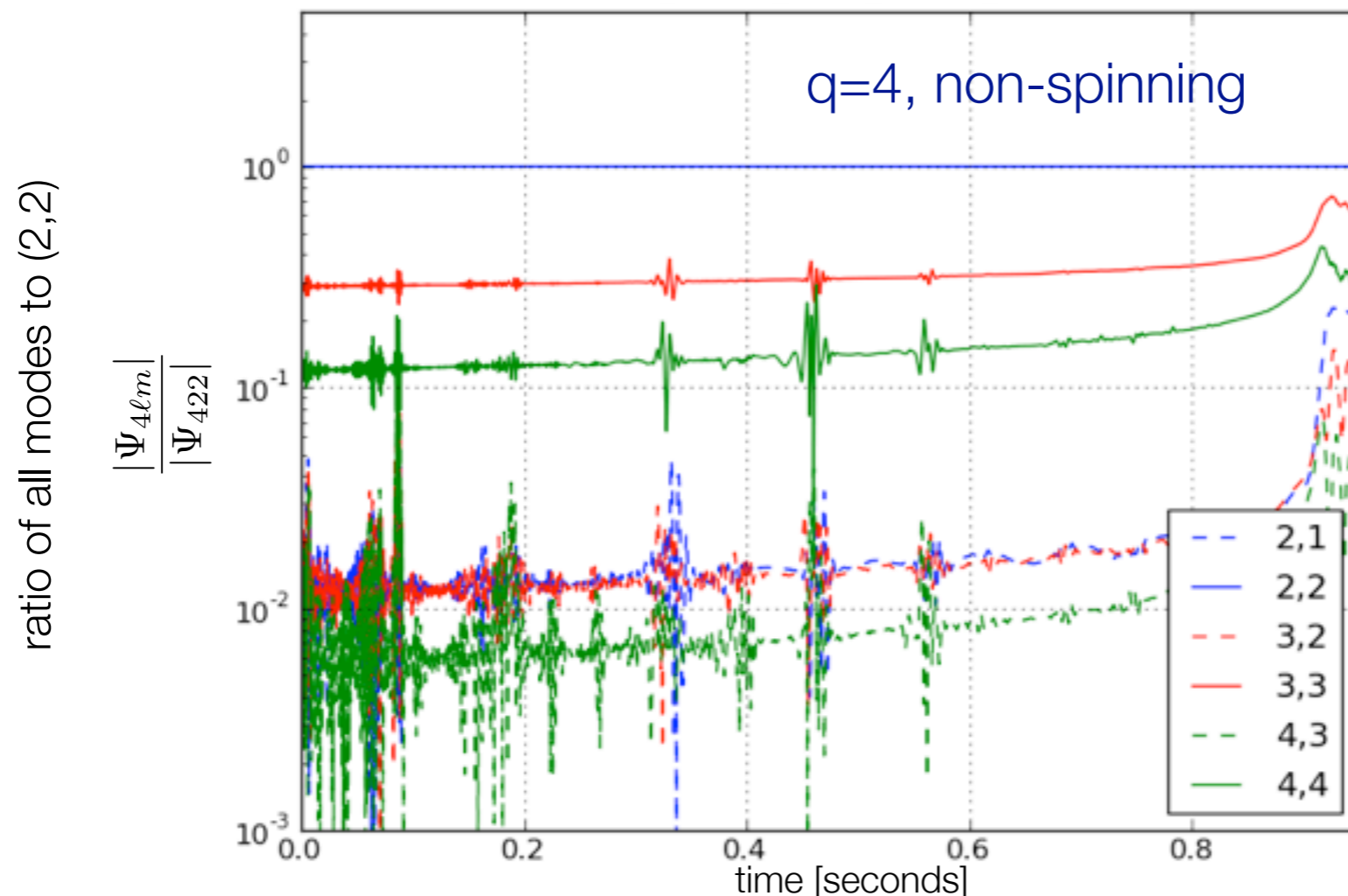
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- Phenomenological Waveforms of BBHs
  - Taracchini et al arXiv:1202.0790, Pan et al PRD84 (2011) 124052
  - Ajith et al PRL 106:241101 (2011), PRD 77:104017 (2009) and Santamaria et al PRD 82:064016 (2010)
  - Sturani et al JPCS 243:012007 (2010)
- Template banks (much harder with precession)
  - Field et al PRL 106:221102 (2011)
  - Cannon et al PRD83:084053 (2011)
- NINJA is testing current algorithms and banks against hybrid BBH signals.
- NRAR is building next generation of phenomenological waveforms.
- What about higher harmonics? Precessing Spins?

# Gravitational-wave Astronomy with NR waveforms

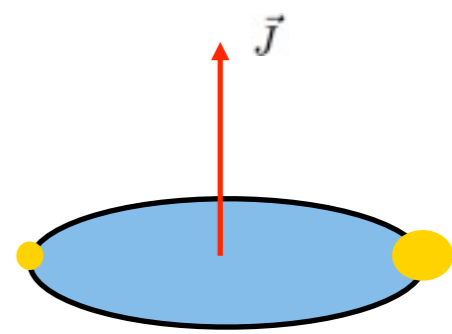
- Binary Black Holes merge in sweet-spot
- Explore the inclusion of higher-modes in templates - will they improve SNRs and hence range and detection rates?
- NR waveforms: a theoretical maximum

$$q = \frac{m_2}{m_1} > 1$$



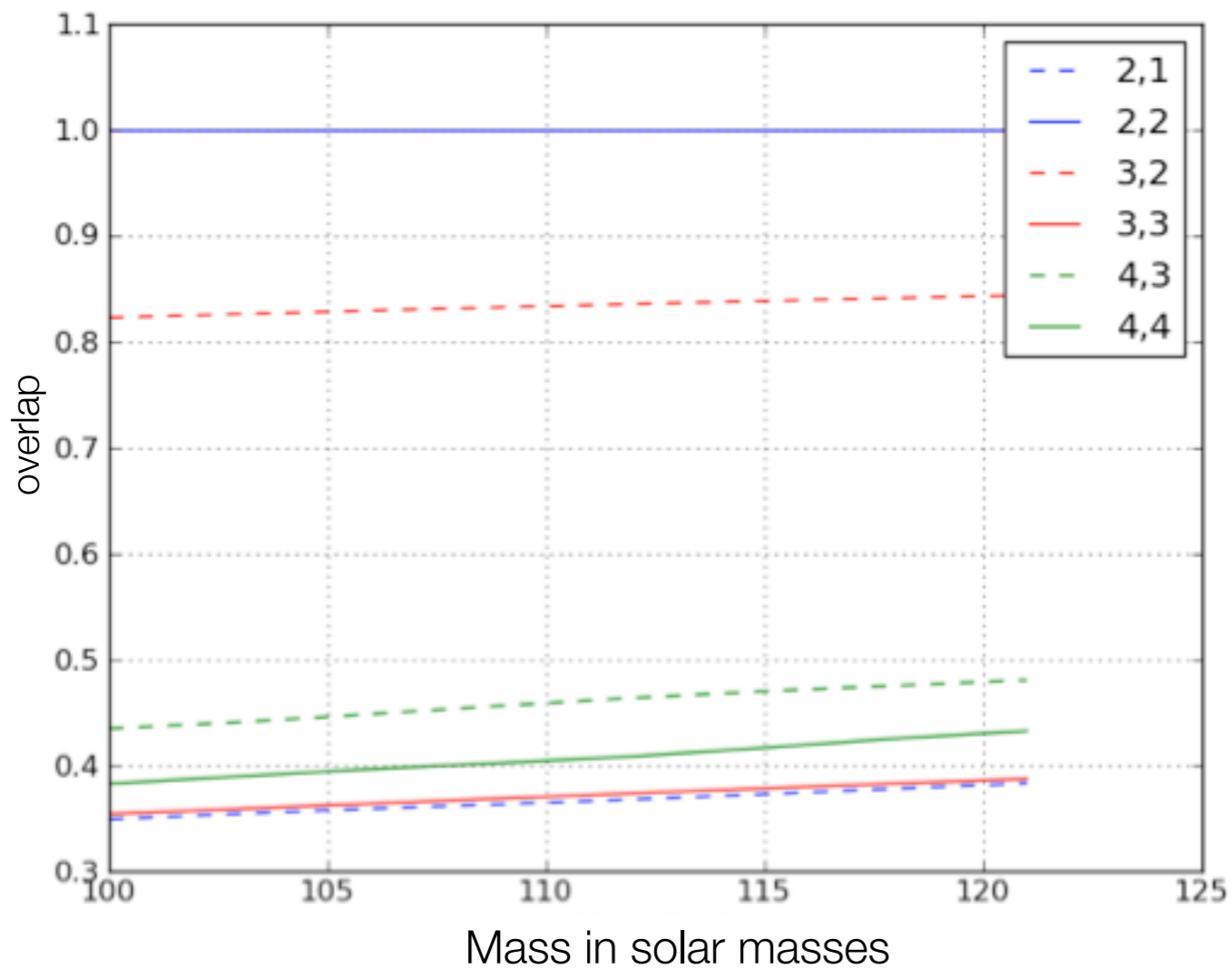
Work with  
Larne Pekowsky  
& James Healy, GT

# Overlap of modes with 2,2

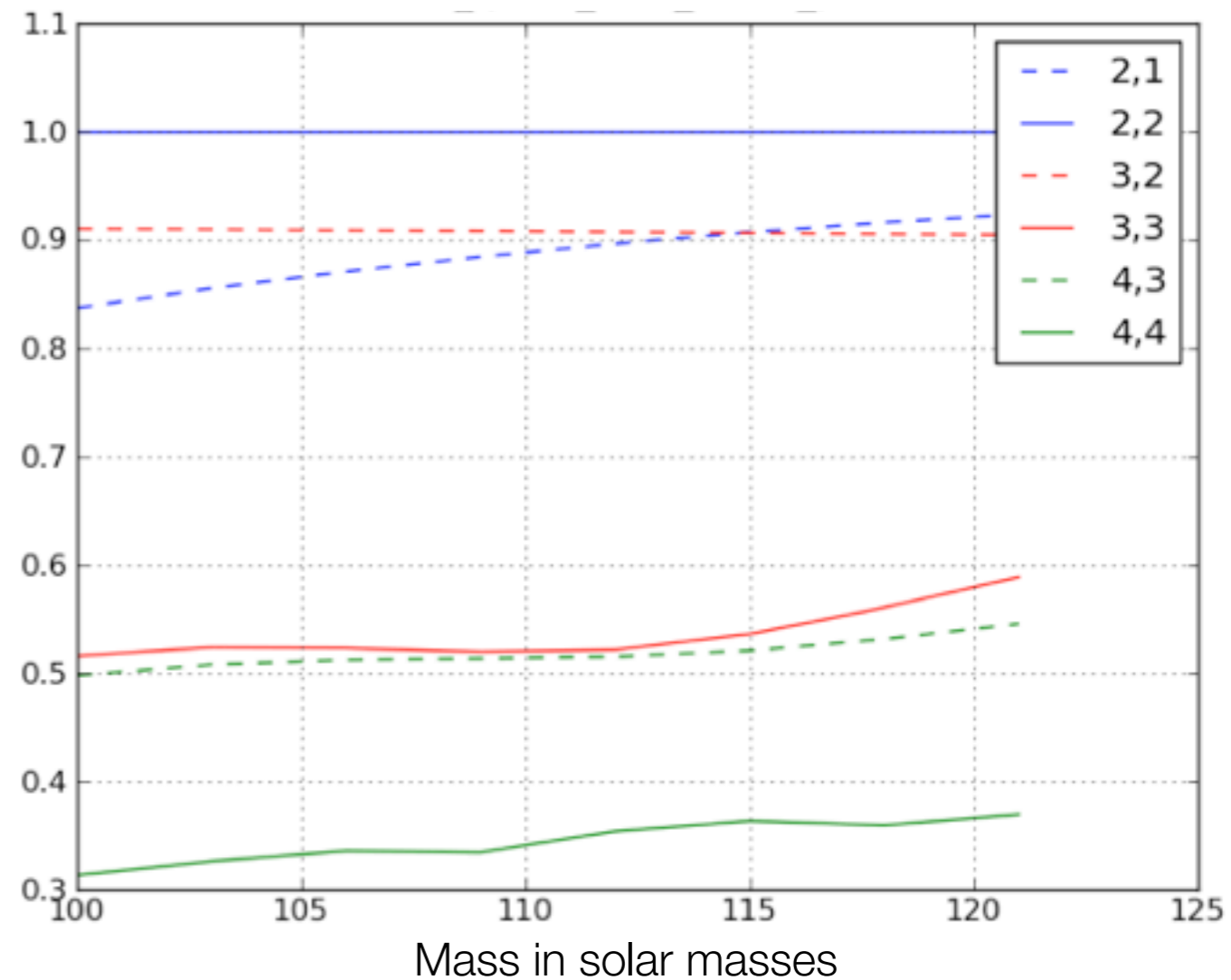


$$\langle s_1 | s_2 \rangle := \max_{\Delta t, \Delta \phi} \frac{(s_1 | s_2)}{\sqrt{(s_1 | s_1) (s_2 | s_2)}}$$

equal-mass, non-spinning



q=4, non-spinning

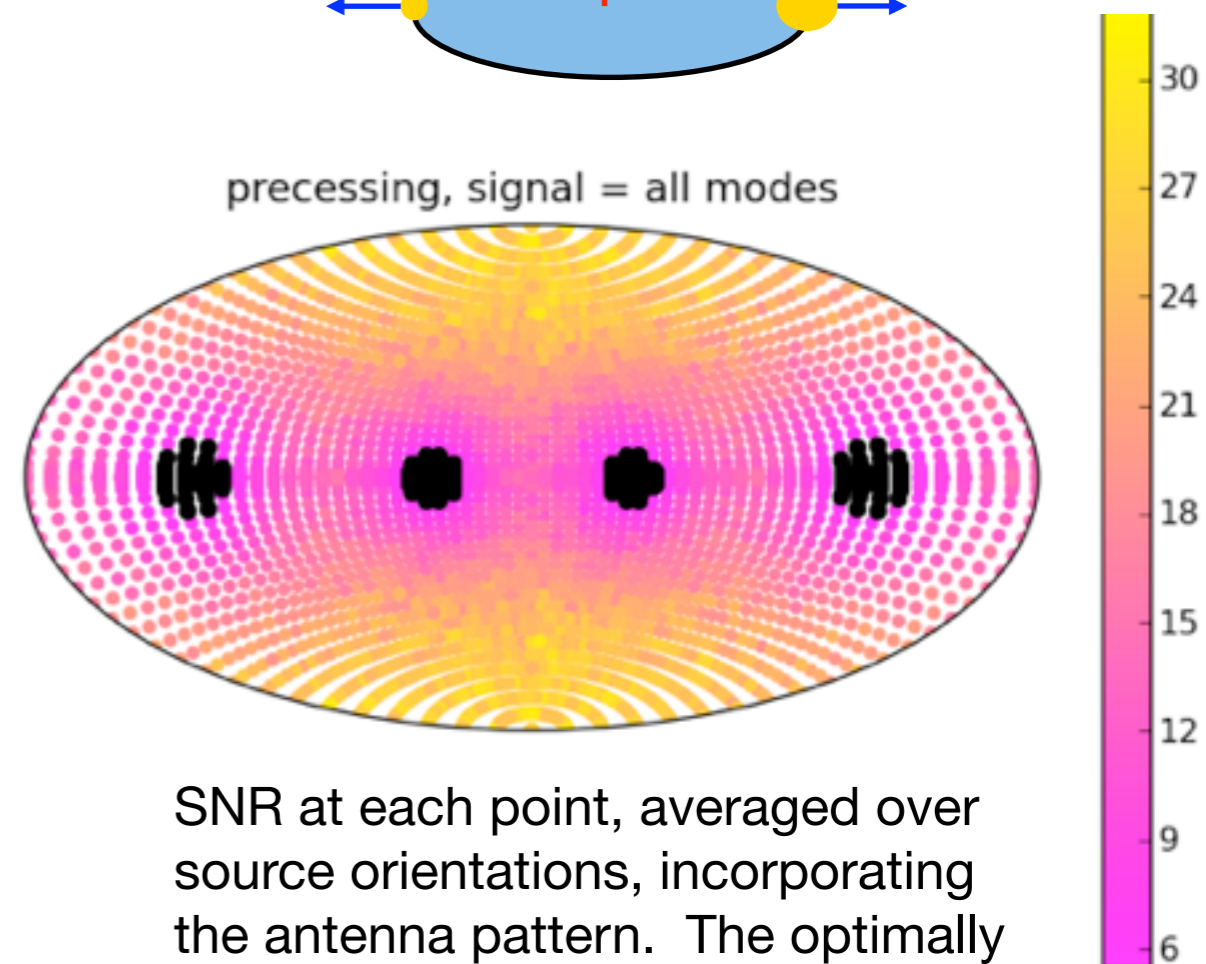
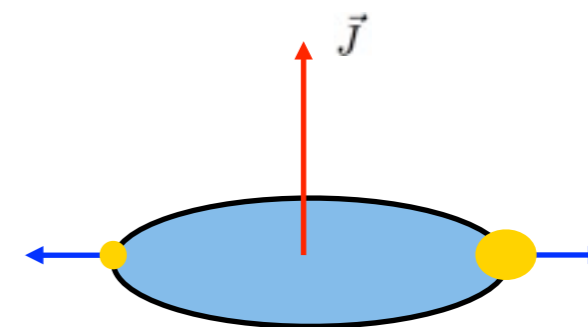
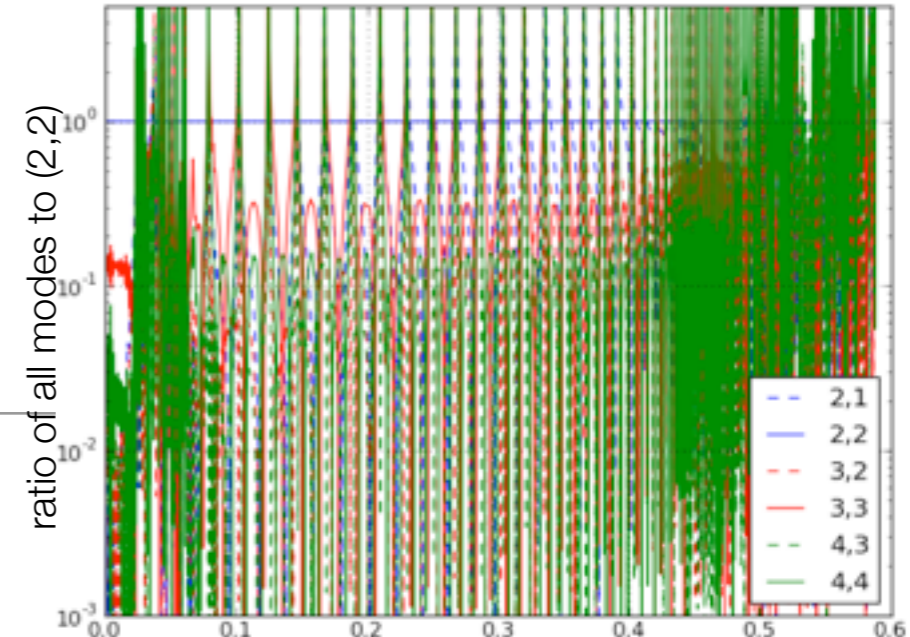




# BBH Waveforms

## ...not that simple ...

- As mass ratio increases we lose SNR (expected from PN studies)
- and we lose sky coverage
- When precessing spins are added, the 2,2 mode becomes much less significant during merger



SNR at each point, averaged over source orientations, incorporating the antenna pattern. The optimally oriented signal has SNR of 50.

# Value Added to SNR with higher modes

$$SNR = \max_{\Delta t, \Delta \phi} \frac{(h|s)}{\sqrt{(h|h)}}$$

Signal	Distance (Mpc)	h(2,2) % SNR>5.5	h(l,m) % SNR>5.5	h(2,2) average SNR	h(l,m) average SNR
q=1	1077	84.8	84.4	10.86	10.90
q=3	812	84.7	85.0	10.85	11.26
q=4	696	84.7	85.3	10.82	11.48
q=7	487	84.1	85.4	10.77	11.72
prec	442	90.8	91.1	16.62	18.06

- 2nd Column: distance at which the signal must be placed for the optimally-oriented case to have an SNR=50 (~power radiated)
- 3rd and 4th Columns: percentage of sky pixels for which SNR > 5.5 using 2,2 and exact waveform as template respectively
- 5th and 6th: average over all source orientations and sky position
- If we knew exactly what we were looking for we could improve average SNRs by about 8.5% for systems with q up to 7
  - Less than might be expected by looking at overlaps.
  - How much better could we do without knowing the exact waveform a priori?
  - How many more templates would it need, how would this affect the computation cost and rate of triggers from noise?
- We must investigate spinning systems (both precessing and non-precessing) very carefully.

# Precession and Frames

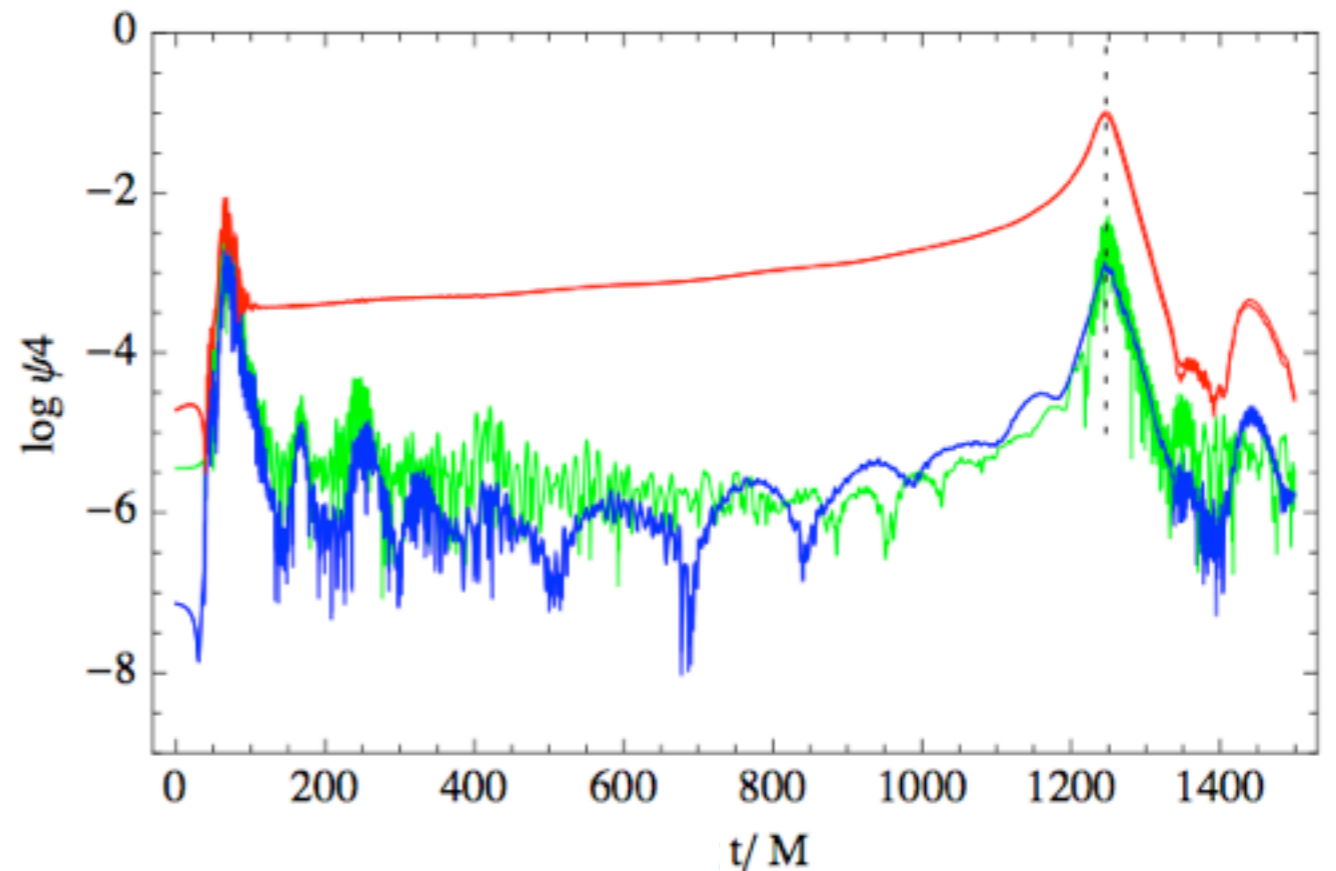
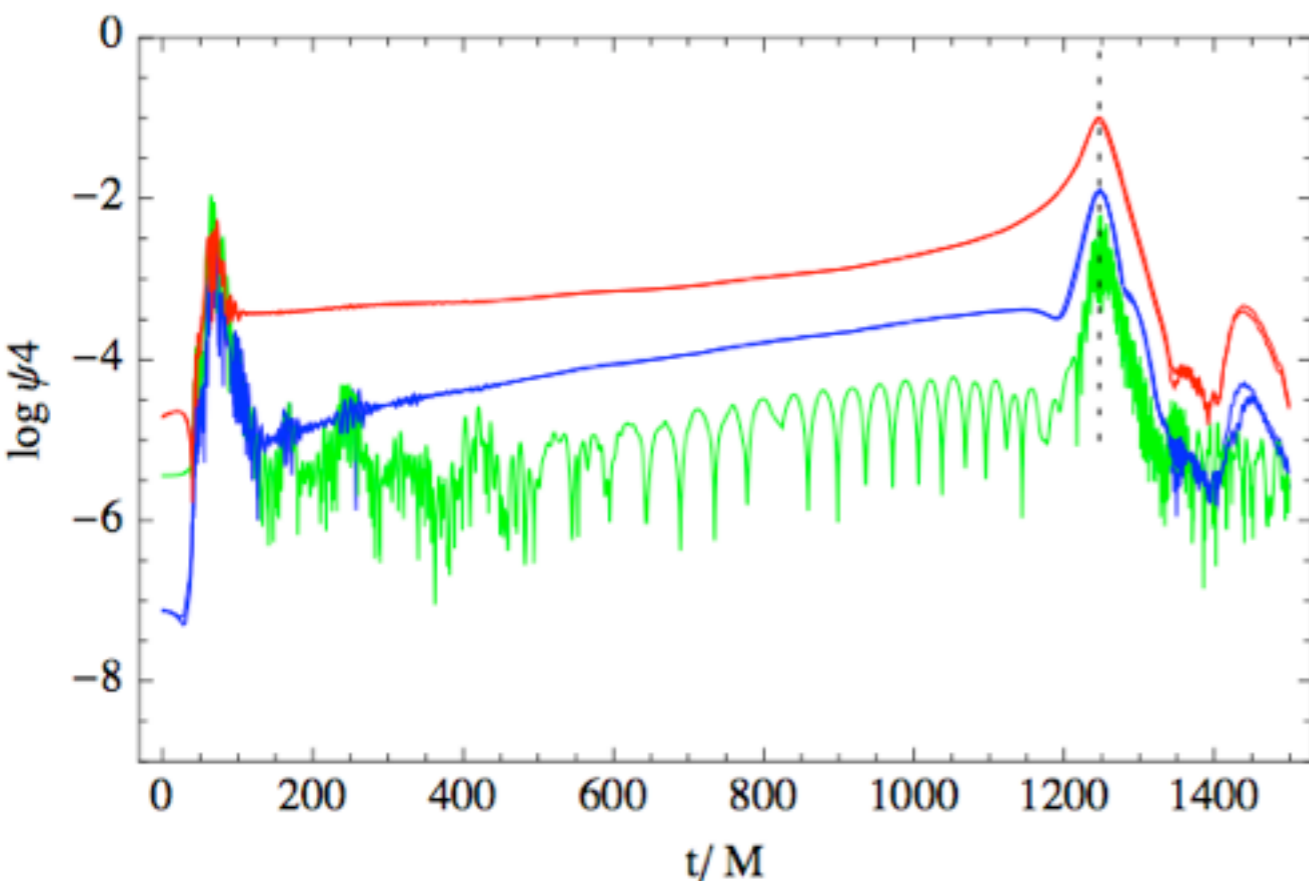
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- Plane of orbital motion inclines and precesses over time.
- PN: arising from spin-orbit couplings.
- Full GR: not easily understood in gauge-unambiguous language  
[Apostolatos et al PRD 49:6274 1994, Kidder PRD 52:821 1995, Gualtieri et al PRD 78:044024 2008]
- Non-precessing simulations:
  - decompose each spherical harmonic amplitude and phase
  - approximated before/after merger as simple/exponential damped polynomials
- Precessing system do not have a preferred frame of inertial coordinates:
  - overall rotation of the inertial coordinates would transform the various waveform modes
  - comparisons between waveforms must account for a rotation between the inertial frames in which they are measured

# A New Frame for Precessing Systems

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- Schmidt et al. (PRD 84: 024046 2011) pointed out that this preferred axis, while no longer fixed in precessing cases, exists
- O'Shaughnessy et al (PRD 84:124002 2011) finds an analytic preferred axis via a rotation operator that maximizes the z component of the angular momentum in the radiation
- Boyle et al (PRD 84:124011 2011) fixed the rotation about the radiation axis resulting in a minimal-rotation frame invariant under rotations of the inertial coordinates in which the precessing waveforms are extracted



# Why not J?

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- GW emitted from a merging binary depends on the orientation of an observer relative to the binary.
- Previous studies suggest that emission along the total initial or total final angular momenta leads to both the strongest and simplest signal from a precessing compact binary.
- In general, this is not true. Information available in just one direction (or mode) does not adequately encode the complexity of orientation-dependent emission for even short signals from merging BBHs (O'Shaughnessy et al PRD 85:084003 (2012)).
- Clever frames may help us to compare waveforms and build phenomenological waveforms.
- Bottom line is that future investigations of precessing, unequal-mass binaries should carefully explore and model their orientation-dependent emission.

# Beyond the scope of this talk

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## Mixed Binaries

- Approaching EMR (Lousto and Zlochower PRL 2011)
- Eccentric black hole-neutron star mergers (East et al arxiv:1111.3055)
- Eccentric binary neutron star mergers (Gold et al arxiv:109.5128)
- EOS BH+NS (Pannarale et al 2011, Markakis et al 2010 ...)

## Supermassive with environments (see Pablo Laguna's talk)

## And beyond...

- NR/HEP: roadmap for the future (Vitor Cardoso et al arXiv:1201.5118)
- Collisions of charged black holes (Miguel Zilhão et al arxiv:1205.1063)
- Dynamics of black holes in de Sitter spacetimes (Miguel Zilhao et al arxiv:1204.2019)
- Detectable seismic consequences of the interaction of a primordial black hole with Earth (Yang Luo et al arXiv:1203.3806)
- Superkicks in ultrarelativistic encounters of spinning black holes (Sperhake et al arXiv:1011.328)
- Head-on collisions of unequal mass black holes in D=5 dimensions (Helvi Witek et al. arXiv:1011.0742)
- Scalar-Tensor Theories in Merging BBHs (Healy et al arXiv:1112:3928)

# Open Questions

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- Is current accuracy of NR and PN good enough for parameter estimation?
- Are we prepared for a low SNR, short, high-mass BBH merger?
- Working toward precessing, unequal BBHs
- Can we determine the constituents at the low-mass regime?
- Can we use higher-modes in templates to improve SNRs and hence range and detection rates?
- How well do existing searches do with signals including higher modes?
- How well will existing searches do on precessing signals?