Striped superconductors from AdS black holes

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OUTLINE

- Striped superconductors
- 'Modulated' black holes
- The critical temperature
- Conclusions

◇ *J. A. Hutasoit, S. Ganguli, G. S., and J. Therrien,*
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◇ *S. Ganguli, J. A. Hutasoit, and G. S., arXiv:1205.3107*

Striped superconductors

◇ normal states of conventional superconductors are well described by Fermi liquid, whose only (weak coupling) instability is to superconductivity.

◇ normal states of high temperature superconductors, such as cuprates and iron pnictides, are highly correlated and thus, exhibit other low temperature orders which interact strongly with superconductivity. One of the prominent orders is the unidirectional charge density wave “stripe” order

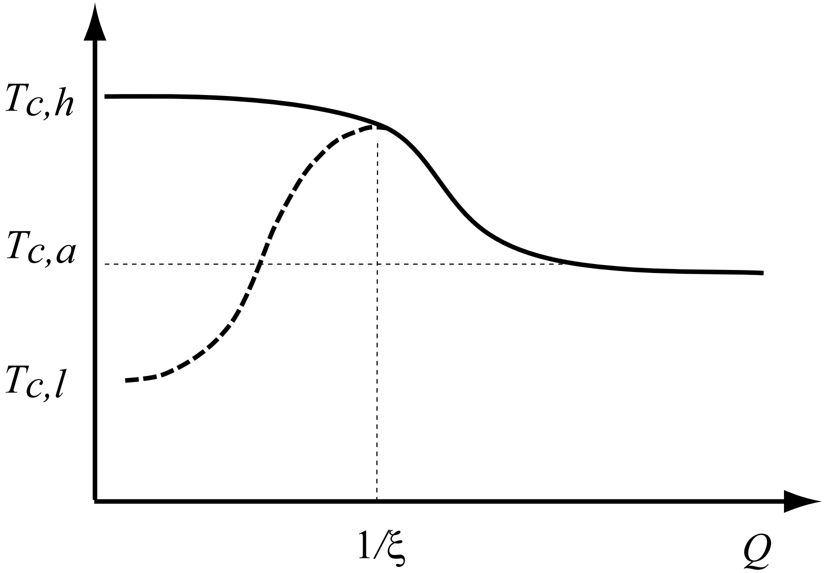
[Hoffman, et al. (2002), Howald, et al. (2003), Vershinin, et al. (2004), Wise, et al. (2008)]

⇒ important to understand nature of interplay between superconductivity and stripe order in the presence of strong correlation.

In gauge/gravity duality (holography), the strongly coupled condensed matter systems are mapped to a weakly coupled Einstein-Maxwell-scalar theory on black hole spacetimes with negative cosmological constant, or the so-called anti de-Sitter (AdS) black holes. Just like the normal states of high temperature superconductors, AdS black holes feature instabilities that lead to the formation of scalar ’hair’ which corresponds to superconductivity

[Gubser (2008), Hartnoll, Herzog, and Horowitz (2008)]

Weak coupling result (BCS theory)



Critical temperature for the inhomogeneous negative U Hubbard model with coupling

$$U(x) = \bar{U} + U_Q \cos(Qx)$$

The thick line denotes the mean-field result, where

$$T_{c,a} = (2\gamma/\pi)\omega_D \exp[-1/N_f\bar{U}] \ , \ \ T_{c,h} = (2\gamma/\pi)\omega_D \exp[-1/N_f(\bar{U} + |U_Q|)]$$

The dashed line shows the critical temperature once phase fluctuations of the order parameter are included. For $Q\xi \ll 1$, the superconductivity is first established locally in regions where $U(x)$ is large, but macroscopic phase coherence is achieved at a lower temperature, bounded from below by

$$T_{c,l} = (2\gamma/\pi)\omega_D \exp[-1/N_f(\bar{U} - |U_Q|)]$$

[Martin, Podolsky, and Kivelson (2005)]

$\xi \sim v_F/T_c$ (v_F is Fermi velocity)

‘Modulated’ black holes

study strongly coupled striped superconductor using gauge/gravity duality.

- $U(1)$ gauge potential A^a, dual to the four-current in CM system
- scalar field ψ charged under A^a, dual to scalar order parameter of superconductor (condensate)
- spacetime with negative cosmological constant $\Lambda = -\frac{3}{L^2}$

Set $L = 1$, $16\pi G = 1$.

study classical gravity theory

$\hookrightarrow$ boundary values of solutions related to the parameters of the superconductor.

action

$$S = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |D_a\psi|^2 - m^2|\psi|^2\right],$$

$D_a = \partial_a - iqA_a$, $F_{ab} = \partial_a A_b - \partial_b A_a$, and $a, b \in \{t, r, x, y\}$.

Field equations

Einstein equations

$$R_{ab} - \frac{1}{2} g_{ab} R - 3 g_{ab} = \frac{1}{2} T_{ab} \,,$$

with stress-energy tensor

$$T_{ab} = F_{ac} F_b{}^c - \frac{1}{4} g_{ab} F^{cd} F_{cd} + D_a \psi (D_b \psi)^* + (a \leftrightarrow b) - g_{ab} \left[|D_a \psi|^2 + m^2 |\psi|^2 \right] \,,$$

Maxwell equations

$$\frac{1}{\sqrt{-g}} \partial_b (\sqrt{-g} F^{ab}) = J^a \,,$$

with $U(1)$ current

$$J^a = -i [\psi^* D^a \psi - \text{c.c.}] \,,$$

and Klein-Gordon equation

$$-\frac{1}{\sqrt{-g}} D_a (\sqrt{-g} g^{ab} D_b \psi) + m^2 \psi = 0 \,.$$

Above critical temperature, $T \geq T_c$, $\psi = 0$.

find static black hole solution of *flat* conformal boundary
- coordinates $(x, y) \in \mathbb{R}$
- sourced by external modulated chemical potential

$$\mu(x) = \mu \sum_{n=0}^{\infty} \delta_n \cos nQx\,, \quad \sum_{n=0}^{\infty} \delta_n = 1.$$

Concentrate on $\delta_0 = 1 - \delta\,, \quad \delta_1 = \delta\,, \quad \delta_n = 0 \ \ (n \geq 2).$

metric *ansatz*:

$$ds^2 = -r^2 e^{-\alpha} dt^2 + e^{\alpha} \frac{dr^2}{r^2} + r^2 e^{-\beta} \left[e^{-\gamma} dx^2 + e^{\gamma} dy^2 \right] ,$$

- α, β, and γ are functions of (r, x).
- boundary at $r \to \infty$, horizon at $r = r_+$ (arbitrary parameter)
- Flat conformal boundary $\Rightarrow \alpha$, β, $\gamma \to 0$, as $r \to \infty$.
  $\hookrightarrow$ we find $\alpha \sim \mathcal{O}(r^{-3})$, and β, $\gamma \sim \mathcal{O}(r^{-4})$.

$U(1)$ potential: $A_r = A_x = A_y = 0$, $A_t = A_t(r, x)$ with $A_t = 0$ at horizon (for finite norm, $A_a A^a < \infty$).
At boundary, $A_t(r, x)\Big|_{r \to \infty} = \mu(x)\,.$

solve Einstein-Maxwell equations *perturbatively* in μ^2.

$\hookrightarrow$ expansion valid for large black holes (small chemical potential), $\mu \lesssim r_+$.

Expand

$$A_t = A_t^{(0)} + \left(\frac{\mu}{r_+}\right)^2 A_t^{(1)} + \dots \qquad \beta = \beta^{(0)} + \left(\frac{\mu}{r_+}\right)^2 \beta^{(1)} + \dots$$

$$\alpha = \alpha^{(0)} + \left(\frac{\mu}{r_+}\right)^2 \alpha^{(1)} + \dots \qquad \gamma = \gamma^{(0)} + \left(\frac{\mu}{r_+}\right)^2 \gamma^{(1)} + \dots$$

$\therefore$ metric, Ricci tensor, $U(1)$ field strength, and stress-energy tensor,

$$g_{ab} = g_{ab}^{(0)} + \left(\frac{\mu}{r_+}\right)^2 g_{ab}^{(1)} + \dots \qquad F_{ab} = \left(\frac{\mu}{r_+}\right)^2 F_{ab}^{(0)} + \dots$$

$$R_{ab} = R_{ab}^{(0)} + \left(\frac{\mu}{r_+}\right)^2 R_{ab}^{(1)} + \dots \qquad T_{ab} = \left(\frac{\mu}{r_+}\right)^2 \mathcal{T}_{ab}^{(0)} + \dots$$

Zeroth order

Einstein-Maxwell equations

$$R^{(0)\,a}{}_{b} + 3\delta^{a}{}_{b} = 0\,, \quad \partial_b\left(\sqrt{-g^{(0)}}F^{(0)ab}\right) = 0\,.$$

Einstein equations decouple, solved by AdS Schwarzschild black hole

$$e^{-\alpha^{(0)}} \equiv h = 1 - z^3\,, \quad \beta^{(0)} = \gamma^{(0)} = 0\,, \quad z = \frac{r_+}{r}$$

boundary at $z = 0$, horizon at $z = 1$.

$U(1)$ potential

$$A_t^{(0)} = \mu\left[(1-\delta)\mathcal{A}_0(z) + \delta\mathcal{A}_1(z)\cos Qx\right]$$

$\therefore$ mode equations

$$\mathcal{A}_0''(z) = 0\ , \quad \mathcal{A}_1''(z) - \frac{Q^2}{r_+^2 h(z)}\mathcal{A}_1(z) = 0$$

boundary conditions: $\mathcal{A}_n(0) = 1$, $\mathcal{A}_n(1) = 0$ ($n = 0, 1$).

Solution: $\mathcal{A}_0(z) = 1 - z$

$$\mathcal{A}_1(z) \approx \frac{\sinh\left[\frac{Q}{r_+}(1-z)\right]}{\sinh\frac{Q}{r_+}}$$ (good analytic approximation)

First order

Lowest-order stress-energy tensor $\mathcal{T}_{ab}^{(0)}$ has modes with $n \leq 2$
$\because$ quadratic in the $U(1)$ potential.
$\hookrightarrow$ same is true for the first-order corrections to the metric

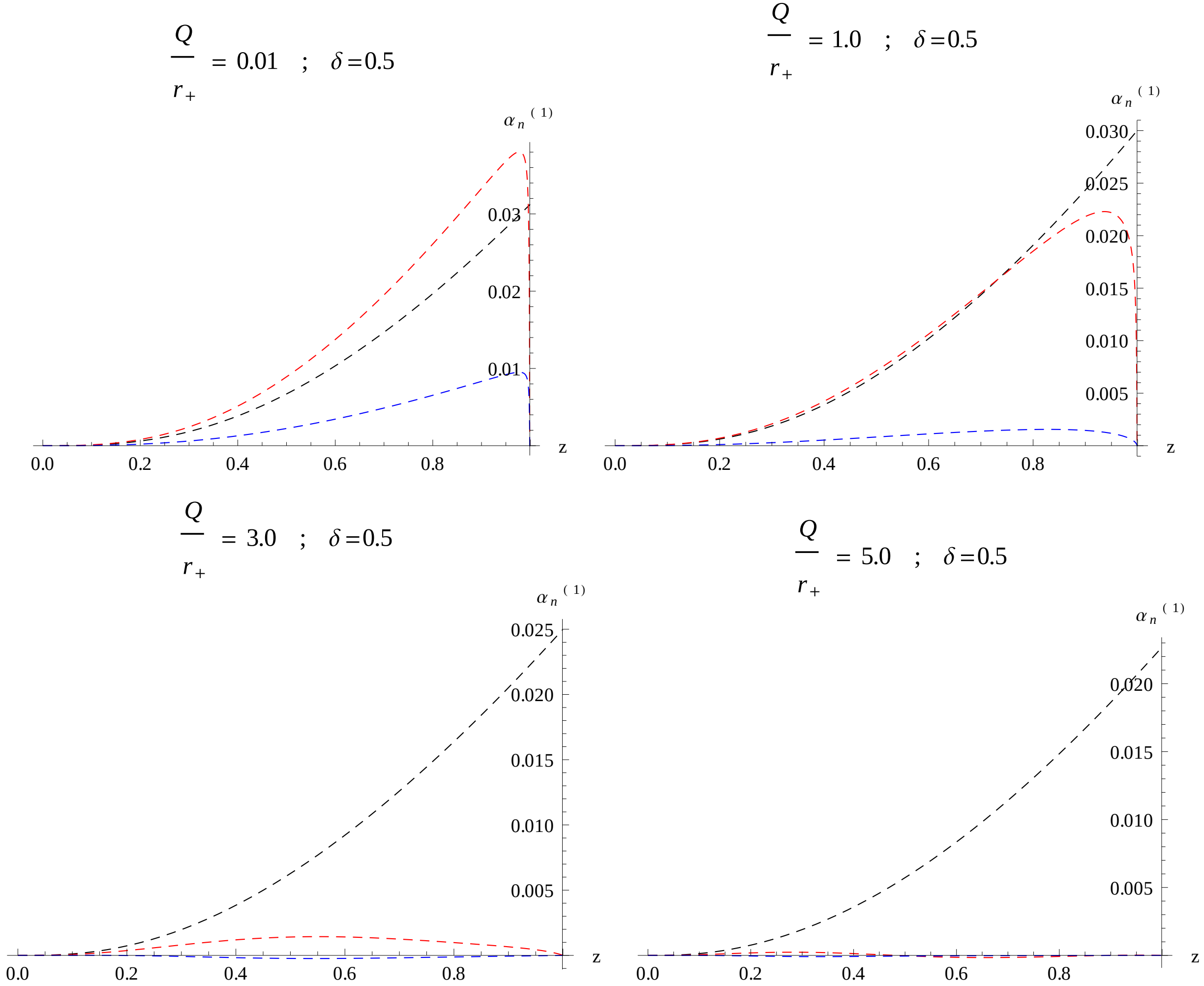
$$\begin{aligned} \mathcal{T}^{(0)\,t}_{\;\;\;t} &= -\mathcal{T}^{(0)\,y}_{\;\;\;y} = -\frac{z^4}{4}\left[\frac{\mathcal{E}_x^2}{h} + \mathcal{E}_z^2\right] , \\ \mathcal{T}^{(0)\,z}_{\;\;\;z} &= -\mathcal{T}^{(0)\,x}_{\;\;\;x} = \frac{z^4}{4}\left[\frac{\mathcal{E}_x^2}{h} - \mathcal{E}_z^2\right] , \\ \mathcal{T}^{(0)\,x}_{\;\;\;z} &= \frac{1}{h}\mathcal{T}^{(0)\,z}_{\;\;\;x} = -\frac{z^4}{2\,h}\mathcal{E}_x\mathcal{E}_z , \end{aligned}$$

in terms of the components of the electric field

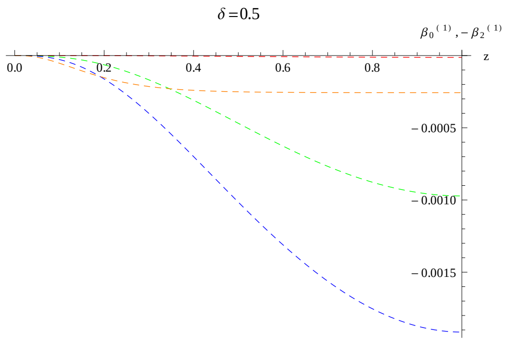
$$\mathcal{E}_x = \frac{\delta Q}{r_+}\mathcal{A}_1 \sin Qx , \quad \mathcal{E}_z = (1-\delta)\mathcal{A}_0'(z) + \delta\mathcal{A}_1'(z)\cos Qx .$$

Einstein equations at first order: $R^{(1)\,a}_{\;\;\;b} = \mathcal{T}^{(0)\,a}_{\;\;\;b}$
$\hookrightarrow$ set

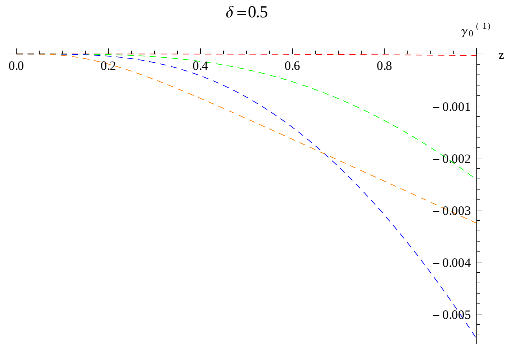
$$\begin{aligned} \alpha^{(1)} &= \alpha_0^{(1)}(z) + \alpha_1^{(1)}(z)\cos Qx + \alpha_2^{(1)}(z)\cos 2Qx , \\ \beta^{(1)} &= \beta_0^{(1)}(z) + \beta_1^{(1)}(z)\cos Qx + \beta_2^{(1)}(z)\cos 2Qx , \\ \gamma^{(1)} &= \gamma_0^{(1)}(z) + \gamma_1^{(1)}(z)\cos Qx + \gamma_2^{(1)}(z)\cos 2Qx . \end{aligned}$$



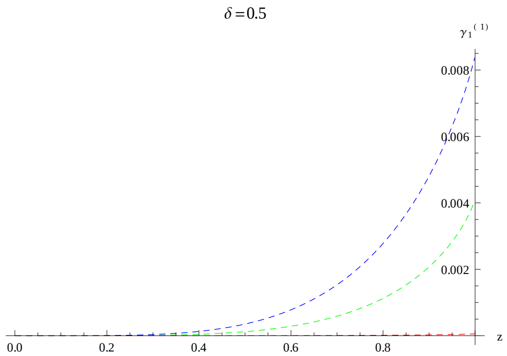
$\alpha_0^{(1)}$ (black), $\alpha_1^{(1)}$ (red) and $\alpha_2^{(1)}$ (blue) for $\delta = 0.5$, and $Q/r_+ = 0.01$, 1, 3 and 5.



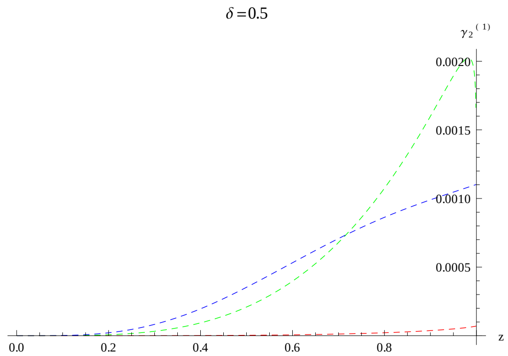
$\beta_0^{(1)} = -\beta_2^{(1)}$ for $\delta = 0.5$, and $QL^2/r_+ = 0.1$ (red), 1.0 (green), 2.0 (blue) and 8.0 (orange).



$\gamma_0^{(1)}$ for $\delta = 0.5$, and $Q/r_+ = 0.1$ (red), 1.0 (green), 2.0 (blue) and 8.0 (orange).



$\gamma_1^{(1)}$ for $\delta = 0.5$, and $Q/r_+ = 0.1$ (red), 1.0 (green) and 2.0 (blue).



$\gamma_2^{(1)}$ for $\delta = 0.5$, and $Q/r_+ = 0.1$ (red), 1.0 (green) and 2.0 (blue).

Hawking temperature

$$T = \frac{3r_+}{4\pi}\left[1 - \frac{\mu^2}{r_+^2}\alpha^{(1)}(1)\right]$$

$\boxed{Q \to 0}$

$$\begin{aligned}
\alpha_0^{(1)} &= \left((1-\delta)^2 + \frac{\delta^2}{2}\right)\frac{z^3}{4\,(1+z+z^2)} + \mathcal{O}\left(\frac{Q^2}{r_+^2}\right), \\
\alpha_1^{(1)} &= \frac{(1-\delta)\,\delta\,z^3}{2\,(1+z+z^2)}(1-z)^{Q^2/6r_+^2} + \mathcal{O}\left(\frac{Q^2}{r_+^2}\right), \\
\alpha_2^{(1)} &= \frac{\delta^2\,z^3}{8\,(1+z+z^2)}(1-z)^{2Q^2/3r_+^2} + \mathcal{O}\left(\frac{Q^2}{r_+^2}\right).
\end{aligned}$$

$\boxed{Q = 0}$

equivalently, $\delta = 0 \Rightarrow$ Reissner-Nordström (homogeneous system)

$$e^{-\alpha} = 1 - \left(1 + \frac{\mu^2}{4r_+^2}\right) z^3 + \frac{\mu^2}{4r_+^2} z^4$$

• Schwarzschild solution in probe limit, $\mu \to 0$.
• as we increase μ, effects of back reaction to the metric more pronounced
• extremality at $\mu/r_+ = 2\sqrt{3}$

N.B.: convergence to homogeneous system not uniform.

$\hookrightarrow$ $\alpha_n^{(1)}(1) = 0$ for $n = 1, 2, \therefore \alpha^{(1)}$ does not converge to its homogeneous counterpart.

$\hookrightarrow$ limits $Q \to 0$ and $z \to 1$ do not commute.

$\Rightarrow$ *enhancement* in temperature upon turning on modulation

$$\frac{\Delta T}{T} = \frac{T}{T_{\delta=0}} - 1 \approx \frac{\mu^2}{12 r_+^2} \delta \left(2 - \frac{3\delta}{2} \right) ,$$

with a maximum enhancement for $\delta = \frac{2}{3}$.

$\boxed{Q \to \infty}$

all functions except $\alpha_0^{(1)}$ become negligible, and

$$\alpha_0^{(1)} \approx \frac{(1-\delta)^2 z^3}{4\left(1+z+z^2\right)} .$$

$\Rightarrow$ another Reissner-Nordström solution, albeit with less charge density,

$$e^{-\alpha} \approx 1 - \left(1 + \frac{\mu^2(1-\delta)^2}{4r_+^2}\right) z^3 + \frac{\mu^2(1-\delta)^2}{4r_+^2} z^4 .$$

coincides with homogeneous solution () if $\delta = 0$, as expected.

Hawking temperature

$$T_{Q\to\infty} \approx \frac{3r_+}{4\pi}\left[1 - \frac{\mu^2(1-\delta)^2}{12r_+^2}\right] .$$

The critical temperature

Solve Klein-Gordon equation for static scalar field $\psi(z,x)$ of mass m and charge q

$$\frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ii}\partial_i\psi\right)+\left(q^2\,A_t^2-m^2\right)\psi=0\,.$$

$m^2=\Delta(\Delta-3)$, Δ: conformal dimension of superconducting order parameter

$\hookrightarrow$ fix parameters $\{\Delta,\delta,q,Q/r_+\}$ and find eigenvalue $\lambda=\frac{q\mu}{r_+}$.

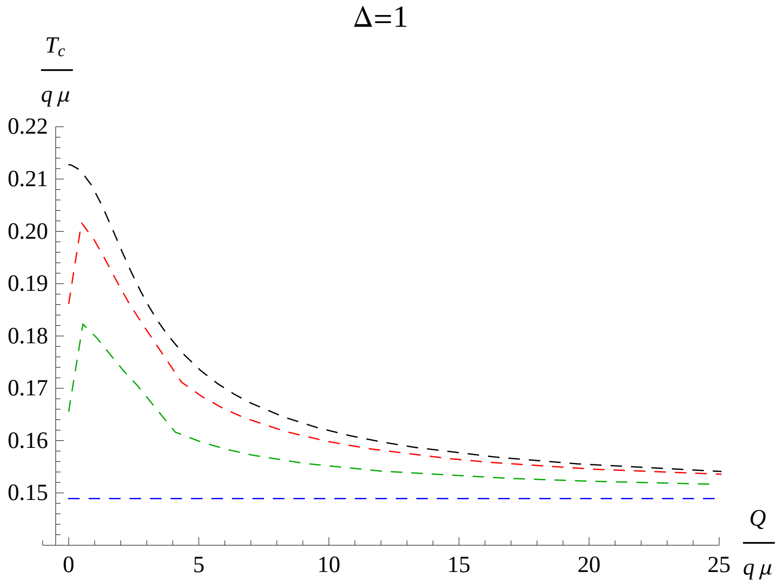
$\therefore$ critical temperature

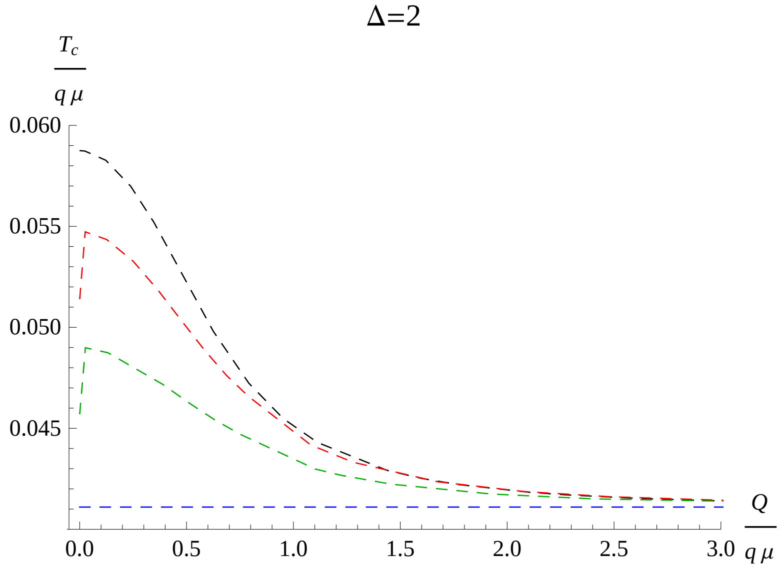
$$\frac{T_c}{q\mu}=\frac{3}{4\pi}\left[\frac{1}{\lambda}-\frac{\lambda}{q^2}\alpha^{(1)}(1)\right]\,.$$

At small Q, enhancement in critical temperature

$$\frac{\Delta T_c}{T_c}\approx\frac{\lambda^2}{12q^2}\delta\left(2-\frac{3\delta}{2}\right)\,,$$

vanishes at probe limit ($q\to\infty$), and becomes significant away from it.
CAUTION: this is only a first-order, $\mathcal{O}(1/q^2)$, result.





From top to bottom: T_c *vs.* Q for $\Delta = 1$ and $(\delta, q^2) = (0.3, \infty)$, $(0.2, 0.411)$ and $(0.1, 0.190)$.

From top to bottom: T_c *vs.* Q for $\Delta = 2$ and $(\delta, q^2) = (0.3, \infty)$, $(0.2, 7.92)$ and $(0.1, 4.22)$.

Conclusions

- ◇ studied effect of inhomogeneity on superconducting transition temperature of strongly coupled striped superconductor beyond the mean field level
  - including backreaction of the electromagnetic field on the geometry of spacetime in the dual gravitational picture.
- ◇ found an enhancement of critical temperature due to inhomogeneity that comes from the stripe order.
- ◇ interesting to study relation between value of Q at maximum T_c and superconducting correlation length scale as seen in the BCS result.