Quasi-normal modes, area spectra and multi-horizon spacetimes

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References

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2. J. Skákala (2011),
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3. J. Skákala (2011), PhD Thesis, [gr-qc/1107.2978]
4. J. Skákala and M. Visser (2010),
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Outline

The black hole quasi-normal modes

- The black hole when perturbed by arbitrary compactly supported perturbation shows specific characteristic damped oscillations called quasi-normal modes (QNM).
- The modes in general form an infinite discrete set and depend on the type of perturbation, its wave mode number and the black hole parameters.
- The modes are described by the quasi-normal mode (QNM) frequencies, which are non-real complex frequencies $\omega = \omega_R + i \cdot \omega_I$. (For the normal mode time dependence convention $e^{i\omega t}$ the frequencies have $\omega_I > 0$.)

- The set of QNM frequencies generally contains frequencies with arbitrary large imaginary parts ω_I. The frequencies with damping going to infinity we call asymptotic QNM frequencies.
- Generically the real part of the asymptotic frequencies may depend on the type of the perturbation and certainly on the black hole parameters, whereas the imaginary part depends only on the black hole parameters.
- For the Schwarzschild black hole the asymptotic QNM frequencies can be explicitly written and for all the types of gravitational perturbations they are given as (everywhere we use Planck units):

$$\omega_n = \frac{\kappa}{2\pi}\ln(3) + i\cdot\kappa\left(n+\frac{1}{2}\right) + O(n^{-1/2}). \qquad (1)$$

Conjectured connection between the asymptotic modes and the black hole thermodynamics

- It was suggested by Hod[1], that due to Bohr's correspondence principle, the real part of the asymptotic frequencies might correspond to the mass quantum emitted in a quantum black hole (BH) state transition, thus

$$\Delta M = \lim_{n \to \infty} \omega_{nR}. \tag{2}$$

- In case of Schwarzschild black hole and gravitational tensor/scalar perturbations (which seem to be the relevant ones) this means $\Delta M = \ln(3)/T$. This leads to the equispaced BH horizon area spectrum with the quantum $\Delta A = 4\ln(3)$.
- The spectrum is of the form $A = 4\ln(M) \cdot n, \quad M \in \mathbb{N}$, which was suggested already before, due to the statistical interpretation of entropy.

[1] S. Hod, Phys.Rev.Lett. **81**, 4293 (1998), [gr-qc/9812002]

- The same results can be obtained by identifying $\omega_{\infty R}$ with a black hole characteristic frequency and using the Bohr-Sommerfeld quantization of adiabatic invariants.
- Also consequences of Hod's conjecture for the value of Immirzi parameter in Loop Quantum Gravity were derived.
- Despite of initial excitement, Hod's conjecture led to many difficulties over the years. To remove most of the difficulties a modification of the conjecture was suggested by Maggiore[2].
- The modification is based on the fact that black hole has to be described as a collection of *damped* oscillators. As a result the link between the mass quantum and the QNM frequencies has to be instead:

$$\Delta M = \lim_{n\to\infty} \Delta_{n,n-1} \sqrt{\omega_{nR}^2 + \omega_{nI}^2}. \tag{3}$$

[2] M. Maggiore, Phys.Rev.Lett. **100**, 141301 (2008), [gr-qc/0711.3145]

- Generically

$$\lim_{n\to\infty} \Delta_{n,n-1} \sqrt{\omega_{nR}^2 + \omega_{nI}^2} = \lim_{n\to\infty} \Delta_{n,n-1}\omega_{nI} \qquad (4)$$

and thus ΔM is given as $n \to \infty$ limit of $\Delta_{n,n-1}\omega_{nI}$.

- In Schwarzschild case this leads to the equispaced BH horizon area spectrum with the area quantum

$$\Delta A = 8\pi. \qquad (5)$$

- This is a black hole area quantization suggested in many works (first time suggested by Bekenstein[3]). The quantum (5) has not a "statistical" form, but this can be explained by the fact that the result is essentially semi-classical.

[3] J. D. Bekenstein, Lett.Nuovo Cim. **11**, 467 (1974)

Asymptotic QNM frequencies of spherically symmetric multi-horizon spacetimes

- In particular, under spherically symmetric black hole multi-horizon spacetimes we mean: Reissner-Nordström spacetime (R-N), Schwarzschild-deSitter spacetime (S-dS) and Reissner-Nordström-deSitter spacetime (R-N-dS).
- The asymptotic frequencies of scalar/vector/tensor type perturbations of R-N, S-dS. R-N-dS generally fulfil equations of the form:

$$\sum_{A=1}^{M} C_A \exp\left(\sum_{i=1}^{N} Z_{Ai} \frac{2\pi\omega}{|\kappa_i|}\right) = 0. \tag{6}$$

Here κ_i are the surface gravities of the N different horizons, and the $M \times N$ matrix Z_{Ai} is composed entirely of values $Z_{Ai} = 0, 1, 2$.

We analysed[4] the behaviour of the solutions of equation (6) and concluded that

- In case all the surface gravities ratios are rational numbers, the solutions split into *finite* number of equispaced families (labeled by a) of the type

$$\omega_{an} = (\text{offset})_a + in \cdot \text{lcm}(|\kappa_1|, |\kappa_2|, ...|\kappa_N|). \tag{7}$$

  By *lcm* we mean the least common multiple of the numbers in the bracket, hence

$$\text{lcm}(|\kappa_1|, |\kappa_2|, ..., |\kappa_N|) = p_1|\kappa_1| = ... = p_N|\kappa_N|,$$

  where $\{p_1, ..., p_N\}$ is a set of relatively prime integers.
- In case ratio of a couple of surface gravities is irrational, there does *not* exist a periodic infinite subset of the solutions (hence the solutions are strongly aperiodic).

[4] J. Skákala and M. Visser, JHEP **1011**, 070 (2010), [gr-qc/1009.0080], J. Skákala, PhD Thesis, (2011) [gr-qc/1107.2978]

Multi-horizon spacetimes and modified Hod's conjecture

What does such behaviour of the frequencies tell us in terms of modified Hod's conjecture? Take the simplest R-N spacetime case[5]:

- One can exactly prove that in case the ratio of the surface gravities is irrational, the limit $n \to \infty$ in $\Delta_{n,n-1}\omega_{nl}$ does *not* exist.
- One can prove that in case of rational ratio of the two surface gravities, the limit $n \to \infty$ $\Delta_{n,n-1}\omega_{nl}$ exists only in case all the solutions of equations (6) are given by equispaced families with the same $(\text{offset})_{al}$ parts. One can also prove that this cannot be the case when $\kappa_+/\kappa_- = p_+/p_-$, where $p_\pm$ are relatively prime and $p_+ \cdot p_-$ is an odd number. Thus one can prove that in case $p_+ \cdot p_-$ odd the limit in $\Delta_{n,n-1}\omega_{nl}$ does *not* exist.

[5] J. Skákala, JHEP **1201**:144, 2012, [gr-qc/1111.4164]

To have some insight in what is happening let us demonstrate why the limit might not exist in case of rational ratios of surface gravities:

- If in such case one has m equispaced families with different $(\text{offset})_{al}$ parts ($m \neq 1$), the limit in $\Delta_{n,n-1}\omega_{nl}$ can exist only in case the gap in the spacing between the modes is $(\text{gap})/m$. (Here (gap) is the spacing in frequencies in each of the equispaced families.) It can be proven that such spacing in the imaginary parts of the offsets cannot be the case. As a result of this fact the gap in the spacing between the frequencies (with respect to ω_I ordered union of all the families) is an oscillating sequence and the limit does not exist.

- These results seem to be no-go theorems for the modified Hod's conjecture in case of multi-horizon spacetimes.
- They were strictly proven only for the simplest R-N spacetime, but it is extremely likely that similar results hold also for the other (S-dS, R-N-dS) spherically symmetric spacetimes.
- On the other hand the general splitting of the frequencies into equispaced families in case the surface gravities ratios are rational seems to indicate something important.
- Is there a way how to reconcile the observed behaviour of the frequencies with Maggiore's (modified Hod's) conjecture?

Let us take the R-N spacetime and suggest the following[6]:

- Let us assume that both of the horizons in the spacetime have the same equispaced area spectra given as $A = 8\pi\gamma \cdot n$. (Suggestions of this type appeared already few times in the literature.)
- Note that the perturbations we are considering are *uncharged*, thus one assumes that the only change of the black hole parameters (such that corresponds to the perturbations) will be in the ADM mass ΔM.
- Then the area changes of the outer horizon ΔA_+ and of the inner Cauchy horizon ΔA_- are given as

$$\Delta A_{\pm} = \frac{8\pi \Delta M}{\kappa_{\pm}}. \tag{8}$$

[6] J. Skákala, accepted for publication by JHEP, (2012), [gr-qc/1204.3566]

- Since $\Delta A_\pm = 8\pi\gamma \cdot m_\pm$, with $m_\pm \in \mathbb{Z}$, then necessarily

$$\Delta M = \gamma \cdot \kappa_\pm m_\pm .$$

- This implies

$$m_+\kappa_+ = m_-\kappa_- \quad \rightarrow \quad \frac{\kappa_+}{\kappa_-} = \frac{m_-}{m_+}, \tag{9}$$

hence the surface gravities ratio must be rational.[7] Thus if one wants the quantum ΔM to be as small as possible, such that it is consistent with area spectra of the two horizons one obtains:

$$\Delta M = \gamma \cdot \mathrm{lcm}(\kappa_+, |\kappa_-|). \tag{10}$$

- Then Maggiore's conjecture suggests that:

$$\lim_{n\to\infty} \Delta_{n,n-1}\, \omega_{nl} = \gamma \cdot \mathrm{lcm}(\kappa_+, |\kappa_-|). \tag{11}$$

[7]Such rational ratios have important consequences for the thermodynamics of multi-horizon spacetime: T.R. Choudhury and T. Padmanabhan, Gen.Rel.Grav. **39**, 1789-1811 (2007), [gr-qc/0404091].

- The relation (11) could be fulfilled if one considers only equispaced families to carry information about mass quanta emitted by the quantum black hole in the semi-classical regime. Thus let us extend Maggiore's (modified Hod's) conjecture to the multi-horizon case by suggesting, that *in general only infinite equispaced subsets of frequencies relate to the mass quantum emitted in the BH quantum transition* (but each frequency is contained in one of the subsets).
- Then the equation (7) fixes $\gamma = 1$, thus both of the horizons have spectra given as $A = 8\pi n$.

- The analysis made for R-N black hole can be repeated in an exact manner also for the S-dS and the R-N-dS black hole.
- So the suggested interpretation of the QNM frequencies works in all the spherically symmetric multi-horizon spacetimes and gives the area spectra of each of the horizons in each of the multi-horizon spacetimes (R-N, S-dS, R-N-dS) to be $A = 8\pi n$.

Conclusions

- We found an interpretation for the asymptotic QNM frequencies of the spherically symmetric multi-horizon BH spacetimes following the proposal by Maggiore.
- The results are consistent with all the spacetime horizons being quantized with the same area spectra given as $8\pi n$.
- The interpretation is similar to the single horizon (Schwarzschild) case, but has certain specific features that were discussed in the talk.
- **Thank you for attention !**