Induced gravity from gauge theories

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Program

- Motivation.
- Yang-Mills theories and gravity.
- The SO(m,n) gauge theory and its contraction.
- The effective gravity.
- Conclusions and perspectives.

(Some) Motivation

- Zanelli's talk.
- Lack of a “complete” description of gravity at quantum level.
- Results from the LHC.
- Predictive power of QFT in Euclidean spacetime.
- Gribov-Zwanziger scenario of confinement in Yang-Mills theories.

- Can gravity be described by a standard gauge theory in 4d?

Background

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Yang-Mills theories and gravity

- 4d-YM: non-Abelian Gauge group G + massless dim 1 gauge conn. Y + 4d Euclidean spacetime.
- Renormalizability.
- Asymptotic freedom.
- Dynamical mass generation – Gribov parameter.

- At IR: Gribov ambiguities = BRST breaking through a mass parameter γ.

- UV unitary YM (γ=0) $\leftarrow$ cont.deform. $\rightarrow$ IR confined ($\gamma\neq 0$).

- Possibility of extra dynamically generated mass parameters.

- Observables = gauge invariant operators.

- Can we apply this idea to gravity?

Some requirements

- **UV**: gravity = pure non-Abelian gauge theory in Euclidean 4d spacetime.
- **UV**: Massless theory. So the gauge field cannot be associated with a metric.
- **IR**: Gauge field must provide the degrees of freedom of a gravity theory (metric and affine degrees).
- **IR:** dynamical mass generation required for metric degrees (dim Y = 1, dim e = 0).
- **IR:** gravity is a geometric theory (in GR way).

SO(m,n) Yang-Mills theory

* $m + n = 5$
* m=0 $\longmapsto$ $SO(5)$
* m=1 $\longmapsto$ $SO(1,4)$
* m=2 $\longmapsto$ $SO(2,3)$

* $m \in \{0, 1, 2\}$
* $\eta^{AB} \equiv \mathrm{diag}(\epsilon, \varepsilon, 1, 1, 1)$

$$\epsilon = (-1)^{(2-m)!}$$

$$\varepsilon = (-1)^{m!+1}$$

Decomposition

$$SO(m,n) \equiv SO(m!-1,n) \otimes S(4)$$

$$\begin{aligned}
\left[J^{ab}, J^{cd}\right] &= -\frac{1}{2}\left[\left(\eta^{ac} J^{bd} + \eta^{bd} J^{ac}\right) - \left(\eta^{ad} J^{bc} + \eta^{bc} J^{ad}\right)\right] \\
\left[J^{a}, J^{b}\right] &= -\frac{\epsilon}{2} J^{ab} , \\
\left[J^{ab}, J^{c}\right] &= \frac{1}{2}\left(\eta^{ac} J^{b} - \eta^{bc} J^{a}\right) ,
\end{aligned}$$

$$\eta^{ab} \equiv \mathrm{diag}(\varepsilon, 1, 1, 1)$$

- Gauge field $$Y = Y^A{}_B J_A{}^B = A^a{}_b J_a{}^b + \theta^a J_a$$

- YM action

$$\begin{aligned} S_{\mathrm{YM}} &= \frac{1}{2}\int F^A{}_B {*} F_A{}^B \\ &= \frac{1}{2}\int \left[\Omega^a{}_b {*} \Omega_a{}^b + \frac{1}{2} K^a {*} K_a - \frac{\epsilon\kappa}{2}\Omega^a{}_b {*} (\theta_a \theta^b) + \frac{\kappa^2}{16}\theta^a \theta_b {*} (\theta_a \theta^b)\right] \end{aligned}$$

$$\Omega^a{}_b = \mathrm{d}A^a{}_b + \kappa A^a{}_c A^c{}_b \qquad K^a = \mathrm{D}\theta^a = \mathrm{d}\theta^a - \kappa A^a{}_b \theta^b$$

Inönü-Wigner contraction

- Mass (at least one mass): γ

- Rescaling (so θ has dimensionless components):

$$A \longmapsto \kappa^{-1} A \,,$$
$$\theta \longmapsto \kappa^{-1} \gamma \theta$$

- Asymptotic freedom:

$$\gamma^2 / \kappa^2 \longmapsto 0$$

- Contracted algebra:

$$\begin{aligned}
\left[J^{ab}, J^{cd}\right] &= -\frac{1}{2}\left[\left(\eta^{ac}J^{bd} + \eta^{bd}J^{ac}\right) - \left(\eta^{ac}J^{bc} + \eta^{bc}J^{ad}\right)\right] \\
\left[J^{a}, J^{b}\right] &= -\frac{\epsilon\gamma^2}{2\kappa^2}J^{ab} \longmapsto 0 \\
\left[J^{ab}, J^{c}\right] &= \frac{1}{2}\left(\eta^{ac}J^{b} - \eta^{bc}J^{a}\right) .
\end{aligned}$$

- Contraction:

$$SO(m,n) \longrightarrow ISO(m! - 1, n)$$

- However, the Poincaré group is not a symmetry of the original action. But:

$$ISO(m! - 1, n) \supset SO(m! - 1, n) \subset SO(m,n)$$

- Implying on a symmetry breaking

$$SO(m,n) \longrightarrow SO(m! - 1, n)$$

- Reduced gauge transformations:

$$\begin{aligned} A^a{}_b &\longmapsto A^a{}_b + \mathrm{D}\alpha^a{}_b \,, \\ \theta^a &\longmapsto \theta^a - \alpha^a{}_b\theta^b \,. \end{aligned}$$

- So *A* is a gauge field for SO(m!-1,n) and θ is a (dimensionless) matter field.

- How do we connect this with gravity?

Gravity

- The reduced theory is invariant under *SO(m!-1,n)* gauge transformations.

- Observables = gauge invariants.

- We can identify gauge inv. LCO with geometry:

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{ab}\theta^a_\mu\theta^b_\nu \ , \\
\Gamma^\alpha_{\mu\nu} &= \theta^\alpha_a\partial_\mu\theta^a_\nu + \theta^\alpha_a A^a_{\mu b}\theta^b_\nu
\end{aligned}
$$

- Formally:

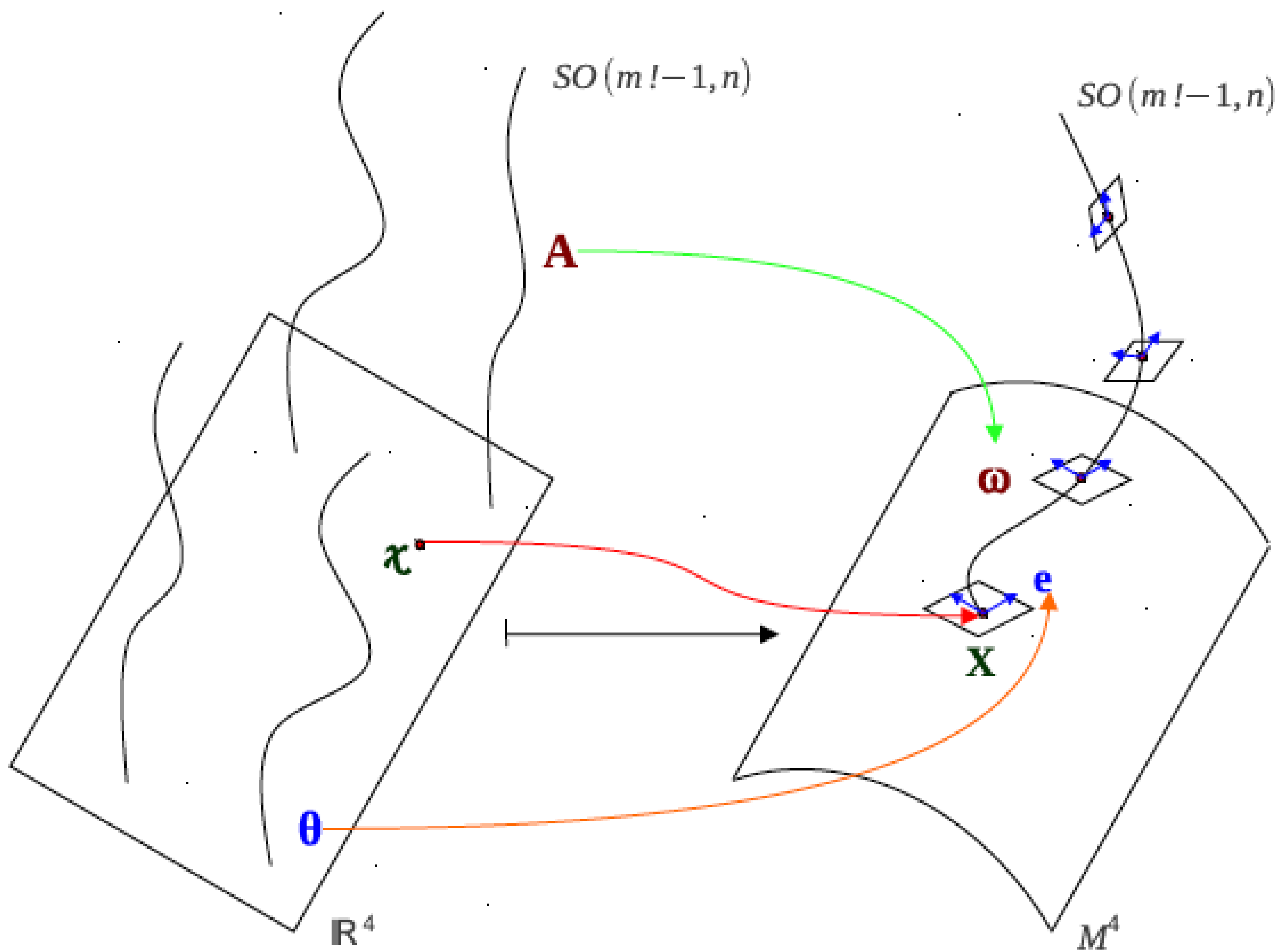
$$\Pi^p \longmapsto \tilde{\Pi}^p \ ,$$
$$*\Pi^p \longmapsto \star\tilde{\Pi}^p$$

- Thus:

$$\omega_\mu^{\mathfrak{ab}}(X)dX^\mu = \delta_a^{\mathfrak{a}}\delta_b^{\mathfrak{b}} A_\mu^{ab}(x)dx^\mu \ ,$$
$$e_\mu^{\mathfrak{a}}(X)dX^\mu = \delta_a^{\mathfrak{a}}\theta_\mu^a(x)dx^\mu \ .$$

- *Frak* indices are related to the tangent space of the deformed space and:

$$X \in \mathbb{M}^4$$



SO(m!−1,n)
SO(m!−1,n)
A
ω
x
e
X
θ
ℝ⁴
M⁴

- A gauge theory in $\mathbb{R}^4$ is mapped into a gravity theory in $\mathbb{M}^4$
- The gravity action is

$$S = \frac{1}{8\pi G}\int \left[\frac{1}{2\Lambda^2} R^{\mathfrak{a}}{}_{\mathfrak{b}} \star R_{\mathfrak{a}}{}^{\mathfrak{b}} + T^{\mathfrak{a}} \star T_{\mathfrak{a}} - \frac{\epsilon}{2}\epsilon_{\mathfrak{abcd}} R^{\mathfrak{ab}} e^{\mathfrak{c}} e^{\mathfrak{d}} + \frac{\Lambda^2}{4}\epsilon_{\mathfrak{abcd}} e^{\mathfrak{a}} e^{\mathfrak{b}} e^{\mathfrak{c}} e^{\mathfrak{d}}\right]$$

where

$$\gamma^2 = \kappa^2/2\pi G \qquad \Lambda^2 = \gamma^2/4$$

Summary

- A **pure** gauge theory in 4d Euclidean space can generate an effective gravity.
- Requirements: asymptotic freedom, dynamical mass parameters, a correct gauge group.
- The local isometries are determined by *SO(m!-1,n)*. Thus, one can interpret this as the rising of the equivalence principle.
- If G is small, than Λ is big. So, there's hope: $\Lambda_{ren} = \Lambda_{obs} - \Lambda_{qft}$
- Mathematical consistency: arXiv:1109.0016 [hep-th].

(Some) Perspectives

- More general groups = dark sectors?

  No dependence on the ratio γ/κ, e.g. SL(5,R) (to appear soon).
- Coupling with matter fields.
- Explicit computations to predict G and Λ. Renormalization Group?
- Find a phase transition. Temperature?
- Use the inverse mapping to study the renormalizability of other gravity theories.

THANK YOU