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The cosmological constant and the vacuum energy: old and new ideas

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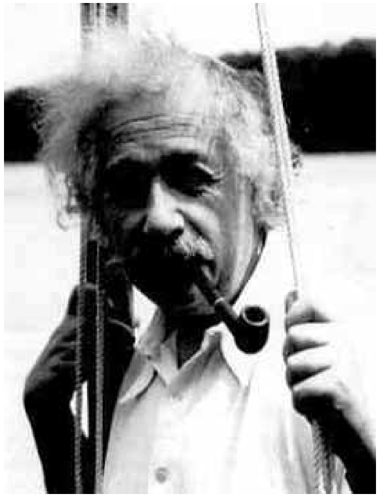
NEB 15, Chania, June 20-23, 2012

Guidelines of the Talk

- Einstein's CC, Vacuum Energy, Dark Energy
- Dynamical CC term in Einstein's equations
- Running CC and Newton's G
- Vacuum energy and and holography
- Conclusions

In a letter to P. Ehrenfest on 4 February 1917 Einstein wrote about his introduction of the CC term:



$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu}\mathbf{R} - \Lambda\, \mathbf{g}_{\mu\nu} = 8\pi \mathbf{G}_N\, \mathbf{T}_{\mu\nu}$$

A. Einstein, Sitzungsber. Konigl. Preuss. Akad. Wiss., phys.-math. Klasse VI, 142 (1917)

"Ich habe wieder etwas verbrochen in der Gravitationstheorie, was mich ein wenig in Gefahr bringt, in ein Tollhaus interniert zu werden"

(I have again perpetrated something relating to the theory of gravitation that might endanger me of being interned to a madhouse.)

- The field equations were proposed by A. Einstein in 1915: "Die Feldgleichungen der Gravitation". Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: 844–847.

- Until about 1930 almost everybody ``knew'' that the universe was static, in spite of the two important papers by Friedmann in 1922 and 1924 and Lemaitre's no less important work in 1927 (recall that Hubble's law is from 1929)

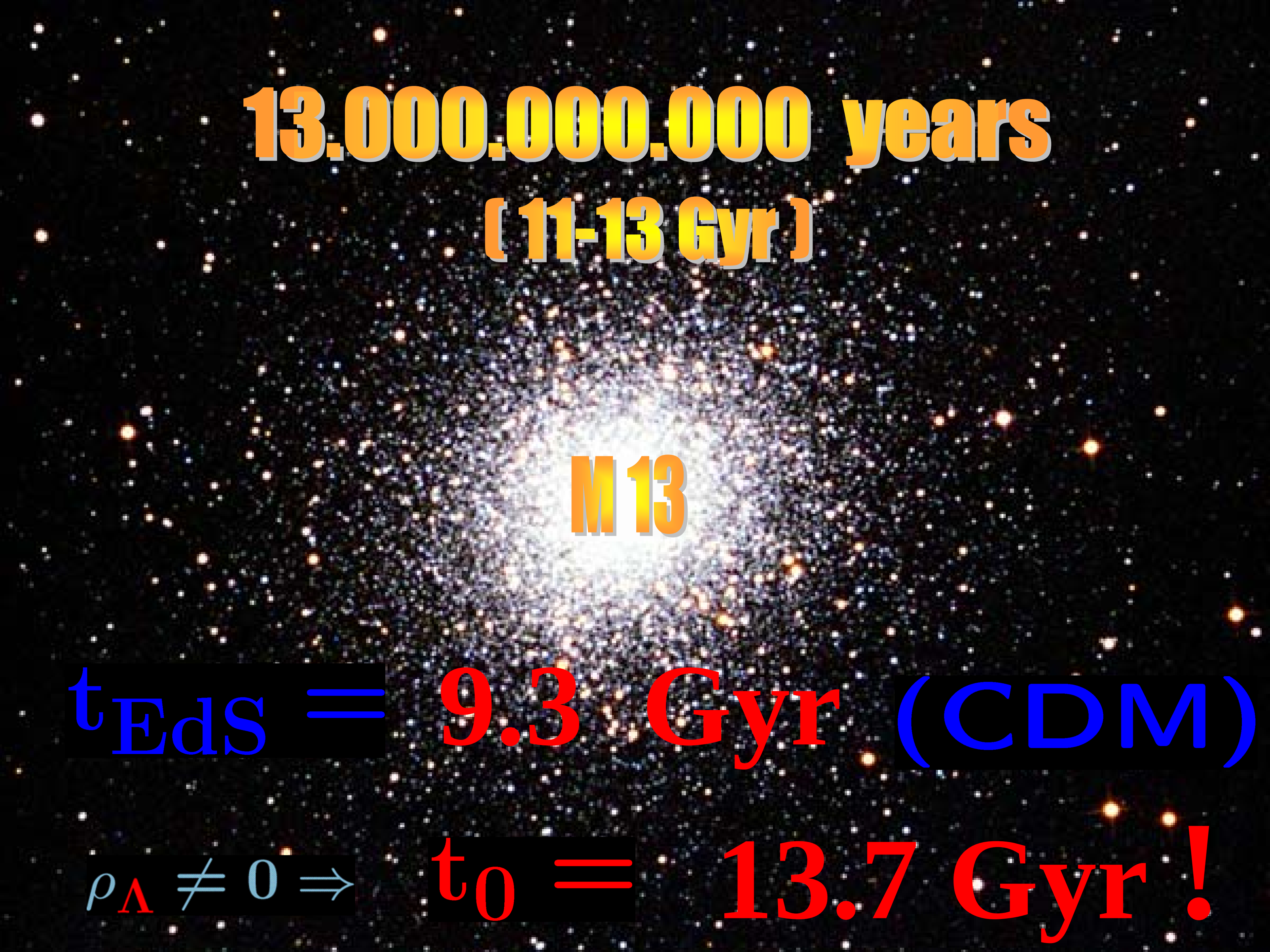
condition for static universe:

$$8\pi G\rho = \frac{1}{a^2} = \Lambda$$

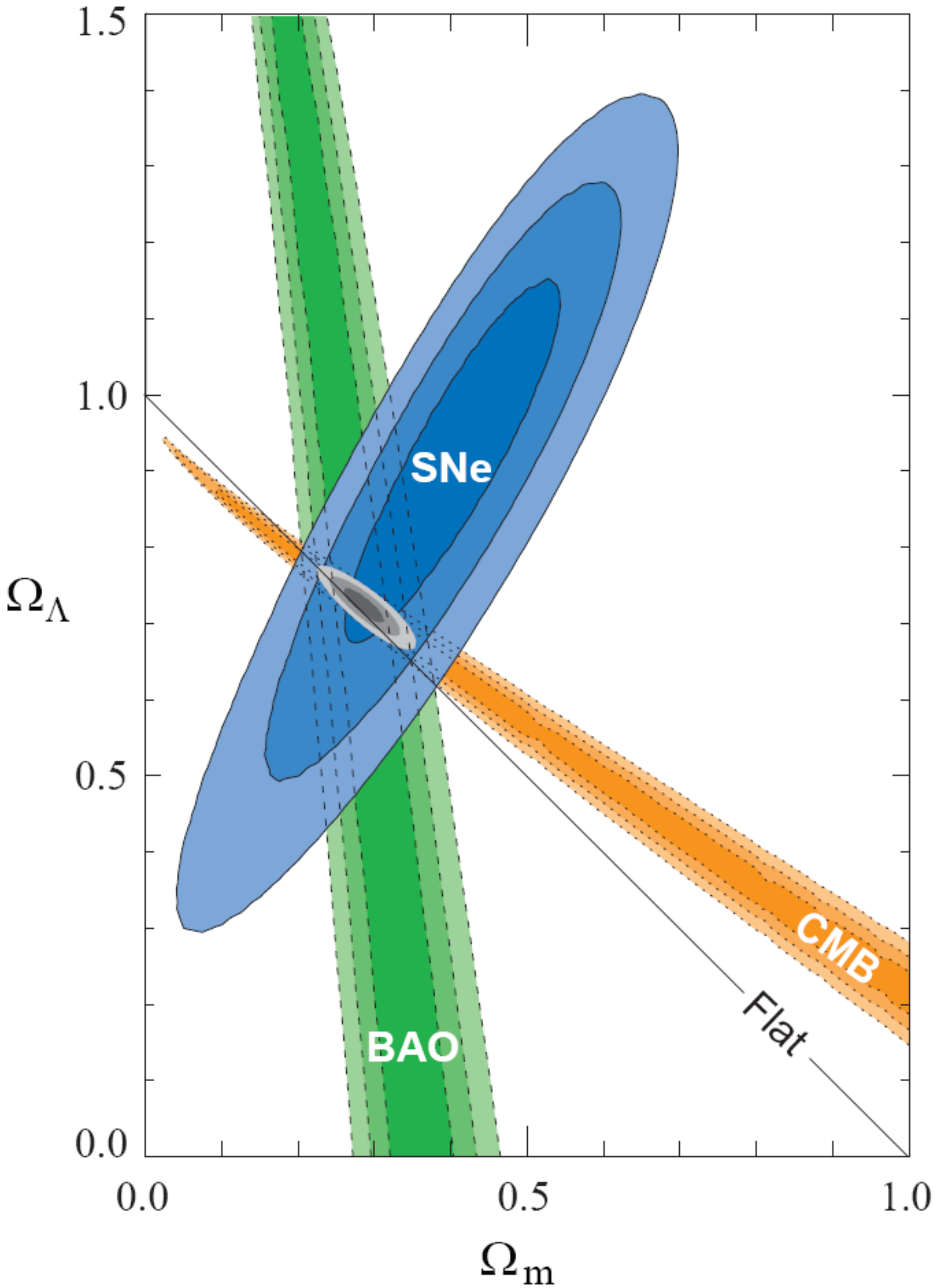
- Einstein himself accepted the idea of an expanding universe only much later

In fact, only after Lemaitre's successful explanation of Hubble's discovery changed the viewpoint of cosmologists. At this point Einstein rejected the cosmological term as superfluous and no longer justified. He published his new view on it in: Einstein A. Sitzungsber. Preuss. Akad. Wiss. (1931) 235-37.

- Funny enough, after the CC was rejected by Einstein, it was later re-introduced by Eddington in order to explain the age of the Universe when H_0 was thought too large (500 Km/s/Mpc) implying t_0 of only 2 billion years (younger than the Earth!). Similar proposals were made later up to the present time to better adjust the modern measurements of H_0 and the ages of stars



13.000.000.000 years
(11-13 Gyr)
M 13
$t_{EdS} = 9.3$ Gyr (CDM)
$\rho_\Lambda \neq 0 \Rightarrow$ $t_0 = 13.7$ Gyr !



$\Omega_\Lambda \simeq 0.73$

$\Omega_M \simeq 0.27$

$\Omega_M + \Omega_\Lambda + \Omega_K = 1$

$\Omega_K \simeq 0$

➢ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\rho_{\Lambda \mathrm{phys}} = \rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}}$$

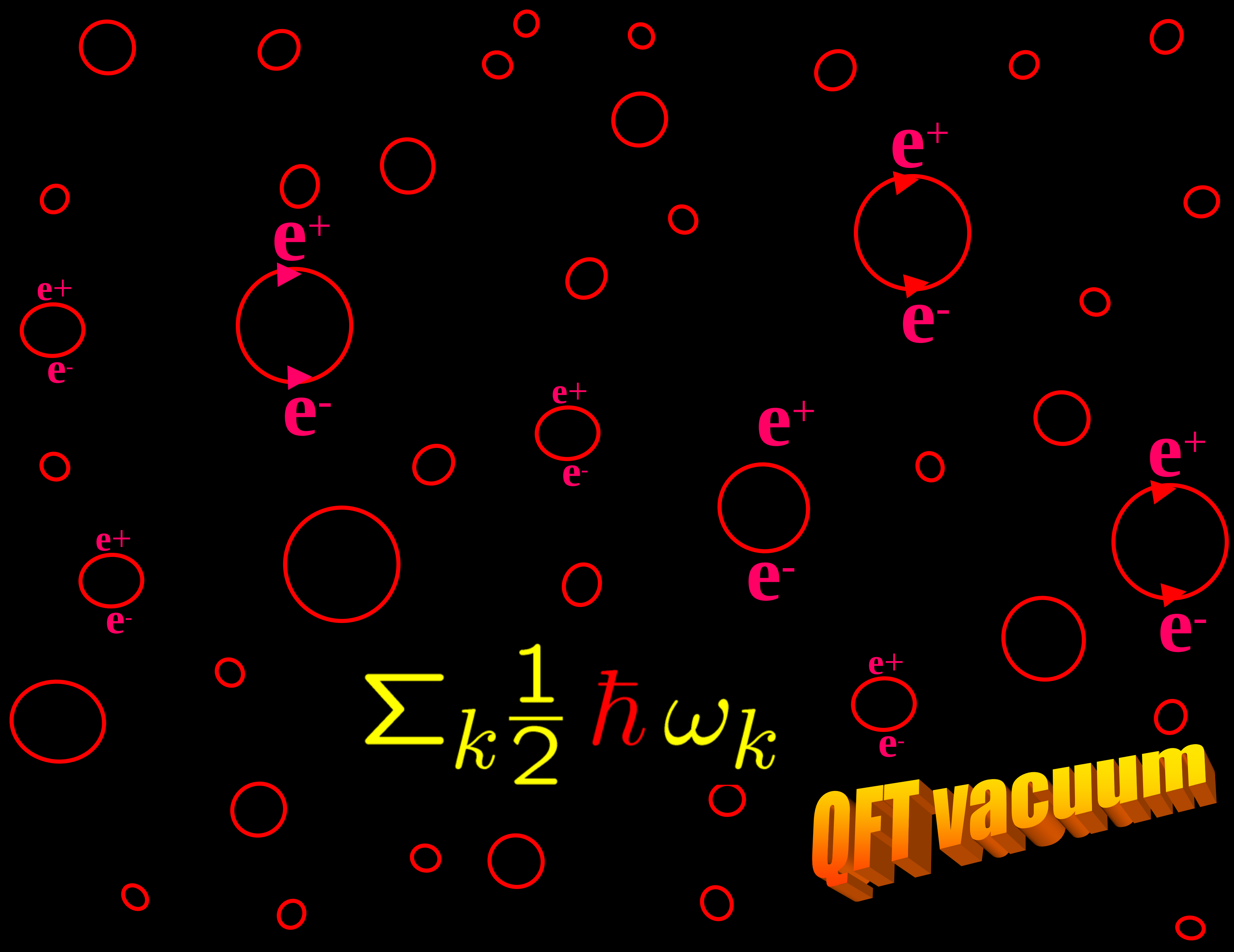
$$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{|g|}\, (R - 2\Lambda_{\mathrm{vac}}) = \int d^4x \sqrt{|g|} \left(\frac{1}{16\pi G_N} R - \rho_{\Lambda \mathrm{vac}} \right)$$

$$\rho_{\Lambda \mathrm{vac}} = \frac{\Lambda}{8\pi G_N}$$

Vacuum bare term in Einstein eqs.

$$R_{ab} - \frac{1}{2} g_{ab} R = -8\pi G_N \left(\langle \tilde{T}_{ab}^{\varphi} \rangle + T_{ab} \right) = -8\pi G_N\, g_{ab} \left(\rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}} + T_{ab} \right)$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi) \rangle + \mathrm{ZPE}$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$\sum_k \frac{1}{2} \hbar \omega_k$
QFT vacuum

Pauli (1933)

(recall that Einstein had already given up the idea in 1931)

zero-point energy of the radiation field
cut off at the classical electron radius

$$8\pi G\rho = \frac{1}{a^2} = \Lambda \quad (1)$$

$$\langle \rho \rangle = \frac{1}{4\pi^2}\int_0^M \mathrm{d}k k^2 \sqrt{k^2+m^2} = \frac{M^4}{16\pi^2}\left(1+\frac{m^2}{M^2}+\cdots\right)$$

In units with $\hbar = c = 1$ the vacuum energy density of the radiation field is

$$<\rho>_{vac} = \frac{8\pi}{(2\pi)^3}\int_0^{\omega_{max}} \frac{\omega}{2}\omega^2 d\omega = \frac{1}{8\pi^2}\omega_{max}^4,$$

(For photons we include a factor of 2)

with

$$\omega_{max} = \frac{2\pi}{\lambda_{max}} = \frac{2\pi m_e}{\alpha}.$$

The corresponding radius of the Einstein universe in Eq.(1) would then be ($M_{pl} \equiv 1/\sqrt{G}$)

$$a = \frac{\alpha^2}{(2\pi)^{\frac{2}{3}}}\frac{M_{pl}}{m_e}\frac{1}{m_e} \sim 31 km.$$

!!

We know now that Pauli's calculation was actually wrong because
he used the unrenormalized result for the ZPE, which depends on the quartic power of the cutoff, whereas QFT tells us that the renomalized result does not depend on any cutof but on the quartic power of the mass (see below)
So indeed photons do NOT contribute to the renormalized ZPE

Renormalized ZPE:

$$V_{\rm ZPE}(\mu) = \hbar\, V_P^{(1)} + \delta\rho_{\Lambda{\rm vac}}$$

$$\rho_\Lambda = \rho_{\Lambda{\rm vac}}(\mu) + \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

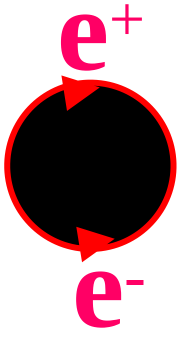
Zero point energy

Planck+Einstein:

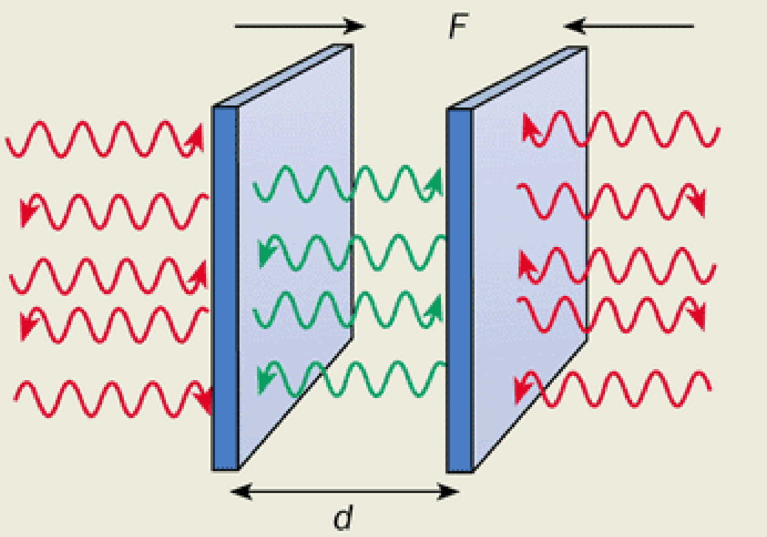
$$E = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}\hbar\omega$$

$$T = 0 \quad ?$$

$$E_0 = \frac{1}{2}\hbar\omega$$



Hendrik Casimir (1909-2000)



$$\omega_n = c\sqrt{k_x^2 + k_y^2 + \frac{n^2\pi^2}{d^2}}$$

Casimir effect

Proposed: 1948

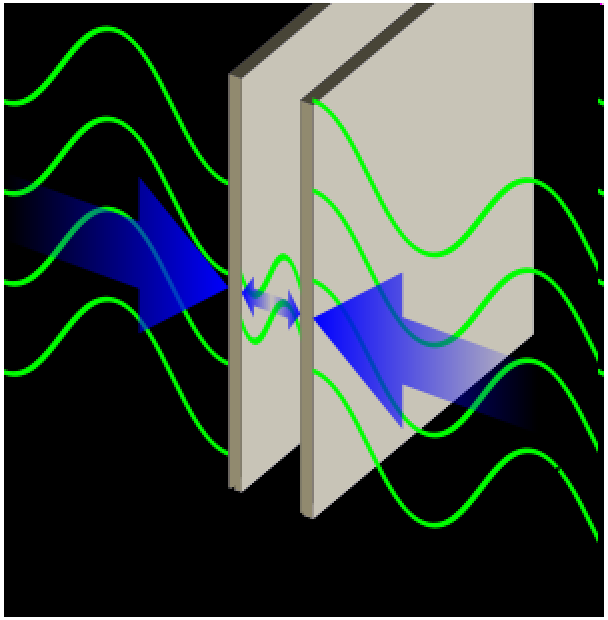
Electrically neutral conducting plates attract each other in the vacuum!

Measuring the energy of the quantum vacuum !!

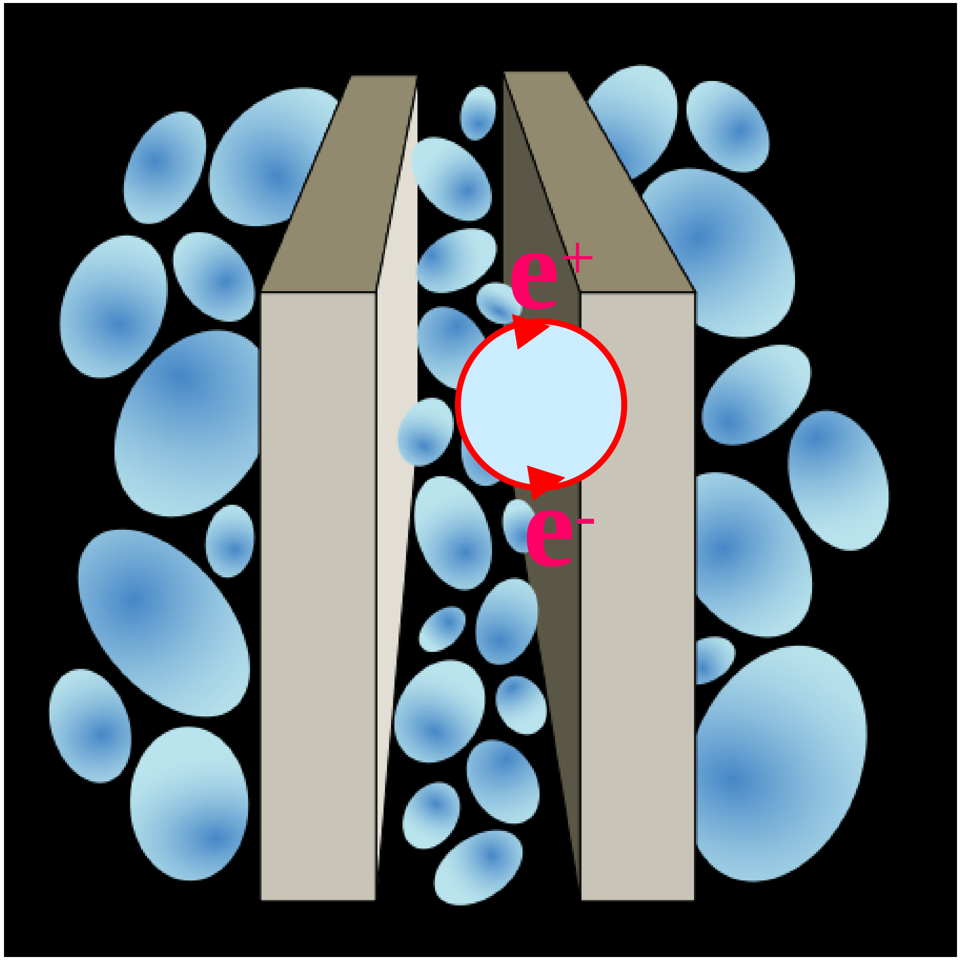
$$E_0(k) = \frac{1}{2}\hbar\,\omega_k$$

$$E_0 = \frac{1}{2}\hbar\sum_k \omega_k \to \frac{1}{2}\hbar\,A\int\frac{dk_x dk_y}{(2\pi)^2}\;\omega_n$$

Casimir effect



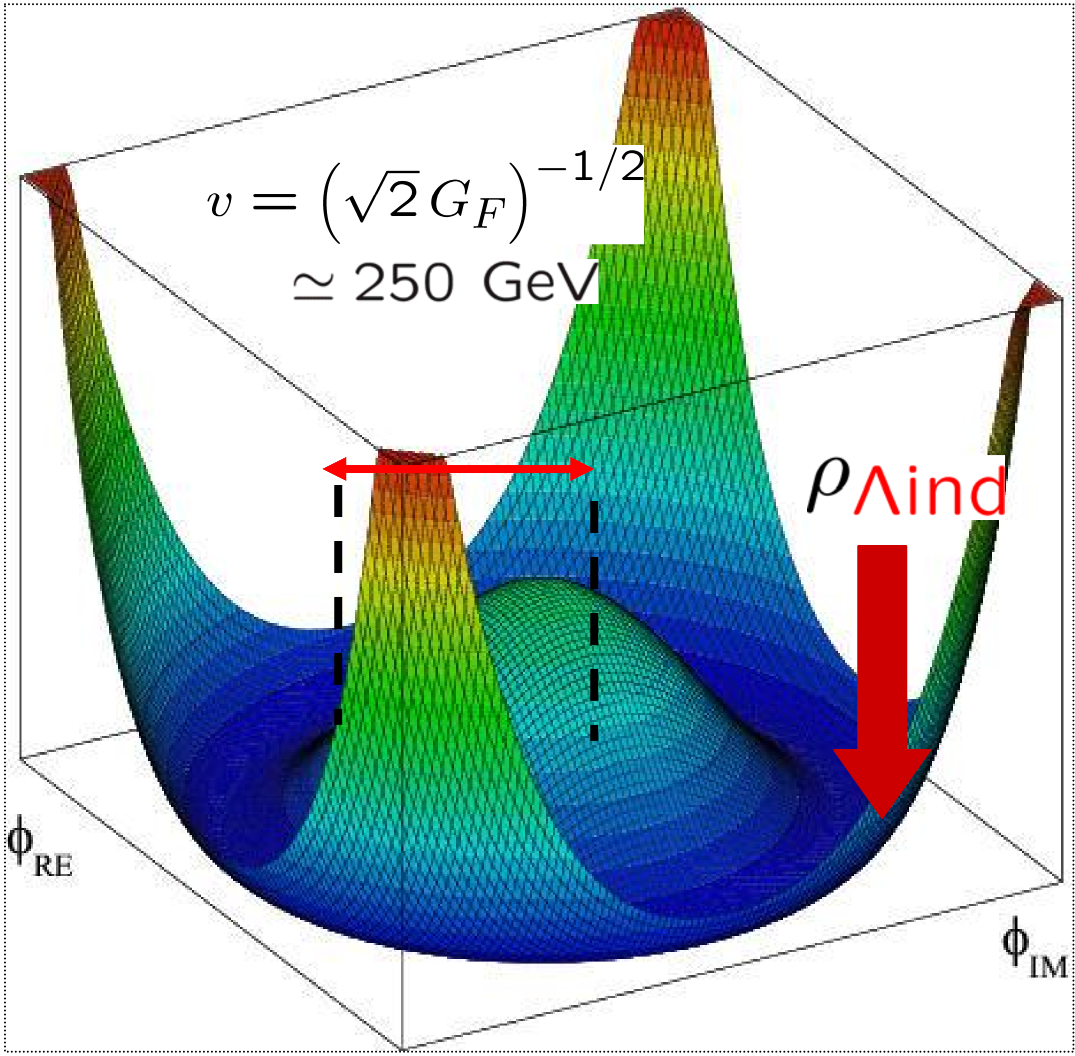
$$\frac{F}{A} = -\frac{\hbar c \pi^2}{240} \frac{1}{d^4}$$ (pressure)



$$F \sim -\frac{\hbar}{d^4}$$

Mesurements: 1958, 1997, 2001…

Higgs Potential → Vacuum Energy



$$G_F/\sqrt{2} = g^2/8M_W^2$$

$$V(\varphi) = \frac{1}{2}\, m^2\, \varphi^2 + \frac{1}{4!}\, \lambda\, \varphi^4$$

$$m^2 < 0 \quad \Rightarrow$$

$$v \equiv \langle \varphi \rangle = \sqrt{\frac{-6\, m^2}{\lambda}}$$

$$\langle V(\varphi) \rangle = -\tfrac{1}{8}\, M_{\mathcal{H}}^2\, v^2$$
$$\sim -10^8\ GeV^4$$

$$M_W = \tfrac{1}{2}\, g\, v$$
$$M_Z = M_W / \cos\theta_w$$
$$m_f = \lambda_f\, v$$

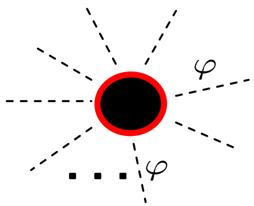
➢ The old CC problem as a fine tuning problem

Take a scalar QFT with effective potential

$$V_{\rm eff} = V + \hbar\, V_1 + \hbar^2\, V_2 + \hbar^2\, V_3 + \ldots$$

where

$$V_1 = V_P^{(1)} + V_{\rm scal}^{(1)}(\varphi)\,, \qquad V_2 = V_P^{(2)} + V_{\rm scal}^{(2)}(\varphi)\,, \qquad V_3 = V_P^{(3)} + V_{\rm scal}^{(3)}(\varphi) \ldots .$$

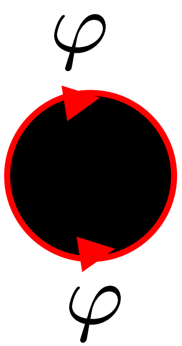


Thus,

$$\boxed{V_{\rm eff}(\varphi) = V_{\rm ZPE} + V_{\rm scal}(\varphi)}$$

with

$$V_{\rm ZPE} = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots .$$



The CC problem (s)

S. Weinberg, Rev.Mod.Phys.61 (1989) 1

Problem 1 : the "old" CC problem

Why all the big contributions to the DE add up to such a small value in Particle Physics units?

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$ $\quad (\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55})$

Problem 2: the cosmic "coincidence" problem

Why the currently observed DE density is so close to the matter density?

coincidence ratio now: $r \equiv \frac{\rho_\Lambda^0}{\rho_M^0} = \frac{\Omega_\Lambda^0}{\Omega_M^0} \simeq \frac{7}{3} = \mathcal{O}(1)$

Λ in the SM and beyond

Source	**Effect** (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

- The first who suggested a possible contribution of vacumm energy to the CC was Zel'dovich

  Y.B.Zel'dovich, JETP letters **6**, 316 (1967); Soviet Physics Uspekhi **11**, 381 (1968).

- He also noticed that in the world with an equal number of bosonic and fermionic d.o.f. having equal masses, the energy of vacuum fluctuations vanishes.

  Ya.B. Zeldovich, Uspekhi Fiz. Nauk, 95, 209 (1968).

- Shortly afterwards it was suggested that there may indeed exist symmetry between bosons and fermions called supersymmetry (**SUSY**), which demands equal number of bosonic and fermionic d.o.f.

  Yu.A. Golfand, E.P. Likhtman, Pis'ma ZhETF, 13, 452 (1971);
  D.V. Volkov, V.P. Akulov Pis'ma ZhETF, 16, 621 (1972);
  J. Wess, B. Zumino, Phys. Lett. 49, 52 (1974).

➢SUSY prediction for the vacuum energy

$Q^{(S=\frac{1}{2})}\,|Fermion\rangle = |Boson\rangle \qquad Q^{(S=\frac{1}{2})}\,|Boson\rangle = |Fermion\rangle$

$$\{Q_\alpha, Q_\beta\} = \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} = 0;$$

SUSY algebra

$$\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu; \qquad [Q_\alpha, P_\mu] = 0.$$

Wess and Zumino (1974)

$$\frac{1}{4}\left(\overline{Q}_1 Q_1 + Q_1\overline{Q}_1 + \overline{Q}_2 Q_2 + Q_2\overline{Q}_2\right) = P^0 \equiv H \geq 0$$

$$E_{\rm vac} \equiv \langle 0|H|0\rangle = 0 \;\Longrightarrow\; \Lambda = 0 \;!!$$

- However our world is NOT supersymmetric !!

If no known symmetry can protect the CC, what else… ?

- Maybe the CC is a dynamical quantity

- Could be mimicked by a scalar field, quintessence and the like

- Or it could be a running quantity in QFT in curved space-time

↓

Running CC and Newton's G…

Time-varying CC models

1) **Power law model** $\Lambda(H) \propto a^{-n}$

M. Ozer and O. Taha (1987),
W. Chen and Y.S. Wu (1990)

2) **Quadratic model** $\Lambda(H) \propto H^2 \propto \rho_T$

J.C. Carvalho, J. Lima, I. Waga (1992),
R.C. Arcuri and I. Waga (1994) etc.

3) **Quantum field vacuum in FLRW universe**

$$\Lambda(H) = n_0 + n_2 H^2$$

I. Shapiro, JS (1999,2000)
JS, H. Stefancic (2005,2006)
JS (2007)
M. Maggiore (2010); N. Bilic (2010)

4) **Linear model** $\Lambda(H) \propto H$

R. Schutzhold, PRL 89 (2002)
S. Carneiro et al. (2008)
F. Klinkhammer, G. Volovik (2009) etc

5) **Power series models:**

$$\Lambda(H) = n_0 + n_1 H + n_2 H^2$$

S. Basilakos, M. Plionis, JS (2009)
F. Costa, J. Lima, F.Oliveira (2012)

$$\Lambda(H) = C_0 + C_{\dot{H}} \dot{H} + C_H H^2$$

S. Basilakos, D. Polarski, JS (2012)

6) **Entropic-holographic model**

$$\Lambda(H) = C_{\dot{H}} \dot{H} + C_H H^2$$

D. Easson, P. Frampton, G. Smoot (2010)

7) **Relaxation model** $\Lambda(H) \propto F(R, G)$

F. Bauer, JS, H. Stefancic (2009,2010,2011),
S. Basilakos, F. Bauer, JS (2011)

...

A semiclassical FLRW with running Λ

I. Shapiro, JS (1999,2000)
JS, H. Stefancic (2005)
JS (2007) ...

$$\rho_\Lambda = C_1 + C_2\, H^2\,.$$

⇓ Bianchi identity

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3\,H(\rho_m + p_m) = 0$$

(matter non-conservation!!)

⇓

$$\rho_m(z) = \rho_m^0\,(1+z)^{3(1-\nu)} \qquad \nu = \frac{M^2}{12\pi\, M_P^2}$$

and dynamical vacuum energy:

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right]$$

Running both... G and Λ ?

Bianchi identity leads to $\nabla^{\mu}\left[G\left(T_{\mu\nu}+g_{\mu\nu}\,\rho_{\Lambda}\right)\right]=0$

$$\frac{d}{dt}\left[G(\rho_m+\rho_\Lambda)\right]+3\,G\,H\left(\rho_m+p_m\right)=0\,.$$

Possible scenario:

$\dot{G}\neq 0$ *and* $\dot{\rho_\Lambda}\neq 0 \Rightarrow \qquad \dot{\rho}_m+3\,H(\rho_m+p_m)=0$

$$(\rho+\rho_\Lambda)\dot{G}+G\dot{\rho_\Lambda}=0$$

Running G logarithmically...

Basic set of equations:
(k=0)

$$\begin{cases} \rho + \rho_\Lambda = \dfrac{3H^2}{8\pi G}\,, \\ \rho_\Lambda = C_1 + C_2\,H^2\,, \\ (\rho + \rho_\Lambda)\,dG + G\,d\rho_\Lambda = 0 \end{cases}$$

$$C_1 = \rho_\Lambda^0 - \frac{3\,\nu}{8\pi} M_P^2\,H_0^2\,, \qquad C_2 = \frac{3\,\nu}{8\pi}\,M_P^2$$

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}$$

Soft decoupling law for the RG evolution of Λ

JS (2007)

$$(\rho_m + \rho_\Lambda)\, dG + G\, d\rho_\Lambda = 0\,,$$

Bianchi identity can be rewritten as follows:

$$\frac{d\rho_\Lambda}{d\ln\mu} = G\,(\rho_m + \rho_\Lambda)\,\frac{d}{d\ln\mu}\left(\frac{1}{G}\right) \;=\; \frac{3}{8\,\pi}\,H^2\,\frac{d}{d\ln\mu}\left(\frac{1}{G}\right)$$

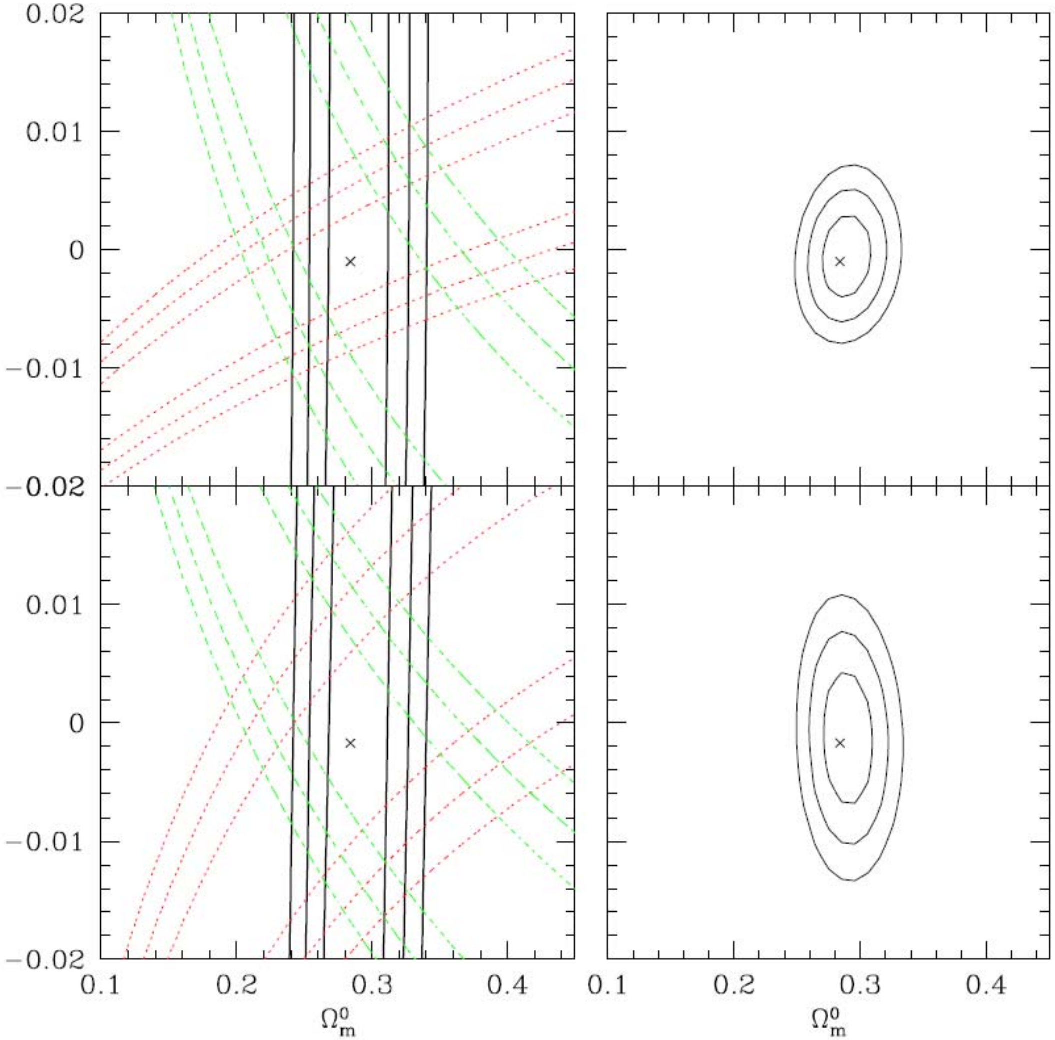
Furthermore, at one-loop we have

$$\frac{d}{d\ln\mu}\left(\frac{1}{G}\right) = \frac{1}{3\,\pi}\sum_F N_F\, m_F^2 + \frac{1}{2\,\pi}\sum_V N_V\, M_V^2$$

As a result we find a realization of the QFT evolving vacuum model :

$$\rho_\Lambda(H) = n_0 + n_2 H^2 \qquad \left(n_2 = \frac{3\nu}{8\pi} M_P^2\right)$$

Confronting running models with recent observations



Fits for model $\Lambda_t G_t CDM$:

$$\nu = -0.001 \pm 0.003$$

$$\Omega_m^0 = 0.283^{+0.012}_{-0.013}$$

Fits for model $\Lambda_t CDM$:

$$\nu = -0.002^{+0.005}_{-0.004}$$

$$\Omega_m^0 = 0.284^{+0.011}_{-0.014}$$

SNIa+BAO+CMB

J. Grande, JS, S. Basilakos, M. Plionis (2011)

Consider a generalized running vacuum model which encompasses entropic force cosmology as a particular case $(C_0 = 0)$:

S.Basilakos, D. Polarski, JS (arXiv:1204.4806)

$$\begin{cases} \left(\dfrac{\dot a}{a}\right)^2 = \dfrac{8\pi G}{3}\displaystyle\sum_i \rho_i + C_0 + C_H H^2 + C_{\dot H}\dot H \\ \dfrac{\ddot a}{a} = -\dfrac{4\pi G}{3}\displaystyle\sum_i (\rho_i + 3p_i) + C_0 + C_H H^2 + C_{\dot H}\dot H \end{cases}$$

with

$$\rho_\Lambda(H,\dot H) = \frac{3}{8\pi G}\left(C_0 + C_H H^2 + C_{\dot H}\dot H\right) = -p_\Lambda(H,\dot H)$$

$$\rho_\Lambda(H) = n_0 + n_2 H^2 \quad \Rightarrow \quad \boxed{\rho_\Lambda(H) = n_0 + n_1 \dot H + n_2 H^2}$$

First, since $C_0 = H_0^2 \left[\Omega_\Lambda^0 - \nu + \left(\Omega_m^0 + \frac{4}{3}\Omega_r^0 \right) \alpha \right]$ $\quad \begin{cases} \xi_m \equiv \frac{1-\nu}{1-\alpha} \\ \xi_r \equiv \frac{1-\nu}{1-\frac{4}{3}\alpha} \end{cases}$

impossible that $|\nu|$ and $|\alpha|$ both small $\Rightarrow$ **no ΛCDM limit** **!!**

Second, if $C_0 = 0$, then in the MDE we find

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = (1 - 3\xi/2) H_0^2 (1+z)^{3\xi}$$

Hence $\ddot{a} > 0 \Longleftrightarrow 3\xi/2 < 1 \Longleftrightarrow \rho_m \sim (1+z)^{3\xi}$ with $3\xi < 2$.

Third, if $C_0 = 0$ $\Rightarrow$ no transition from deceleration $\rightarrow$ acceleration

⇩

Entropic-force models are ruled out **!!**

Running CC models with $C_0 \neq 0$
compatible with observations

Summarized conclusions

- **Dynamical DE** could be explained by a **variable** Λ term or a **variable** $\Lambda + X$ model

(X is in general **not** a field but a non-trivial part of the effective action)

- Expect dynamical terms of the form

$$\delta\rho_\Lambda \sim \nu H^2 M_P^2, \qquad \delta G/G \sim \nu \ln H.$$

- Running vacuum models with $C_0 \neq 0$ are candidates for dynamical DE

- Entropic-force models ($C_0 = 0$) are ruled out

- It is possible to construct ΛXCDM models in which the effective X adjusts **dynamically** the value of the vacuum energy ($\Rightarrow$ no fine-tuning)