Thermodynamic phase structure of charged anti-de Sitter scalar-tensor black holes

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20 June 2012

Plan of the talk

- Charged AdS black holes in STT
- Thermodynamics of charged AdS black holes in STT

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The action of STT

Jordan frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left(F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right) + S_m$$

Eistein frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} \left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi) \right) + S_m[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}]$$

where

$$V(\varphi) = \frac{1}{2}\Lambda \quad and \quad \mathcal{A}(\varphi) = e^{\alpha\varphi}$$

The action of Born-Infeld NLED

$$S_m = \frac{1}{4\pi G_*}\int d^4x\sqrt{-g}\mathcal{A}^4(\varphi)L(X,Y),$$

where $L(X, Y)$ is the Lagrangian of the nonlinear electrodynamics. The equations defining the functions X and Y are

$$X = \frac{\mathcal{A}^{-4}(\varphi)}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1}$$

$$Y = \frac{\mathcal{A}^{-4}(\varphi)}{4}F_{\mu\nu}(\star F)^{\mu\nu}, \tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor and $\star$ stands for the Hodge dual with respect to the metric $g_{\mu\nu}$.
The Lagrangian of the Born-Infeld nonlinear electrodynamics is

$$L = 2b\left[1 - \sqrt{1 + \frac{X}{b} - \frac{Y^2}{4b^2}}\right], \tag{3}$$

Properties of the black hole solutions

- Asymptotically anti-de Sitter
- Single, non-degenerate event horizons (no extremal black holes)
- Purely magnetically charged (metric, scalar field and thermodynamics preserved under electric-magnetic duality rotations)
- Free parameters: α, b, P

Thermodynamic quantities

Temperature

$$T = \left. \frac{f'(r)\, e^{-\delta(r)}}{4\pi} \right|_{r=r_H}$$

Entropy

$$S_J = \frac{1}{4G_*}\int d^2x\sqrt{-^{(2)}\tilde{g}}F(\Phi) = \frac{1}{4G_*}\int d^2x\sqrt{-^{(2)}g} = S_E = S$$

Free energy

$$F(T,P) = M - TS$$

Pair of conjugate variables

$$T(S) \quad \text{or} \quad T(r_H)$$

Effect of b and α on phase structure

- For $b > b_{\rm crit}$ – just one stable phase; no phase transitions.
- For $b < b_{\rm crit}$ – two stable phases; zeroth order and first order phase transitions.

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Effect of the magnetic charge P on phase structure

The function $T = T(r_H, P)$ has two inflection points $P_{\rm crit}^{(1)}$ and $P_{\rm crit}^{(2)}$ defined by

$$\frac{\partial T}{\partial r_H} = \frac{\partial^2 T}{\partial r_H^2} = 0$$

- Low charge $P < P_{\rm crit}^{(1)}$
- Middle charge $P_{\rm crit}^{(1)} < P < P_{\rm crit}^{(2)}$ (three subintervals)

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- High charge $P_{\rm crit}^{(2)} < P$

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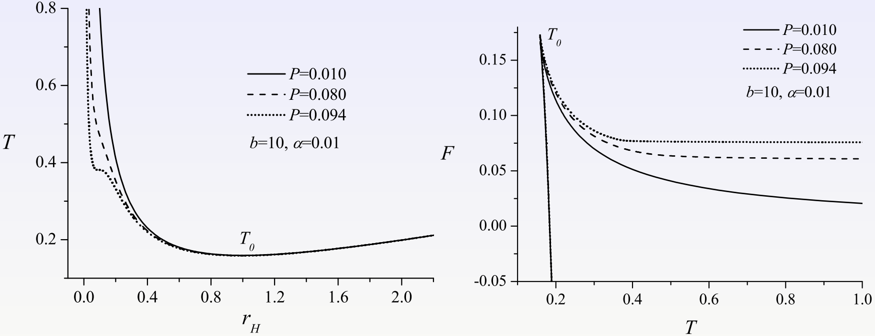
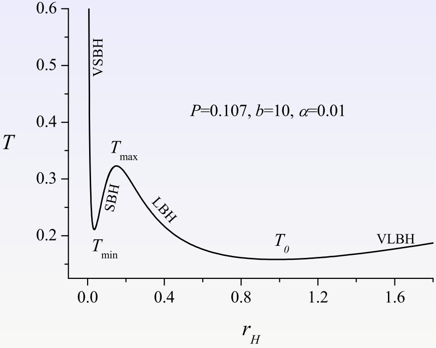
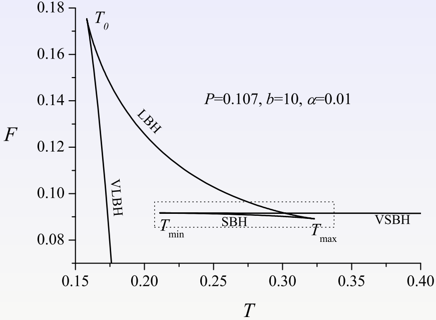
- Low charge $P < P^{(1)}_{\mathrm{crit}}$
- Middle charge $P^{(1)}_{\mathrm{crit}} < P < P^{(2)}_{\mathrm{crit}}$ (three subintervals)
- High charge $P^{(2)}_{\mathrm{crit}} < P$

Low charge


0.8
0.6
0.4
0.2
T
0.0
0.4
0.8
1.2
1.6
2.0
r_H
T_0
P=0.010
P=0.080
P=0.094
b=10, α=0.01
0.15
0.10
0.05
0.00
-0.05
F
0.2
0.4
0.6
0.8
1.0
T
T_0
P=0.010
P=0.080
P=0.094
b=10, α=0.01

Middle charge – subinterval I

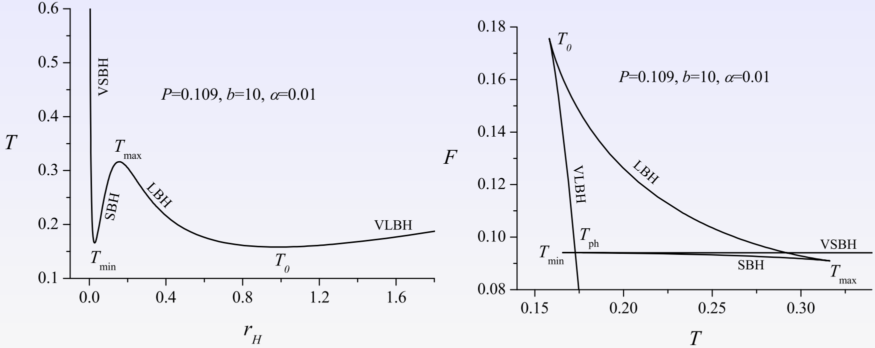




$$T_0 < T_{\rm min};$$

For all values of T, $F_{\rm VLBH} < F_{\rm SBH}$

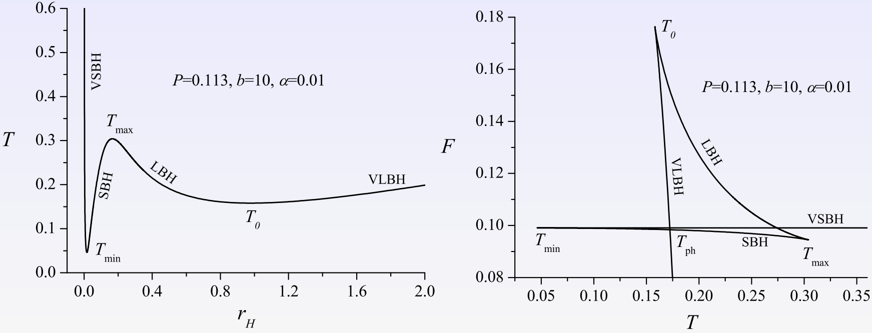
Middle charge – subinterval II



$$T_0 < T_{\rm min};$$

- $T_0 < T < T_{\rm min}$, only VLBH;
- $T_{\rm min} < T < T_{\rm ph}$, $F_{\rm SBH} < F_{\rm VLBH}$, 0-th order phase transition;
- $T > T_{\rm ph}$, $F_{\rm VLBH} < F_{\rm SBH}$, 1-st order phase transition.

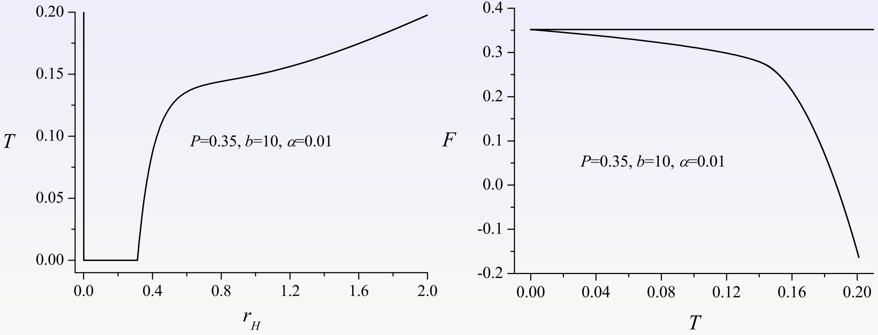
Middle charge – subinterval III



$$T_0 > T_{\min}$$

- $T_{\min} < T < T_0$, only SBH;
- $T_0 < T < T_{\mathrm{ph}}$, $F_{\mathrm{SBH}} < F_{\mathrm{VLBH}}$;
- $T > T_{\mathrm{ph}}$, $F_{\mathrm{VLBH}} < F_{\mathrm{SBH}}$, 1-st order phase transition.

High charge


0.20
0.15
0.10
0.05
0.00
T
0.0
0.4
0.8
1.2
1.6
2.0
r_H
P=0.35, b=10, α=0.01
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
F
0.00
0.04
0.08
0.12
0.16
0.20
T
P=0.35, b=10, α=0.01

Off-equilibrium consideration

Off-shell free energy

$$F_{\rm off} = M(r_H) - T_{\rm th} S(r_H) = F_{\rm off}(r_H, T_{\rm th})$$

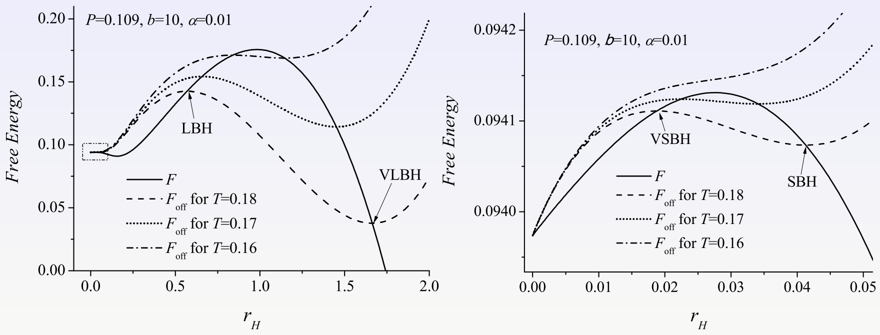
Equilibrium states occur at the extrema of the off-shell free energy

$$\left. \frac{\partial F_{\rm off}(r_H)}{\partial r_H} \right|_{T_{\rm th}={\rm const}} = 0$$

This relation defines r_H as an implicit function of $T_{\rm th}$ which, in the general case, may have several branches $r_{H,a}(T_{\rm th}), \quad a = 0, 1, 2....$

$$F_a(T) = F_{\rm off}(r_{H,a}(T_{\rm th}),\, T_{\rm th})|_{T_{\rm th}=T} \,.$$

Off-equilibrium consideration



Summary of results

- Low charge $P < P^{(1)}_{\rm crit}$: 2 phases, no phase transitions;
- Middle charge $P^{(1)}_{\rm crit} < P < P^{(2)}_{\rm crit}$: 4 phases
  - subinterval I: 2 stable phases, no phase transitions;
  - subinterval II: 2 stable phases, 2 phase transitions,
    VLBH $\rightarrow$ SBH 0-th order phase transition,
    SBH $\rightarrow$ VLBH 1-st order phase transition
  - subinterval III: 2 stable phases, 1 phase transition,
    SBH $\rightarrow$ VLBH 1-st order phase transition;
- High charge $P^{(2)}_{\rm crit} < P$: 2 phases, no phase transitions.

Suggestions for further research

- Is there connection between thermodynamical and dynamical stability?
- Under what conditions are these phase transitions realized?

Results published in....

- Daniela D. Doneva, Stoytcho S. Yazadjiev, Kostas D. Kokkotas, Ivan Zh. Stefanov, Michail D. Todorov, "Charged anti-de Sitter scalar-tensor black holes and their thermodynamic phase structure", Phys. Rev. D 81, 104030 (2010).

THANK YOU!