Hořava–Lifshitz Gravity: Detailed Balance Revisited

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Hořava's Proposal

- *In 2009, Hořava proposed an UV completition to GR modifying the graviton propagator by adding to the gravitational action higher order spatial derivatives without adding higher order time derivatives*
- *This prescription requires a splitting of spacetime into space and time and leads to Lorentz violations*
- *Lorentz invariance can be recovered at low-energy, or at least Lorentz violations in the infrared (IR) are requested to stay below current experimental constraints*

P. Hořava, Phys. Rev. D **79**, *084008 (2009)*

Foundations of the Theory

The theory is constructed using the full ADM metric

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx_i + N_i dt)(dx_j + N_j dt) \tag{1}$$

The most general action is

$$S_H = S_K - S_V \tag{2}$$

where the kinetic term, which contains all of the time derivatives, is given by

$$S_K = \frac{M_{\rm pl}^2}{2} \int dt d^3x \sqrt{g} N \left(K_{ij}K^{ij} - \lambda K^2 \right) \tag{3}$$

K_{ij} *is the extrinsic curvature of the spacelike hypersurfaces,*

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \tag{4}$$

Foundations of the Theory

and the potential term is

$$S_V = \frac{1}{2M_{\rm pl}^2}\int dtd^3x\sqrt{g}N\, V\left[g_{ij}, N\right] \tag{5}$$

- *Power counting renormalizability requires as a minimal prescription at least 6th order spatial derivatives in V*
- *The most general potential V with operators of dimensions up to 6 (the operators with odd dimensions are forbidden by spatial parity) contains tens of terms*
- *The theory propagates both a 2-spin and a 0-spin graviton*

Detailed Balance with Projectability

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2. *We can impose as an additional symmetry to the theory the so called "**Detailed Balance**": it requires that V should be derivable from a superpotential W as follows*

$$V = E^{ij} G_{ijkl} E^{kl} \tag{7}$$

where E^{ij} is given in term of a superpotential W as

$$E^{ij} = \frac{1}{\sqrt{g}} \frac{\delta W\left[g_{kl}\right]}{\delta g_{ij}} \tag{8}$$

The Superpotential W and the Action

The most general superpotential containing all of the possible terms up to third order in spatial derivatives is

$$W = \frac{M_{\rm pl}^2}{2M_6^2}\int \omega_3(\Gamma) + \frac{M_{\rm pl}^2}{M_4}\int d^3x\sqrt{g}\left[R - 2\xi(1-3\lambda)M_4^2\right] \qquad (9)$$

where $\omega_3(\Gamma)$ *is the gravitational Chern-Simons term. Then the action is*

$$\begin{aligned} S_H &= \frac{M_{\rm pl}^2}{2}\int dt d^3x\sqrt{g}N\Bigg[K_{ij}K^{ij} - \lambda K^2 + \xi R - 2\Lambda - \frac{1}{M_4^2}R_{ij}R^{ij} \\ &+ \frac{1-4\lambda}{4(1-3\lambda)}\frac{1}{M_4^2}R^2 + \frac{2}{M_6^2 M_4}\epsilon^{ijk}R_{il}\nabla_j R^l_k - \frac{1}{M_6^4}C_{ij}C^{ij}\Bigg] \end{aligned} \qquad (10)$$

where the (bare) cosmological constant is

$$\Lambda = \frac{3}{2}\xi^2(1-3\lambda)M_4^2 \qquad (11)$$

Problems

Obvious:

1. *There is a parity violating term (the term which is fifth order in derivatives)*
2. *The scalar mode does not satisfy a sixth order dispersion relation and is not power-counting renormalizable (see later)*
3. *The (bare) cosmological constant has the opposite sign and it has to be much larger than the observed value*

T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. **102**, *251601 (2009)*

C. Appignani, R. Casadio and S. Shankaranarayanan, JCAP **1004**, *006 (2010)*

Less obvious:

1. *The infrared behavior of the scalar mode is plagued by instabilities and strong coupling at unacceptably low energies*

C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, JHEP **0908**, *070 (2009)*

D. Blas, O. Pujolas and S. Sibiryakov, JHEP **0910**, *029 (2009)*

Projectable Version of the Theory without DB

T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. **102**, *251601 (2009)*

$$
\begin{aligned}
S_p \;=\; & \frac{M_{\rm pl}^2}{2}\int d^3x dt N\sqrt{g}\Big\{K^{ij}K_{ij}-\lambda K^2-g_0\,M_{\rm pl}^2-g_1 R-g_2\,M_{\rm pl}^{-2}\,R^2 \\
& -g_3\,M_{\rm pl}^{-2}\,R_{ij}R^{ij}-g_4\,M_{\rm pl}^{-4}\,R^3-g_5\,M_{\rm pl}^{-4}\,R(R_{ij}R^{ij}) \qquad (12)\\
& -g_6\,M_{\rm pl}^{-4}\,R^i{}_jR^j{}_kR^k{}_i-g_7\,M_{\rm pl}^{-4}\,R\nabla^2R-g_8\,M_{\rm pl}^{-4}\,\nabla_iR_{jk}\,\nabla^iR^{jk}\Big\}
\end{aligned}
$$

- *Parity violating terms have been suppressed*
- *Power-counting renormalizability is achieved*
- *The cosmological constant is controlled by g_0 and it is not restricted*
- *Strong coupling and instabilities plaguing the scalar mode at low energies*

K. Koyama and F. Arroja, JHEP **1003**, *061 (2010)*

T. P. Sotiriou, M. Visser and S. Weinfurtner, JHEP **0910**, *033 (2009)*

A Possible Solution for these Problems

- *Abandoning Projectability: one can use not only the Riemann tensor of g_{ij} and its derivatives in order to construct invariants under foliation preserving diffeomorphisms, but also the vector $a_i = \partial_i lnN$*

- *In the version without detailed balance this leads to a proliferation of terms ($\sim 10^2$), while here there is, remarkably, only one 2-dim operator one can add to the superpotential W in the version with detailed balance: $a_i a^i$*

New Superpotential and Action

The superpotential W then becomes

$$W=\frac{M_{\rm pl}^2}{2M_6^2}\int\omega_3(\Gamma)+\frac{M_{\rm pl}^2}{M_4}\int d^3x\sqrt{g}\left[R-2\xi(1-3\lambda)M_4^2\right]+\beta\int d^3x\sqrt{g}\,a_ia^i \tag{13}$$

The total action now looks as

$$S_H=\frac{M_{\rm pl}^2}{2}\int dtd^3x\sqrt{g}N\Bigg\{K_{ij}K^{ij}-\lambda K^2+\xi R-2\Lambda+\eta\,a^ia_i$$
$$-\frac{1}{M_4^2}R_{ij}R^{ij}+\frac{1-4\lambda}{4(1-3\lambda)}\frac{1}{M_4^2}R^2+\frac{2\eta}{\xi M_4^2}\left[\frac{1-4\lambda}{4(1-3\lambda)}Ra^ia_i-R_{ij}a^ia^j\right] \tag{14}$$
$$-\frac{\eta^2}{4\xi^2M_4^2}\frac{3-8\lambda}{1-3\lambda}(a^ia_i)^2+\frac{2}{M_6^2M_4}\epsilon^{ijk}R_{il}\nabla_jR^l_k+\frac{2\eta}{\xi M_6^2M_4}C^{ij}a_ia_j-\frac{1}{M_6^4}C_{ij}C^{ij}\Bigg\}$$

where $\Lambda=\frac{3}{2}\xi^2(1-3\lambda)M_4^2$ *as before.*

D. Vernieri and T. P. Sotiriou, Phys. Rev. D **85***, 064003 (2012)*

Linearization at Quadratic Order in Perturbations

Assuming that some resolution to the cosmological constant problem were to be found, we simply set $\Lambda = 0$ in the action and we perturb it to quadratic order, considering only scalar perturbations

$$N = 1 + \alpha, \qquad N_i = \partial_i y, \qquad g_{ij} = e^{2\zeta}\delta_{ij} \tag{15}$$

Finally the quadratic action reads

$$\begin{aligned} S_H^{(2)} &= \frac{M_{\rm pl}^2}{2}\int dt d^3x \left\{ \frac{2(1-3\lambda)}{1-\lambda}\dot{\zeta}^2 + 2\xi\left(\frac{2\xi}{\eta} - 1\right)\zeta\partial^2\zeta \right. \\ & \left. - \frac{2(1-\lambda)}{1-3\lambda}\frac{1}{M_4^2}(\partial^2\zeta)^2 \right\} \end{aligned} \tag{16}$$

Dispersion Relation and Stability

The dispersion relation for the scalar is then given by

$$\omega^2 = \xi\left(\frac{2\xi}{\eta} - 1\right)\frac{1-\lambda}{1-3\lambda}p^2 + \frac{1}{M_4^2}\left(\frac{1-\lambda}{1-3\lambda}\right)^2 p^4 \qquad (17)$$

For the scalar to have positive energy as well as for the spin-2 graviton one needs

$$\lambda < \frac{1}{3} \qquad \text{or} \qquad \lambda > 1 \qquad (18)$$

whereas classical stability requires that

$$c_\zeta^2 = \xi\left(\frac{2\xi}{\eta} - 1\right)\frac{1-\lambda}{1-3\lambda} > 0 \qquad (19)$$

Dispersion Relation and Stability

- *For the spin-2 graviton to be stable $\xi > 0$ must be required, then one has*

$$2\xi > \eta > 0 \tag{20}$$

- *The coefficient of the p^4 term cannot lead to an instability at higher energies.*

- *The scalar satisfies a fourth, and not a sixth, order dispersion relation:*
  The renormalizability properties of the theory are compromised!

Recovering Power-Counting Renormalizability

- *Adding fourth order terms in W would lead to both sixth and eight order terms for the scalar, rendering the theory power-counting renormalizable.*
  The fourth order terms one could add in the superpotential W are

$$R^2\,, \quad R^{\mu\nu}R_{\mu\nu}\,, \quad R\nabla^i a_i\,, \quad R^{ij}a_i a_j\,,$$
$$R a_i a^i\,, \quad (a_i a^i)^2\,, \quad (\nabla^i a_i)^2\,, \quad a_i a_j \nabla^i a^j\,. \tag{21}$$

- *After adding these terms and imposing parity invariance in total there would be 12 free couplings in the theory.*
  This is roughly an order of magnitude less than the number of couplings in the theory without detailed balance.

Conclusions

- *Hořava–Lifshitz gravity as an UV complete gravity theory*
- *Problems of the Projectable theory with or without Detailed Balance*
- *Solutions: Abandoning Projectability; Adding fourth order terms in W*
- *Results: Improved behaviour of the Scalar Graviton in the IR; Power-counting renormalizability of the Theory*
- *Shortcomings: Magnitude and Sign of the Bare Cosmological Constant*
- *Perhaps there could be a cancellation between Bare Cosmological Constant and Vacuum Energy, leaving behind a tiny residual that would account for the observed value*