Causal Sets Dynamics: Progress and Outlook

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Contents

Causal Set: Definition

(a) Partially Ordered Set

(b) The order relation is the *causal* relation.

(c) Locally finite

- Partially Ordered Set: a set $\mathcal{P}$ with relation $\prec$, such that $\forall \quad x, y, z \in \mathcal{P}$:

(a) $x \prec y$ and $y \prec z \Rightarrow x \prec z$: Transitivity

(b) $x \prec y$ and $y \prec x \Rightarrow x = y$: Acyclicity

- Locally Finite: $\{z | x \prec z \prec y\}$ is a finite set for all $x, y \in \mathcal{P}$. (ie. Discrete rather than Continuous)

Motivation

Why Discrete?

- Indications from several other approaches to QG of some space(time) discretness.

- Infinities + Singularities in GR, QFT and black hole thermodynamics.

- Suggestions for modifying Gravity (see e.g. cosmological constant problem).

Why Causal?

- Direct physical motivation (cause-effect). Standard spacetime, is less natural (assuming topology+differential structure+metric).

- Causal relations include most of the information of a Lorentzian manifold (apart from conformal info $\sqrt{-g}d^4x$).

How can a Causal Set replace Spacetime?

Theorem(Malament): The metric of a globally hyperbolic spacetime can be reconstructed uniquely from its causal relations up to a conformal factor.*

• For a discrete spacetime, one can fix the remaining degree of freedom, by equating the volume V with the number of elements N (we can fix the units, by assuming that a single element corresponds to a unit Plank volume).

Order+Number=Geometry

*David Malament, J. Math. Phys 18: 1399 (1977)

Central Conjecture: Two distinct, non-isometric spacetimes cannot arise from a single causal set.

A natural question that arises, is when can we say that a particular causal set is well approximated by a manifold. For this we use the concept of *faithful embedding*

Faithful Embedding: A map ϕ from a causal set to a manifold M such that

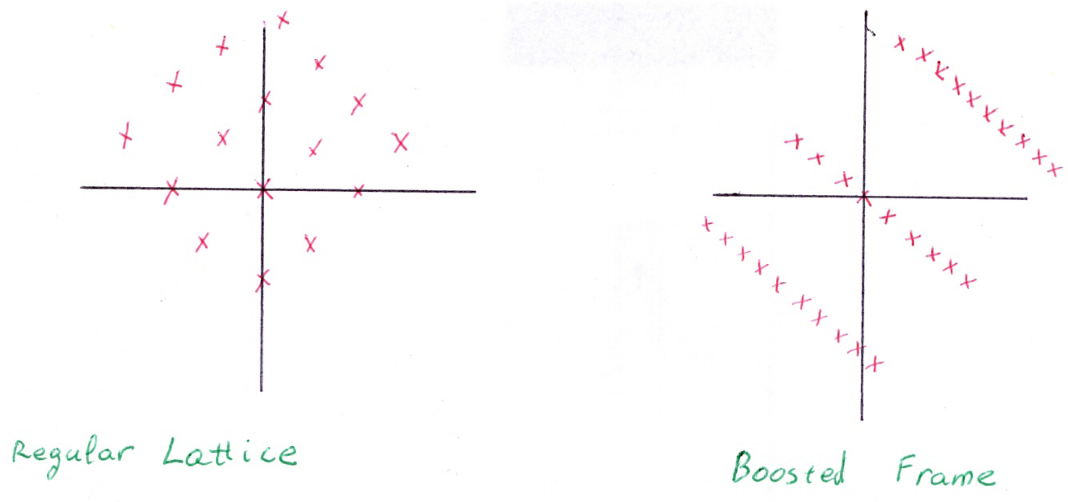
(a) Preserves causal relations (i.e. $x \prec y \iff \phi(x) \prec \phi(y)$).

(b) The number of elements N mapped into *any* Alexandrov neighborhood, is equal to its spacetime volume V up to poisson fluctuations,

i.e. $N = V \pm O(\sqrt{V})$

(c) M doesn't possess curvature at scales smaller than that of the "intermolecular spacing" of the embedding.

Using this definition, we can easily see that a regular lattice of (say) $2-d$ Minkowski spacetime does NOT embed faithfully to $\mathbf{M}^2$.



Instead, we need a *random* lattice. (e.g. one generated, by sprinkling elements in spacetime, randomly, with probability $P(n) = \frac{(\rho V)^n e^{-\rho V}}{n!}$ i.e. a Poisson sprinkling.)

Kinematics and Phenomenology

- Links: $x \prec y$ are linked if $\nexists \quad z| \quad x \prec z \prec y$.

- Chain C: $\quad \forall \quad x, y \in C \quad x \prec y$ or $y \prec x$.

Dimension of causal set: ‘Midpoint-scaling’, compare the ‘proper time’ (longest chain) of two related elements, with the volume (number of elements between the two). Other measures exists that give fractal dimension for causets non-embedable to manifolds.

Topology of the causal set: one needs to consider the analogue of spacelike surface, which is a *maximal anti-chain* (collection of elements, that are all un-related). One considers the (immediate) future and past ('thickened' anti-chain). Get neighborhoods for each element, and construct a simplicial complex that gives the homology of the slice. Agreement with continuous topology.

Geometry: Timelike distance: *:

$d_t(x,y) := \max |C_i \cap J^+(x) \cap J^-(y)|$

and $|A|$ is set cardinality. It is the maximum steps needed to go from x to y.

Spacelike distance: more subtle. Using relations one can recover the spacelike distance for causets embedded in Minkowski †. Can use this to obtain lengths of curves in curved spacetime.

*G. Brightwell and R. Gregory, Phys. Rev. Lett. 66: 260-263 (1991)

†D. Rideout and P. Wallden, Class. Quant. Grav. 26 (2009) 155013

Phenomenology: Elements of causal set nature of spacetime could be apparent even prior constructing full quantum dynamics. Indicatively:

Cosmological Constant Problem: Heuristic Argument, how discreteness of spacetime along with Lorentz invariance leads to a non-zero cosmological constant of the order of magnitude of the critical density and thus in agreement with the actual value. Importantly that prediction was made as early as 1991 (prior the experiments)‡

Entropy bounds and BH entropy: Fundamental discreteness can account for the finite value of entropy bounds and BH entropy, as counting fundamental degrees of geometry crossing the

‡R. Sorkin, in Relativity and Gravitation: Classical and Quantum, pp. 150-173. World Scientific, Singapore, 1991.
M. Ahmed, S. Dodelson, P. Greene and R. Sorkin, Phys. Rev. D69 (2004) 103523.

horizon. Counting links that cross the horizon is one attempt[§].

Deviations of motion on Causet: Possible deviation of a particle moving on a discrete random background rather than a continue spacetime. Indeed leads to a diffusion type equation, however deviations are beyond observation in current experiments.

[§]D. Dou and R. Sorkin, Found. Phys. 33 (2003) 279

Dynamics: Introduction

- What is the effect on quantum matter and fields, if we replace continuum spacetime with a causal set.

- What dynamics could a pure classical causal set have, that would be intrinsic to the definition of a causet

- How could one formulate quantum dynamics of a causal set and how one would interpret it. Two approaches: (a) bottom-up (starting from fundamental relations of causal set) (b) top-down (get motivation from continuous spacetime and make analogy for causet)

- "Entropy" problem. Most causets are not manifold like. By counting, the vast majority of causets, are 3-layers with $n/4$ in layer 1, $n/2$ at layer 2 and $n/4$ at layer 3 (Kleitman-Rothschild). Dynamics should select causal sets that are manifold like.

Dynamics: Quantum Matter on Classical Causet

- Matter can appear on a causet in two ways:

(1) From the fundamental relations. Matter degrees of freedom can appear for example in a Kaluza-Klein way.

(2) Model matter ON a causal set (this is explored here)

- Causal set analogue of Green's function for given field is a property of the causet. It can been shown that $G_{ret} + G_{adv} = 1/2(L + L^T)$ in 4d, where L is the link matrix.

  The d'Alembertian can be recovered from symmetrising and inverting*.

*S. Johnston, Class. Quant. Grav. 25, 202001 (2008)

- Alternatively one considers a slowing varying (at some frame) field $\phi(x)$ and uses $\Box_d\phi(u,v) = \frac{1}{a^2}(\phi(u,v) - \phi(u-a,v) - \phi(u,v-a) + \phi(u-a,v-a))$ to define the operator:

$$
\begin{aligned}
B\phi(x) &= \frac{4}{a^2}(-\frac{1}{2}\phi(x) + \sum_{y\in N_1(x)} \phi(y) \\
&\quad -2 \sum_{y\in N_2(x)} \phi(y) + \sum_{y\in N_3(x)} \phi(y))
\end{aligned}
$$

which can be shown (confirmed by simulations) to agree in average value with the d'Alembertian for flat space in the suitable limit. N_i is the set of elements i steps distance from x. Further care is needed to guarantee the variations are also controlled[†].

[†]R. Sorkin, in "Towards Quantum Gravity", Cambridge University press, 2007

- Curved spacetime:

  Using expressions for the Ricci scalar in terms of volume and proper distance of small causal intervals[‡] the expression for the d'Alembertian in flat space (B) changes by a term:

$$B\phi(x) = (\Box - \frac{1}{2}R)\phi(x)$$

  Applying this to a constant filed $\phi(x) =$ constant gives an expression of the Ricci scalar for causet.

[‡]Gibbons and Solodukhin, Phys. Lett. B652 (2007)

This leads to the causal sets Einstein-Hilbert action for causets in curved spacetime in 2 and 4 dimensions (has been generalised for arbitrary dimensions)[§]:

Benincasa-Dowker Action

$$\frac{1}{\hbar}S(2) = N - 2N_1 + 4N_2 - 2N_3$$

$$\frac{1}{\hbar}S(4) = N - N_1 + 9N_2 - 16N_3 + 8N_4$$

[§]D. Benincasa and F. Dowker, PRL 104, 181301 (2010)

Classical (stochastic) Dynamics

Sequential Growth (growing a causet by giving 'births' element by element)*

(a) General Covariance ('time' labeling is pure 'gauge')

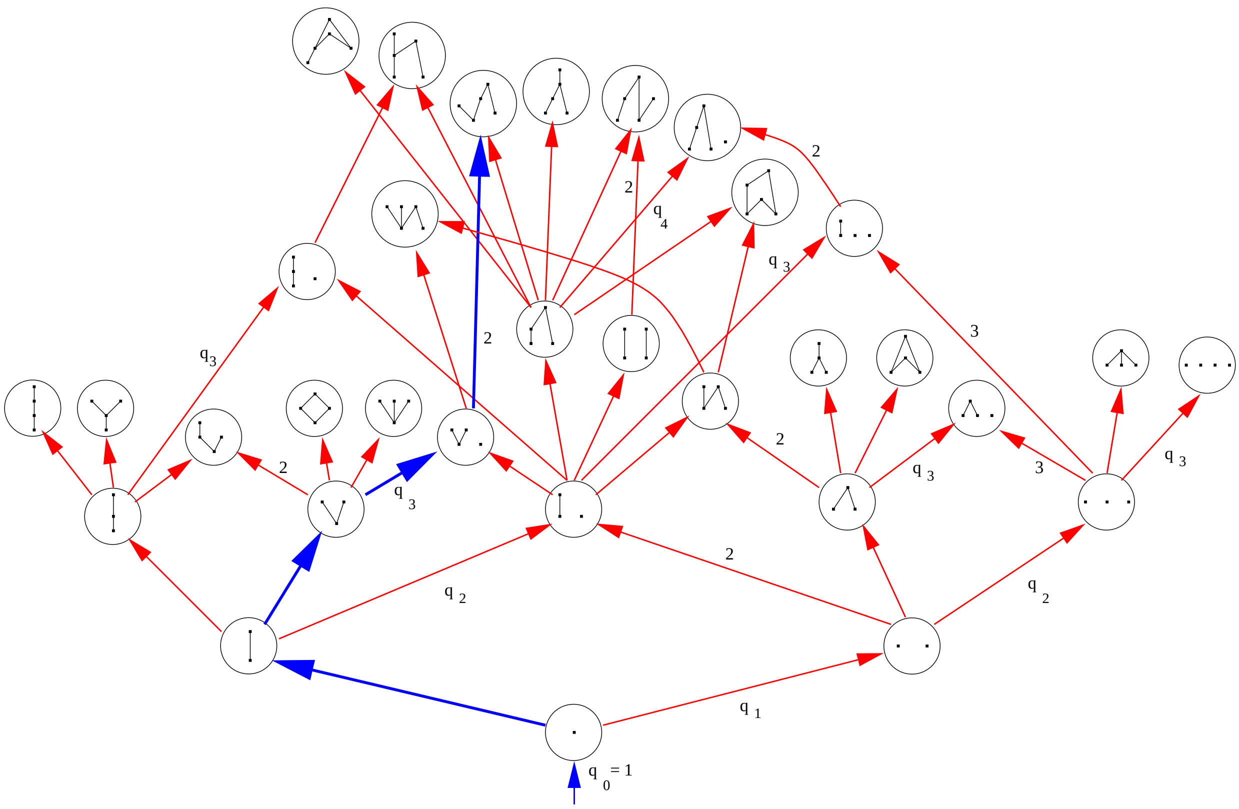
(b) 'Bell's Causality' ('births' in spacelike regions do not affect each other)

Set of solutions parametrized by some constants.

Not in general *manifold* like.

*D. Rideout and R. Sorkin, Phys. Rev. D 61 (2000) 024002.

INTRINSIC to causal set (bottom-up).



The poset of finite causal sets

Quantum Dynamics & Interpretation

- Purpose: To assign a quantum amplitude to each of the possible causal sets

- CANNOT have canonical formulation, because of the fundamental spacetime nature. Needs a *histories* formulation.

(1) Either extend a "quantum" measure, by a growth process like the Rideout-Sorkin model

(a) Have a quantum rather than classical measure (issues extending the measure on the full histories space)

(b) Use a weakened "Bell's locality" condition to incorporate relevant quantum violations

(2) Or assign a weight on causets e.g. by mimicking the Einstein-Hilbert Action for causets.

In both cases, one should be able to interpret the quantum measure (see below).

In (1) progress for defining the quantum measure and extending it to all measurable subsets has been made. No suitable generalisation of the Bell's locality condition is present.

In (2) we have the following directions:

-Consider equal weights, BUT restrict sum over some particular sub-class of causal sets. For 2-D partial orders (restriction of sum), the dominant contribution (at large volume limits) comes from causal sets that correspond to 2-D Minkowski spacetime*.

*G. Brightwll. J. Henson and S. Surya, Class. Quant. Grav. 25 (2008) 105025.

- Attempt to mimic the Einstein-Hilbert action (to adjust weights). Write down an analogue of the Lagrangian density for causal set, using only the causal order. Main recent development is the Benincasa-Dowker action and generalisations.

In all this we are able to assign an amplitude to each causal set (histories) of the system. This defines a *quantum measure* on the space of all causal sets. It is not a proper measure, since it fails to obey the additivity condition due to interference.

How to Interpret the Quantum Measure?

(1) Find re-labelling invariant questions (diffeomorphism's invariant)

(2) Have a way to understand the quantum measure without resorting to (a) external observers and (b) repetitions of experiments

- Novel interpretation of QT, the "Co-event Interpretation", based on consistent histories. Interprets the quantum measure, is realistic (no-external observer). Pioneered by R. Sorkin and collaborators[†].

[†]R.D. Sorkin, J. Phys. Conf. Ser. 67 (2007) 012018; Y. Ghazi-Tabatabai and P. Wallden, J. Phys. A: Math. Theor. 42 (2009) 234303; S. Surya and P. Wallden, Found. Phys. 40 (2010) 585; K. Clements, F. Dowker and P. Wallden arXiv:1201.6266

Summary and Conclusions

- We introduced causal sets and how they are expected to replace spacetime
- We briefly explored the kinematics and phenomenology of the theory.
- The behaviour of quantum matter and fields on a classical causal set was examined.
- The classical stochastic dynamics of causets were explored as a growth stochastic process.
- Finally, directions to full quantum dynamics of causets and its interpretation was explored. A top-down approach is more developed, and the co-event interpretation of the quantum measure is considered more suitable.