BLACK HOLE UNIQUENESS THEOREMS IN HIGHER DIMENSIONS

Stoytcho Yazadjiev

Department of Theoretical Physics
Sofia University

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The story in 4 dimensions

- Under some natural physical conditions the horizon topology in 4D is S^2 (**Hawking 1973**)
- Black holes in 4 dimensions are fully specified by their conserved asymptotic charges. – **[Israel (1967), Muller zum Hagen (1974), Carter (1971), Robinson (1975), Mazur (1982); Bunting (1983)]**,
- **Theorem** [**Mazur (1982); Bunting (1983)**] The Kerr-Newman solution with parameters M, $a = J/M$ and Q is the unique black hole solution with regular event horizon ($M^2 > a^2 + Q^2$) and stationary, axisymetric and asymptotically flat domain of outer communications.
- **This uniqueness (classification) theorem is a key and central result in the theory of the 4D black holes !**

Surprises in 5D: Black rings and ... non-uniqueness

- 5D analogues of the Kerr solution - **Myers-Perry** black holes - asymptotically flat solutions with S^3 horizon topology
- Black Rings [**Emparan and Reall (2001)**]–asymptotically flat black 5D solutions with $S^2 \times S^1$ horizon topology
- In 5 dimensions, the horizon topology can be different from S^3 !
- In 5 dimensions, the black objects are not uniquely specified by their conserved asymptotic charges !

Non-uniqueness diagram

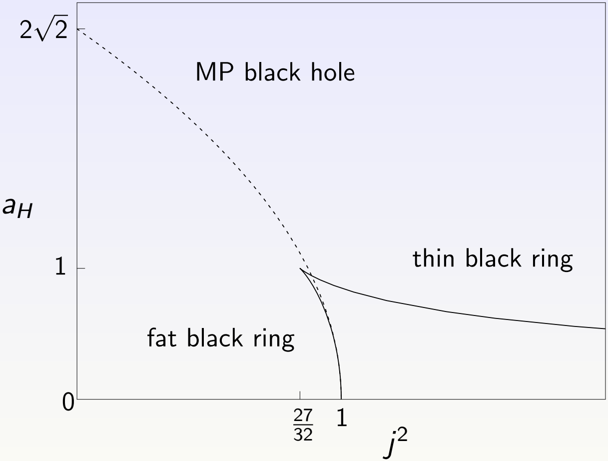

Figure: Horizon area a_H vs j^2, for given mass.

Other interesting solutions in 5D

- Black Saturn – a solution of the 5D vacuum Einstein equations describing a black hole surrounded by a rotating black ring (**Elvang and Figueras, 2007**)

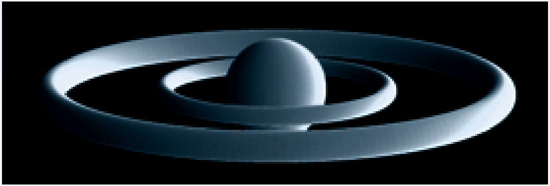

Figure: Black Saturn and concentric black rings

- Concentric rotating black rings (**Iguchi and Mishima, 2008**)
- Orthogonal rotating black rings (**Izumi, 2009)**

Black hole uniqueness problem in 5D

- The question is whether some kind of uniqueness theorem could be formulated in higher dimensions. The most difficult question, however, is what kind of suitable further parameters associated with the black solutions should be specified in addition to the conserved asymptotic charges.
- We will show that the black solutions in 5D are uniquely determined in terms of their mass, angular momenta, and a datum called "**INTERVAL STRUCTURE**" . The interval structure encodes information about the relative position of various axis and the horizon, and gives a measure of their lengths. Another very important fact is that the interval structure determines the topology of the horizon. Actually, the topology of the horizon can be either S^3, $S^2 \times S^1$, or a Lens-space $L(p, q)$.

Interval structure

- Isometry group – $\mathcal{G} = \mathcal{R} \times U(1)^2$

Theorem [S. Hollands and S.Y. (2007)] Let (M_{ext}, g_{ab}) be the exterior of a 5-dimensional stationary, asymptotically flat, vacuum black hole spacetime with 2 mutually commuting independent axial Killing fields ψ_1^a and ψ_2^2. Then the orbit space $\hat{M} = M_{ext}/\mathcal{G}$ by the isometry group $\mathcal{G} = \mathcal{R} \times U(1)^2$ is a simply connected, 2-dimensional manifold with boundaries and corners. Points in the interior of $\hat{M}$ correspond to point in M where all Killing fields t^a, ψ_1^a, ψ_2^a are linearly independent. Points on the i-th 1-dimensional boundary segment of $\partial\hat{M}$ correspond to either the horizon H, or points where a linear combination $\nu_i^1\psi_1^a + \nu_i^2\psi_2^a = 0$, where $v_i = (\nu_i^1, \nu_i^2)$ is a vector of integers. Points in the corners of $\partial\hat{M}$ correspond to points in M_{ext} where $\psi_1^a = \psi_2^a = 0$. The boundary $\partial\hat{M}$ is connected.

Interval structure

In the realization of $\hat{M}$ as the upper complex half plane, the line segments of $\partial\hat{M}$ correspond to intervals

$$(-\infty, z_1), (z_1, z_2), \ldots, (z_k, z_{k+1}), (z_{k+1}, \infty) \tag{1}$$

of the real axis forming the boundary of the upper half plane. Evidently, if the horizon is connected as we assume, precisely one interval (z_h, z_{h+1}) corresponds to the horizon. The other intervals correspond to rotation-axis, while the points z_j correspond to the intersection points of the axis, except for the boundary points of the interval (z_h, z_{h+1}) representing the horizon. The k positive real numbers

$$l_1 = z_1 - z_2, \quad l_2 = z_2 - z_3, \quad \ldots \quad l_k = z_k - z_{k+1} \tag{2}$$

are invariantly defined, i.e., are the same for any pair of isometric stationary black hole spacetimes of the type we consider. Thus, they may be viewed as global parameters ("moduli") characterizing the given solution in addition to the mass m and the two angular momenta J_1, J_2.

Interval structure

Furthermore, with each I_j, there is associated a label in $\{v_j \in \mathbb{Z}^2, H\}$ according to whether we are on the horizon, or which rational linear combination $v_j^1 \psi_1^a + v_j^2 \psi_2^a$ vanishes. The labels corresponding to the "outmost" intervals $(-\infty, z_1)$ and (z_k, ∞) must be $(0,1)$ respectively $(1,0)$, because this is the case for Minkowski spacetime, and we assume that our solutions are asymptotically flat.

For 4 dimensional black holes, there is only the trivial interval structure $(-\infty, z_1), (z_1, z_2), (z_2, \infty)$, with the middle interval corresponding to the horizon, and the first and third corresponding to single axis of rotation of the Killing field. Furthermore, the interval length l_1 may be expressed in terms of the global parameters m, J of the solution. By contrast, in 5 dimensions, the interval structure can be non-trivial, and in fact differs for the Myers-Perry and Black Ring solutions. For these cases, the interval structure is summarized in the following table:

Interval structure

	Intervals	Int. Vectors	Topology
Myers-Perry	$(-\infty, z_1)(z_1, z_2)(z_2, \infty)$	$(1,0)H(0,1)$	S^3
Black Ring	$(-\infty, z_1)(z_1, z_2)(z_2, z_3)(z_3, \infty)$	$(1,0)H(1,0)(0,1)$	$S^2 \times S^1$
Flat Spacetime	$(-\infty, z_1)(z_1, \infty)$	$(1,0)(0,1)$	—

Classification of the horizon topologies in 5D

Theorem [**S. Hollands and S.Y. (2007)**] In a black hole spacetime of dimension 5 with 2 commuting, independent axial Killing fields, the horizon cross section H must be topologically either a ring $S^1 \times S^2$, a sphere S^3, or a Lens-space $L(p,q)$, with $p, q \in Z$ where $p = \det(v_{h-1}, v_{h+1})$. For $p = 0$ the topology is $H = S^2 \times S^1$, $H = S^3$ for $p = \pm 1$ and $H = L(p,q)$ for other values of p.

Remark The Lens-spaces $L(p,q)$ are the spaces obtained by glueing the boundaries of two solid tori together in such a way that the meridian of the first goes to a curve on the second which wraps around the longitude p-times and which wraps around the meridian q-times. A Lens-space may also be obtained by factoring the unit sphere S^3 in $\mathcal{C}^2$ by the group action $(z_1, z_2) \to (e^{2\pi i/p} z_1, e^{2\pi i q/p} z_2)$ $(p \neq 0, q \neq 0)$. If $q = 0$ modulo pZ, then $p = \pm 1$ and $L(\pm 1, 0\, modulo\, pZ) = S^3$. If $p = 0$ modulo qZ, then $q = \pm 1$ and $L(0,\ modulo\, qZ, \pm 1) = S^2 \times S^1$.

Black lenses

- Possible existence of new type black objects–Black Lenses

	Intervals	Int. Vectors	Topology
Black Lens	$(-\infty, z_1)(z_1, z_2)(z_2, z_3)(z_3, \infty)$	$(1,0)H(1,n)(0,1)$	$L(n,1)$

- So far known exact black lens solutions are not completely regular **[Evslin (2008), Chen and Teo (2008)]**

Uniqueness theorem

Theorem [**S. Hollands and S.Y. (2007)**] Consider two stationary, asymptotically flat, vacuum non-degenerate black hole spacetimes of dimension 5, having two commuting axial Killing fields that commute also with the time-translation Killing field. Assume that both solutions have the same interval structure, and the same values of angular momenta J_1, J_2. Then they are isometric.

Extensions of the Uniqueness theorem

- Extensions to nonvacuum gravity - Einstein-Maxwell, Einstein-Maxwell-dilaton, 5D minimal supergravity
- Extensions to spacetimes with Kaluza-Klein asymptotic

THANK YOU FOR THE ATTENTION!