Hawking radiation from chirality and the principle of effective gravity

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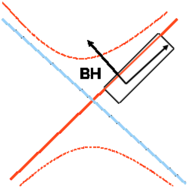
Purpose

- discuss the method of gravitational anomalies in Schwarzschild spacetimes
- discuss problems of the method in general spacetimes with horizons
- modify the method to resolve these problems
- calculate radiation from the horizon in de Sitter and Rindler spacetimes

Theory near the horizon

1. Near the horizon, a d-dimensional quantum field can be seen as a 2-dimensional massless free field



Outgoing modes $\equiv$ Right-moving fields
Incoming modes $\equiv$ Left-moving fields

2. Exclusion of the incoming modes from the horizon leads to an effective 2-dimensional theory near the horizon

S. P. Robinson and F. Wilczek. Phys. Rev. Lett., 95:011303, 2005

S. Iso, H. Umetsu, and F. Wilczek. Phys. Rev. Lett., 96:151302, 2006.

Conservation Equations

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 \tag{1}$$

- Far from the horizon

$$\nabla_\mu T^\mu_{\nu(o)} = 0 \tag{2}$$
$$\partial_r T^r_{t(o)} = 0 \tag{3}$$
$$T^r_{t(o)}(r_0) - T^r_{t(o)}(r \to \infty) = 0 \tag{4}$$

- Near the horizon

$$\nabla_\mu T^\mu_{\nu(H)} = \mathcal{A}_\nu = \frac{1}{96\pi\sqrt{-g}} \epsilon_{\nu\mu} \partial^\mu R(r) \tag{5}$$
$$\partial_r T^r_{t(H)} = \partial_r N^r_t = \partial_r \frac{1}{192\pi}(2ff'' - f'^2) \tag{6}$$
$$T^r_{t(H)}(r) - N^r_t(r) = T^r_{t(H)}(r_H) - N^r_t(r_H) \tag{7}$$

Boundary conditions and Hawking flux

- $$T^r_{t(o)}(r \to \infty) = a_0 \tag{8}$$
- $$T^r_{t(H)}(r_H) = 0 \tag{9}$$
- $$N^r_t(r = r_0 \to \infty) = 0 \tag{10}$$

Thus the flux from the horizon is

$$\Phi = a_0 = \lim_{r\to\infty} [T^r_t(r) - N^r_t(r)] = \frac{f'^2(r_H)}{192\pi} = \frac{\pi}{12} T_H^2$$

Generalization

$$ds^2 = -f(r)dt^2 + h^{-1}(r)dr^2 + K(r)d\sigma^2$$

- $f(r_H) = h(r_H) = 0$
- Surface gravity $\kappa = \frac{1}{2}\sqrt{f'(r_H)h'(r_H)}$
- Dimensional reduction is applicable
- $\sqrt{-det(g_{\mu\nu})(r_H)} = \sqrt{\frac{f(r_H)}{h(r_H)}} \neq 0$ or ∞
- $K(r) \neq 0$ or ∞

Examples

- Spherical spacetimes
  - Charged black hole

    S. Iso, H. Umetsu, and F. Wilczek. Phys. Rev. Lett., 96:151302, 2006
  - Rotating black hole

    S. Iso, H. Umetsu, and F. Wilczek. Phys. Rev., D74:044017, 2006
- Non-spherical topologies
  1. Topological black holes

     E. Papantonopoulos and P. Skamagoulis. Phys. Rev., D79:084022, 2009
  2. (2+1)-black holes *R. Li, S. Li, and J.-R. Ren. Class. Quant. Grav., 27:155011, 2010*

Problems of the method

- Separated region outside the black hole in two parts, near and far from the horizon which leads to two different equations for the E-M tensor $\rightarrow$ Can we unify them?
- Unable to calculate radiation fron spacetimes with R =const since $\partial_r T^r_t = 0$, i.e. no existence of chiral anomalies

Resolution

Instead of considering the gravitational anomalies as fundamental, we consider the following

- Chirality near the horizon region taken as a symmetry of the effective manifold (it precedes the anomalies equations)
- Principle of effective theory of gravity: Physical theories in a given coordinate system are formulated entirely in terms of the variables that an observer using that coordinate system can access

The above two principles allows us to

- Define a new effective tensor for this part of the manifold

$$\tilde{T}^{\mu}_{\nu} = T^{\mu}_{\nu} - N^{\mu}_{\nu} \tag{11}$$

  Then, the equations of non-conservation of E-M tensor for the whole manifold can actually be written as conservation equations for the part of the manifold we are concerned for, i.e. the part that the observer has access and there is no need to split the manifold in two regions
- Justify the application of the method to spacetimes with $R = \text{const}$ since we do not have anomalies but chirality

The conservation equation for the effective E-M tensor is

$$\nabla_\mu \tilde{T}^\mu_\nu = 0 \tag{12}$$

$$\tilde{T}^r_t(r_0) - \tilde{T}^r_t(r_H) = N^r_t(r_0) - N^r_t(r_H) \tag{13}$$

Again we impose

$$\tilde{T}^r_t(r_H) = 0 \tag{14}$$

$$\tilde{T}^r_t(r_0) = a_0 = \Phi \tag{15}$$

Thus the flux is

$$\Phi = N^r_t(r_0) - N^r_t(r_H) \tag{16}$$

Constant curvature spacetimes

1. de Sitter spacetime

$$ds^2 = -(1 - \frac{r^2}{a^2})dt^2 + (1 - \frac{r^2}{a^2})^{-1}dr^2 \tag{17}$$

- $r_H = a$
- $N_t^r(r \to 0) = \frac{1}{96\pi}(-\frac{2}{a^2})$
- $\Phi = T_t^r(r_H) + N_t^r(r_H) = N_t^r(r_H) = -\frac{f'^2(r_H)}{192\pi} = -\frac{\pi}{12}T_{HG}^2, T_{HG} = \frac{1}{2\pi a}$

2. Rindler spacetime (static coordinates)

$$ds^2 = -(1 + 2ar)dt^2 + (1 + 2ar)^{-1}dr^2 \tag{18}$$

- $r_H = \frac{1}{2a}$
- $N_t^r(r = 0) = -\frac{a^2}{48\pi}$
- $\Phi = T_t^r(r_H) - N_t^r(r_H) = \frac{f'^2(r_H)}{192\pi} = \frac{\pi}{12}T_H^2, T_H = \frac{a}{2\pi}$

Conclusions

- Started with the gravitational anomalies method and stressed its problems
- Argued that a natural modification was needed to include some important cases
- Also no need to split the external to the horizon region
- Generalise to time-dependent spacetimes?
- Extend to calculate entropy?