A new kind of Supersymmetry

Jorge Zanelli
CECs-Valdivia

NEB 15 - Recent Developments in Gravity
21 June 2012, Chania, Greece

Pedro Álvarez (CECs) and Mauricio Valenzuela (Mons)
arXiv:1109.3944 [hep-th]; JHEP04(2012)058

1. Why supersymmetry?

Supersymmetry has many blessings:

- Only nontrivial way to combine Poincaré and internal symmetries
- Natural unification of fermions and bosons as parts of the same reality
- Fewer arbitrary constants
- $H = \{Q^\dagger, Q\}$ → Positivity of energy, stability of ground states (BPS states)

Practical advantages:

- Cancellation of infinities; Renormalizability; Hierarchy; Convergence of coupling constants...
- Spacetime + internal symmetry → natural extension of gravity: Supergravity
- Superstrings
- Dark matter candidate
- SUSY Standard model(s)
- Every known particle has a superpartner

Good testing signal

Unfortunately, it has never been observed...

Selected for a Viewpoint in *Physics*

PRL **107,** 221804 (2011) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2011

Search for Supersymmetry at the LHC in Events with Jets and Missing Transverse Energy

S. Chatrchyan *et al.**

(CMS Collaboration)

(Received 12 September 2011; published 21 November 2011)

A search for events with jets and missing transverse energy is performed in a data sample of pp collisions collected at $\sqrt{s} = 7$ TeV by the CMS experiment at the LHC. The analyzed data sample corresponds to an integrated luminosity of 1.14 fb^{-1}. In this search, a kinematic variable α_T is used as the main discriminator between events with genuine and misreconstructed missing transverse energy. No excess of events over the standard model expectation is found. Exclusion limits in the parameter space of the constrained minimal supersymmetric extension of the standard model are set. In this model, squark masses below 1.1 TeV are excluded at 95% C.L. Gluino masses below 1.1 TeV are also ruled out at 95% C.L. for values of the universal scalar mass parameter below 500 GeV.

Is Supersymmetry Dead?

The grand scheme, a stepping-stone to string theory, is still high on physicists' wish lists. But if no solid evidence surfaces soon, it could begin to have a serious PR problem

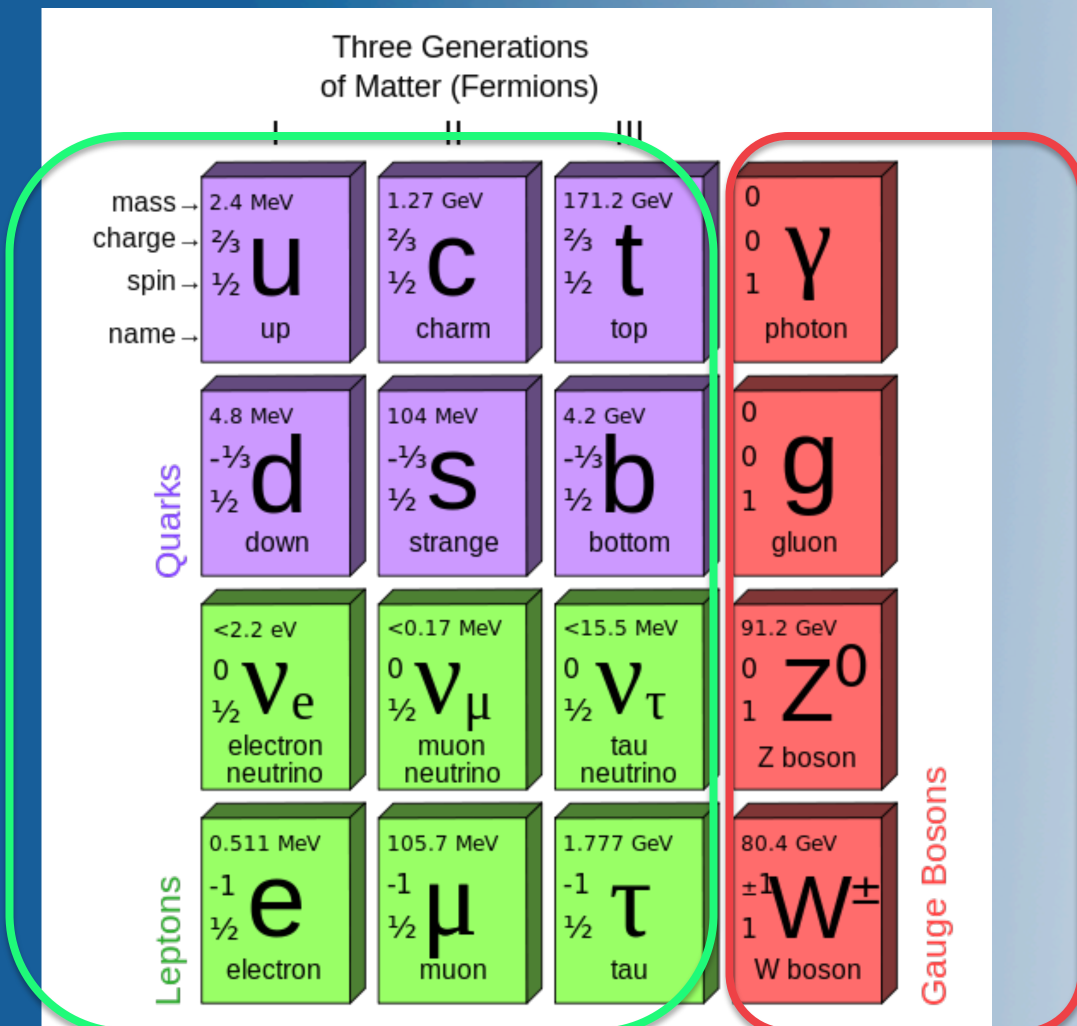
By Davide Castelvecchi | April 25, 2012 | 29

See Inside

Is SUSY a *natural* idea?

F

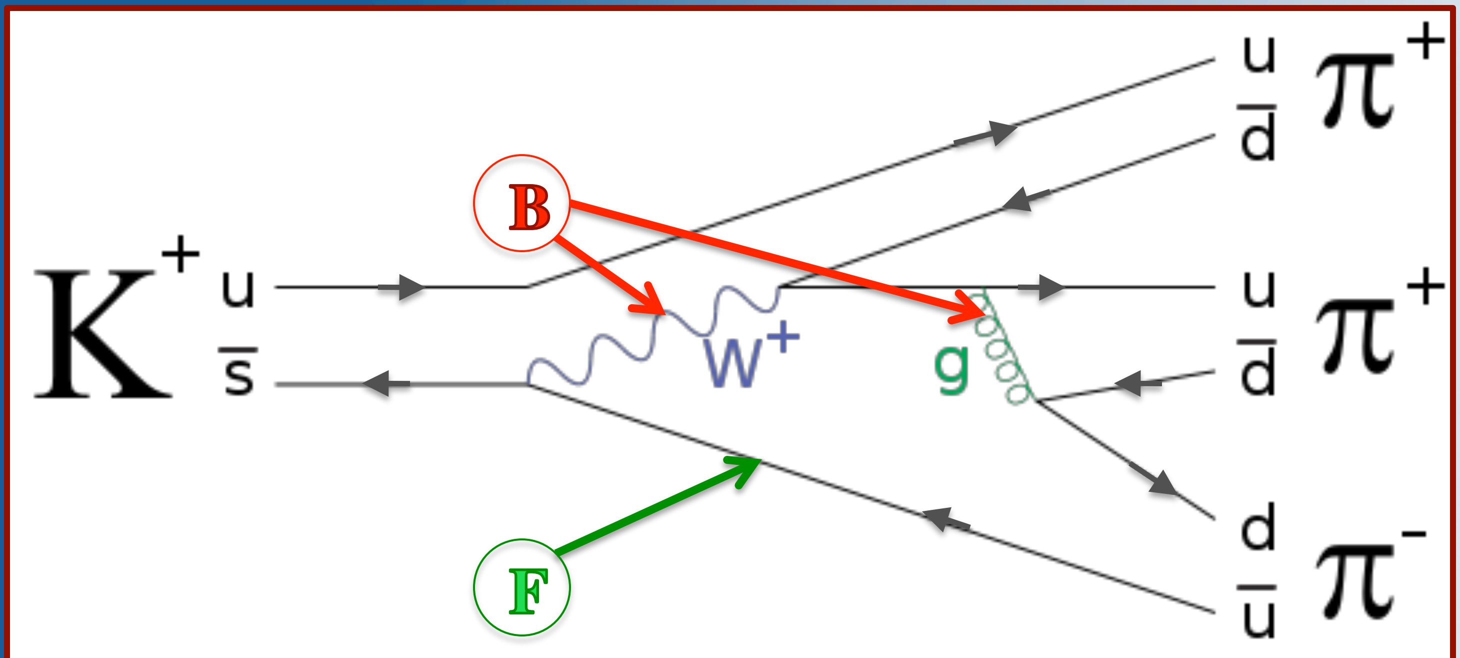
Building blocks of matter



B

Interaction carriers

Bosons and fermions play very different roles in particle physics (Standard Model):



Fermions: Building blocks of matter

Bosons: Interaction carriers

Fermions: $e^{\pm}$, $\mu^{\pm}$, ν, $\bar{\nu}$, q, $\bar{q}$, ... [Matter]

- Carry conserved charges (electric, lepton, baryon)
- Conserved currents: sources of interactions
- Spin ½ (ir-rep. of Lorentz group)
- Zero-form sections in the fiber bundle
- Vectors under gauge transformations
- Scalars under general coordinate transformations
- First order field equations

$$\psi_\alpha^a$$

Under gauge transformations:

$$\psi'_\alpha(x) = g^{-1}(x)\psi_\alpha(x)$$

Bosons: γ, Z, $W^{\pm}$, gluon, graviton [Interactions]

- Manifestations of local (gauge) symmetry
- Not necessarily conserved
- Spin 1 (ir-rep. of Lorentz group)
- Connections under gauge transformations
- Lie-algebra valued 1-forms in the fiber bundle
- Covariant vectors under GCT
- Second order field equations

$$A^{a}{}_{b\,\mu}$$

Under gauge transformations:

$$A'(x) = g^{-1}(x)[A(x) + d]g(x), \quad A = A_\mu dx^\mu$$

$$\psi'_\alpha(x) = g^{-1}(x)\psi_\alpha(x)$$

$$A'_\mu(x) = g^{-1}(x)(A_\mu(x) + \partial_\mu)g(x)$$

Can such different objects be part of a single entity?

Standard (rigid/global) SUSY

Under supersymmetry, fields transform as vectors:

$$\begin{bmatrix} B' \\ F' \end{bmatrix} = Q \begin{bmatrix} B \\ F \end{bmatrix} = \begin{bmatrix} s_{BB} & s_{BF} \\ s_{FB} & s_{FF} \end{bmatrix} \begin{bmatrix} B \\ F \end{bmatrix}$$

Since the supersymmetric charge is conserved,

$$[H, Q] = 0,$$

the masses come in degenerate pairs:

$$m_B = m_F$$

(a conspicuously absent signal so far).

Supersymmetry trick: For each field include another of the opposite statistics: “MSSSM”

quark → *squark*		photon → *photino*,	
electron → *selectron*		gluon → *gluino*,	
neutrino → *sneutrino*		graviton → *gravitino*,...	

Should this duplication be called a unification?
Is this all really necessary?

“If the Lord had consulted my opinion, I would have suggested something considerably simpler...”

Alfonso X *the Wise*, commenting on Ptolemaeus’ cosmology. (ca.1270)

“There are two ways of doing calculations in theoretical physics. One way, is to have a clear physical picture of the process that you are calculating. The other way is to have a precise and self-consistent mathematical formalism. You have neither.”

Fermi on Dyson’s model of strong interactions (1953)

Can one do better?

- Combine fermions and bosons respecting their different mathematical roles in a gauge theory:

  B = connections   **F** = sections

- Allow for different masses in the same multiplet

- Implement SUSY as a true gauge symmetry (local)

Let's try.

2. Idea: Combine fermions and bosons in a different way

Curious coincidence

$$\bar{\psi}\partial\!\!\!/\psi \Longleftrightarrow AdA$$

Dirac 3*D* Chern-Simons

There must be a way to combine a connection A_μ and a Dirac spinor ψ^α as components of a single connection $\mathcal{A}$, respecting their transformation rules in the gauge group.

Ansatz:

$$\mathcal{A} \sim \mathrm{AK} + \bar{Q}\,\psi + \bar{\psi}Q + \cdots,$$

U(1) generator (AK)

Susy generators ($\bar{Q}$, Q)

SUSY transformations should mix A and ψ. Let's look for the simplest algebra containing a *U(1)* generator (K) and two spinor charges (Susy generators $\bar{Q}^{\alpha}$, Q_{α}).

The smallest semi-simple algebra of this type requires also 3 rotation generators J^a, in order to close the algebra. The resulting super algebra is:

$$K = i\left[\begin{array}{c|c} I & 0 \\ \hline 0 & 2 \end{array}\right], \quad Q^1 = \left[\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right], \quad Q^2 = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \hline 0 & 0 & 0 \end{array}\right]$$

$$J^a = \frac{1}{2}\left[\begin{array}{c|c} \gamma^a & 0 \\ \hline 0 & 0 \end{array}\right], \quad \bar{Q}_1 = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 1 & 0 & 0 \end{array}\right], \quad \bar{Q}_2 = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 1 & 0 \end{array}\right]$$

The (anti-) commutator algebra is

$$\left[J_a, Q^\alpha\right] = -\tfrac{1}{2}(\gamma_a)^\alpha{}_\beta Q^\beta, \quad \left[J_a, \overline{Q}^\alpha\right] = \tfrac{1}{2}\overline{Q}_\beta(\gamma_a)^\beta{}_\alpha$$

$$\left[J_a, J_b\right] = -\varepsilon_{ab}{}^c J_c, \qquad \left[\mathrm{K}, J_a\right] = 0$$

$$\left\{Q^\alpha, \overline{Q}_\beta\right\} = J_a(\gamma_a)^\alpha{}_\beta - i\tfrac{1}{2}\delta^\alpha_\beta \mathrm{K},$$

$$\left\{Q^\alpha, Q^\beta\right\} = \left\{\overline{Q}_\alpha, \overline{Q}_\beta\right\} = 0,$$

$$\left[\mathrm{K}, Q^\alpha\right] = iQ^\alpha, \quad \left[\mathrm{K}, \overline{Q}_\beta\right] = -i\overline{Q}_\beta$$

$osp(2|2)$

With this graded algebra, one can define a gauge connection:

$$\mathcal{A} = A\mathrm{K} + \overline{Q}\Gamma\psi + \overline{\psi}\Gamma Q + \tfrac{1}{2}\omega^a J_a,$$

where $A = A_\mu dx^\mu, \ \omega^a_\mu dx^\mu, \ \Gamma = \gamma_a e^a_\mu dx^\mu.$

Vielbein: projects Clifford algebra onto the background manifold

3. Action

Given a connection $\mathcal{A}$ for some gauge group, the Chern Simons form defines a gauge invariant action,

$$L = \tfrac{1}{2}\left\langle \mathcal{A} d\mathcal{A} + \tfrac{2}{3}\mathcal{A}\mathcal{A}\mathcal{A} \right\rangle$$

where the bracket is the super trace in the algebra.

$$L = AdA + \tfrac{1}{2}[\omega^a{}_b d\omega^b{}_a + \tfrac{2}{3}\omega^a{}_b \omega^b{}_c \omega^c{}_a]$$
$$- 2\bar{\psi}(\overleftrightarrow{\partial\!\!\!/} + iA\!\!\!/ - \tfrac{1}{4}\gamma_a \omega\!\!\!/^{ab}\gamma_b)\psi\, |e|\, d^3x$$
$$- \bar{\psi}\psi\, e^a T_a$$

new term

$$= L^{CS}_{U(1)} + L^{CS}_{Gravity} + L_{Dirac} + [\mathit{torsional\ coupling}]$$

The field equations are:

$$\delta A \; : \qquad F = \tfrac{i}{2}\varepsilon_{abc}\bar{\psi}\gamma^{a}\psi e^{b}e^{c} \qquad (1)$$

$$(F_{\mu\nu} = \varepsilon_{\mu\nu\lambda} j^{\lambda})$$

$$\delta\omega^{ab} : \qquad R^{ab} = -2\bar{\psi}\psi e^{a}e^{b} \qquad (2)$$

$$\delta\bar{\psi} : \qquad [\not{\partial} + i\not{A} - \tfrac{1}{4}\gamma^{a}\not{\omega}_{ab}\gamma^{b} + \mu]\psi = 0 \qquad (3)$$

$$\delta e^{a} : \qquad \bar{\psi}\varepsilon_{abc}\gamma^{c}[\overleftarrow{d}e^{b} - e^{b}\overrightarrow{d} + 2iAe^{b}]\psi = 2\bar{\psi}\psi T_{a} \qquad (4)$$

"Dark energy"

Mass

- Standard equations for CS electrodynamics, gravity & spin ½ in 2+1 dimensions.

- ψ gets "mass" from torsion: $\mu(x) \sim e^{a}T_{a}$

- Covariantly const. torsion: $R^{ab} \sim e^{a}e^{b} \;\Rightarrow\; DT^{a} = 0$

4. What does this describe?

1. Vacuum (matter-free) solutions: $\psi = 0$

$$(1) \qquad F = \tfrac{i}{2}\varepsilon_{abc}\bar{\psi}\gamma^a\psi e^a e^b = 0$$

$$(2) \qquad R^{ab} = -2\bar{\psi}\psi e^a e^b = 0$$

$$(3) \qquad [\partial\!\!\!/ + iA\!\!\!/ - \tfrac{1}{4}\gamma^a \omega\!\!\!/_{ab}\gamma^b + \mu]\psi = 0 \quad ✔$$

$$(4) \qquad \bar{\psi}\varepsilon^a{}_{bc}\gamma^c[\overleftarrow{d}e^b - e^b\overrightarrow{d} + 2iAe^b]\psi = 2|e|\bar{\psi}\psi T^a \quad ✔$$

$\psi = 0 \Rightarrow$

- No electromagnetic field ?
- Flat spacetime

... not quite :

(1) $F = 0 \Rightarrow A = d\Omega$ (Not necessarily trivial)

(2) $R^{ab} = -2\bar{\psi}\psi e^a e^b \Rightarrow DT^a = 0$

$$\Rightarrow T^a = \mu \varepsilon^{abc} e_b e_c ,$$

$$e_a T^a = 6\mu |e| d^3x, \quad \mu = const.$$ Fermion mass

If $T^a \neq 0$, R^{ab} *is not* the Riemann tensor

$\therefore R^{ab} = 0 \not\Rightarrow$ flat spacetime:

$$\omega^{ab} = \bar{\omega}^{ab} + \kappa^{ab}$$

$$R^{ab} = \bar{R}^{ab} + \bar{D}\kappa^{ab} + \kappa^a{}_c \kappa^{cb}$$

Contorsion

Torsion-free part

Torsion-free (Riemann tensor)

Here $DT^a = 0 \Rightarrow T^a = \mu \varepsilon^{abc} e_b e_c \ (\neq 0)$

$$\Rightarrow \kappa^{ab} = -\mu \varepsilon^{abc} e_c$$

$$\Rightarrow \overline{R}^{ab} = -\mu^2 e^a e^b \ (\neq 0)$$

- Spacetime is AdS_3, constant T and massive fermion

$$\Lambda = -\mu^2 = -l^{-2}$$ (Integration const.)

- This spacetime has nontrivial solutions (black holes of mass M, ang. momentum J, magnetic charge q.)
- BPS states:

  4 Killing spinors if M =-1, $J = q$ =0 (AdS)

  2 Killing spinors if $M = J = q$ =0 (0 mass BH)

  1 Killing spinors if $M = J$ =N $q \neq 0$ (extremal BH)

2. Perturbative matter (small $\psi \neq 0$)

To first order, the equations for A, ω^{ab} and e^a do not change.

3. Flat background ($\omega^{ab} = 0 = A$)

The dreibein is determined by the fermionic field:

$$[E_a^\mu \gamma^a \partial_\mu + \mu]\psi = 0$$

$$2\overline{\psi}\psi\, de^a = \overline{\psi}\varepsilon^a{}_{bc}\gamma^c[\overleftarrow{d}e^b - e^b\overrightarrow{d}]\psi$$

5. Where is the gravitino?

Under an infinitesimal $OSp(2|2)$ transformation,

$$g \approx 1 + \Lambda,$$

where $\Lambda = \alpha K, + \bar{\varepsilon} Q + \overline{Q}\, \varepsilon + \lambda^a J_a$, the connection changes by $\delta_\Lambda \mathcal{A} = d\Lambda + [\mathcal{A}, \Lambda]$.

The resulting action is invariant under local

$OSp(2\|2)$	$U(1)$	A	electromagnetism
	$SO(2,1)$	ω^{ab}	gravity
	SUSY	ψ	"electron"

Under $U(1)$ and $SO(2,1)$ all fields behave as they should, and under supersymmetry...

$$SUSY: \Lambda = \bar{\varepsilon}(x)Q - \bar{Q}\,\varepsilon(x)$$

$$\delta A = -\tfrac{i}{2}(\bar{\varepsilon}\gamma^a\psi + \bar{\psi}\gamma^a\varepsilon)e_a$$

$$\delta\psi = \tfrac{1}{3}\gamma^\mu(\partial_\mu - \mathrm{i}A_\mu + \tfrac{1}{2}\omega^a{}_\mu\gamma_a)\varepsilon$$

$$\delta\omega^a = (\bar{\psi}\varepsilon + \bar{\varepsilon}\psi)e^a - \varepsilon^a{}_{bc}e^b(\bar{\psi}\gamma^c\varepsilon - \bar{\varepsilon}\gamma^c\psi)$$

$$\delta e^a = 0$$

← *Metric is SUSY-invariant*

* The metric is a composite ($g_{\mu\nu} = \eta_{ab}e^a{}_\mu e^b{}_\nu$) –not a fundamental field-, associated with the torsion-free part of the connection:

$$\bar{\omega}^a{}_{b\mu} = e^a{}_\lambda(\partial_\mu E^\lambda{}_b + \Gamma^\lambda_{\mu\nu}E^\nu{}_b)$$

* SUSY-invariant metric ➔ No gravitini

The connection is

~'gravitino'

$$\mathcal{A} = (A_\mu \mathrm{K} + \overline{Q}\gamma_\mu\psi + \overline{\psi}\gamma_\mu Q + \tfrac{1}{2}\omega^a{}_\mu J_a)dx^\mu$$

Under *SUSY*, $\delta(\gamma_\mu\psi) = \nabla_\mu\varepsilon.$

Here $\delta e^a = 0 \Rightarrow \delta\gamma_\mu = \delta(e^a{}_\mu\gamma_a) = 0$ and therefore,

$$\gamma_\mu\delta\psi = \nabla_\mu\varepsilon \Rightarrow \delta\psi = \tfrac{1}{3}\nabla\!\!\!/\,\varepsilon.$$

In this way, the spin 3/2 part of $\delta\psi$ is projected out:

$$(\delta^\mu_\nu - \tfrac{1}{3}\gamma_\nu\gamma^\mu)\nabla_\mu\varepsilon = 0.$$

Is this consistent?

This condition is consistent because the projector

$$P^{\mu}_{\nu} = (\delta^{\mu}_{\nu} - \tfrac{1}{3}\gamma_{\nu}\gamma^{\mu})$$

is susy-invariant.

- In general, $P^{\nu}_{\mu} = \left(\delta^{\nu}_{\mu} - \tfrac{1}{D}\gamma_{\mu}\gamma^{\nu}\right)$ kills the spin 3/2 piece in *D* dimensions: consistent truncation.
- *This trick can also be played in higher D...*

CS recipe works in higher odd dimensions, e.g.,

$$L_5 \sim \left\langle \tfrac{1}{3} \mathcal{A} d\mathcal{A} d\mathcal{A} + \tfrac{1}{2} \mathcal{A}^3 d\mathcal{A} + \tfrac{1}{5} \mathcal{A}^5 \right\rangle$$

etc.

6. Summary

Different paths (representations) lead to SUSY:

i) Standard approach:

★ Fermions and bosons form multiplets of equal mass in a vector representation or global SUSY.

★ Local (gauge) supersymmetry (SUGRA), brings in a S=3/2 gravitino.

★ Realistic model building requires SUSY breaking mechanisms.

ii)Alternative approach:

- ★ SUSY extension of a standard gauge symmetry, with all fields as parts of a connection
- ★ **F** (matter) = sections
  B (interactions) = connections } *in a standard gauge bundle*
- ★ Fermion and bosons are not SUSY partners
- ★ Local SUSY includes gravity, without gravitini
- ★ Bosons remain massless, while Fermions can be massive, (brought to you by Torsion)

Lessons:

- Gravity is brought in by fermions:

  Charged fermions + local SUSY

  ⇒ Gravitation

- No need for fundamental s=3/2 fields

$$\chi_\mu^\alpha = e_\mu^a (\gamma_a)^\alpha_\beta \psi^\beta \quad \sim \text{“gravitini”}$$

- No need for fundamental metric field

$$g_{\mu\nu} = e_\mu^a \eta_{ab} e_\nu^b \quad \sim \text{“graviton”}$$

no fermions? ⇒ *no metric required.*

More lessons:

- No matching necessary or implied between bosons and fermions (no duplications)
- No mass degeneracy
- Fermion mass comes from coupling to torsion
- Fermion mass is related to cosmological constant

$$\Lambda = -\mu^2$$

and they are integration constants, not fundamental parameters in the action.

Next ...

- Applications to condensed matter physics?
- Non-relativistic limit
- Higher dimensions
- Other gauge groups
- etc...

Ευχαριστω!