# Effective Equations of Motion, Decoherence and Backreaction

(In Preperation)

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**Question**: What is the Nature of Coupling Quantum to Classical? Is it Possible to Construct a Consistent Model for Dynamical Coupling Between Quantum and Classical Systems?

## This Issue is Related to

- Quantum Measurement
- Systems Lying in the Domain Between The Fully Classical and Quantum Regimes

- Semiclassical Theories (e.g. Semiclassical Gravity)

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$$

(Traditional approach: Couple to mean values)

## Our Aims:

- Derive from First Principles the Backreaction of Quantum Degrees of Freedom to Classical ones in the Context of Environmentally Induced Decoherence.

- Test Features of Decoherence in Relation to Classicality

# The Model

Particle(Classical)(could be interpreted as a measuring device) couples to a Thermal Bath of Harmonic Oscillators and to Another Particle (Quantum). Quantum Brownian Motion (QBM) Models.

We Expect that coupling to the Environment induces decoherence and hence Classicality (??)

# The Technique

Path Integral Representation for the Density Operator and the Influence Functional Technique

# Detailed Description of the Model

- Whole System is Closed: A Thermal Bath of Harmonic Oscillators couples Linearly in Position with the Classical Particle which in turn Interacts with the Quantum Particle linearly in position. Dynamics governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{P}_x^2}{2M_x} + \frac{1}{2}M_x\omega_x^2\hat{X}^2 + \frac{\hat{p}_y^2}{2m_y} + \frac{1}{2}m_y\omega_y^2\hat{y}^2 + \lambda\hat{X}\hat{y} + \sum_n \left(\frac{\hat{p}_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2\hat{q}_n^2\right) + \hat{X}\sum_n c_n\hat{q}_n \tag{1}$$

- We Trace out the Environmental Degrees of Freedom:

# Some Machinery...

## Full Density Operator Evolution

$$\rho(\widehat{t}) = J(t,\ t_0)\rho(\widehat{t_0})$$

In position representation the Propagator $J$ is:

$$\begin{aligned} J(x, q, x', q', t | x_i, q_i, x'_i, q'_i, 0) = \\ & \mathcal{K}(x, q, t | x_i, q_i, 0)\mathcal{K}^*(x', q', t | x'_i, q'_i, 0) \\ & = \int_{x_i}^{x} Dx \int_{q_i}^{q} Dq \,\text{exp}\left[\frac{i}{\hbar} S[x,\ q]\right] \\ & \times \int_{x'_i}^{x'} Dx' \int_{q'_i}^{q'} \text{exp}\left[-\frac{i}{\hbar} S[x', q']\right] \end{aligned}$$

with operator $\mathcal{K}$ the evolution operator for the wave functions.

Reduced Density Operator: Suppose Full Density Operator takes the form, $\rho(x, q;\ x', q')$, in position representation, then the Reduced Density Operator for the $x$ degrees of freedom (recall marginal distribution in probability theory) is given by,

$$\rho_r(x, x') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(x, q;\ x', q') \delta(q - q')$$

# Model Continued...

Time Evolution for the Reduced Density Operator of Classical+Quantum System:

$$\hat{\rho}_t(X_f y_f ; X'_f y'_f) =$$
$$\int_0^t dX_0 dy_0 dX'_0 dy'_0$$
$$\mathcal{J}(X_f y_f, X'_f y'_f, t | X_0 y_0, X'_0 y'_0, 0) \; \hat{\rho}_0(X_0 y_0, X'_0 y'_0)$$

Assumptions

– The initial density operator is taken to be factored

– The environment is taken to be Ohmic and is assumed to start in a Thermal State.

– We work in the Fokker-Planck limit and neglect dissipation.

$\mathcal{J}(X_f y_f, X'_f y'_f, t|X_0 y_0, X'_0 y'_0, 0)$ May then be evaluated but Most Important can be expressed as:

$$\mathcal{J}(X_f y_f, X'_f y'_f, t|X_0 y_0, X'_0 y'_0, 0) = \int \mathcal{D}\overline{x}\ \mathcal{K}_{\overline{x}}(X_f y_f, t|X_0 y_0, 0)\ \mathcal{K}^*{}_{\overline{x}}(X'_f y'_f, t|X'_0 y'_0, 0)$$

Then

$$\widehat{\rho}_t(X_f y_f; X'_f y'_f) = \int \mathcal{D}\overline{x}\ \Psi_{\overline{x}}(X_f\ y_f, t)\ \Psi^*_{\overline{x}}(X'_f\ y'_f, t)$$

With the unnormalized

$$\Psi_{\overline{x}}(X_f\ y_f,t) = \int dX_0 dy_0\ \mathcal{K}_{\overline{x}}(X_f y_f, t|X_0 y_0, 0)\ \Psi_0(X_0\ y_0, 0)$$

Expression for $\mathcal{K}_{\overline{x}}(X_f y_f, t|X_0 y_0, 0)$:

$$\mathcal{K}_{\overline{x}}(X_f y_f, t|X_0 y_0, 0) =$$

$$\int \mathcal{D}X\mathcal{D}y\ \exp\left(-\frac{\alpha^2}{2}\int dt\ \ (X-\overline{x})^2\right)$$

$$\times \exp\left(\frac{i}{\hbar}\int dt(\frac{1}{2}M_x\dot{X}^2 - \frac{1}{2}M_x\omega_x^2 X^2)\right)$$

$$\times \exp\left(\frac{i}{\hbar}\int dt(\frac{1}{2}m_y\dot{y}^2 - \frac{1}{2}m_y\omega_y^2 y^2 - \lambda X y)\right)$$

Asymptotic Behavior: Evaluation of Path Integral shows that in the Long Time Limit

$$\mathcal{K}_{\overline{x}}(X_f y_f, t | X_0 y_0, 0) \rightarrow$$

$$\exp\Bigg[ + \frac{i}{\hbar} C_{11}(X_f^2 + X_0^2) + \frac{i}{\hbar} C_{22}(y_f^2 + y_0^2) - \frac{i}{\hbar} C_{12}(X_f y_f + X_0 y_0) + \frac{i}{\hbar} C_{31}\, X_f + \frac{i}{\hbar} C_{32}\, y_f + \frac{i}{\hbar} C_{41}\, X_0 + \frac{i}{\hbar} C_{42}\, y_0 - \frac{i}{\hbar} C_5 - \frac{\alpha^2}{2} \int_0^t d\tau \overline{x}^2 \Bigg]$$

where

$C_{ij}, C_5 - -- >$ parameters...

$\alpha^2 \propto KT/\hbar^2 - - >$ constant

Most Important Feature of $\mathcal{K}_{\overline{x}}(X_f y_f, t | X_0 y_0, 0)$:

Initial and Final Degrees of Freedom

are Decoupled!

With Redefinitions:

$$\mathcal{K}_{\overline{x}}(X_f y_f, t | X_0 y_0, 0) =$$

$$N \boxed{\langle X_f y_f | \overline{P}_x \, \overline{Q}_x \, \overline{p}_y \, \overline{q}_y \rangle} \, \exp(-\frac{i}{\hbar} C_{11} \, \overline{Q}^2_{x_f} - \frac{i}{\hbar} C_{22} \, \overline{q}^2_{y_f})$$

$$\times \exp\left(\frac{i}{\hbar}(C_{11} \, X_0^2 + C_{22} \, y_0^2 - C_{12} \, X_0 y_0 + C_{41} \, X_0 + C_{42} \, y_0)\right)$$

$$\times \exp\left(-\frac{i}{\hbar} C_5 - \frac{\alpha^2}{2} \int_0^t d\tau \, \overline{x}^2\right)$$

Where

$$\langle X_f y_f | \overline{P}_x \, \overline{Q}_x \, \overline{p}_y \, \overline{q}_y \rangle =$$

$$\exp\left(\frac{i}{\hbar} C_{11} (X_f - \overline{Q}_x)^2 + \frac{i}{\hbar} C_{22} (y_f - \overline{q}_y)^2 - \frac{i}{\hbar} C_{12} (X_f - \overline{Q}_x)(y_f - \overline{q}_y) + \frac{i}{\hbar} \overline{P}_x X_f + \frac{i}{\hbar} \overline{p}_y y_f\right)$$

$\langle X_f y_f | \overline{P}_x \, \overline{Q}_x \, \overline{p}_y \, \overline{q}_y \rangle$ is a 2-Dimensional Gaussian.

It Satisfies a Generalized Uncertainty Relation

$$
\begin{aligned}
(\Delta(\widehat{X}))^2(\Delta(\widehat{P}_x))^2 - R_x^2 &= (\Delta(\widehat{y}))^2(\Delta(\widehat{p_y}))^2 - R_y^2 \\
&= \frac{\hbar^2}{4}\left(1 + \frac{|C_{12}|^2}{\Delta_c}\right)
\end{aligned}
$$

$R_x \equiv \frac{1}{2}\langle \widehat{X}\widehat{P}_x + \widehat{P}_x\widehat{X}\rangle - \langle \widehat{X}\rangle\langle \widehat{p_X}\rangle$ ( similarly for $R_y$) and $\Delta_c = 4Im(C_{11})Im(C_{22}) - Im^2(C_{11})$. The generalized uncertainty relation is Constant with Time but Not Necessarily Minimized.

# The Density Operator

Calculations yield that

$$\widehat{\rho}_t(X_f y_f; X'_f y'_f) = \int dP_x dQ_x dp_y dq_y \, f(P_x \, Q_x \, p_y \, q_y, t) \times \langle X_f y_f | P_x \, Q_x \, p_y \, q_y \rangle \langle P_x \, Q_x \, p_y \, q_y | X'_f y'_f \rangle$$

Where $f(P_x \, Q_x \, p_y \, q_y, t)$ is Positive by Construction.

## THIS IS THE MAIN RESULT

# Discussion

The Density operator Acquires a Stationary Form, prerequisite for a Classical Regime.

Its form guarantees damping of initial intereference.

- Both of these properties are characteristic of Decoherence

BUT

Both Classical and Quantum Particle Decay into a Mixture of States With Each State Expressing Correlations Between the Two Systems, i.e. Quantum Entanglement!. This means

- We cannot assign properties to individual subsystems.
- The Stationarity Property and damping of intereference Expected with Decoherence is not Enough to Ensure Classicality.

THUS

Decoherence Cannot Solve the Measurement Problem!

# To be Done...

Can we at least say something about the effective equations of Motion?

- Effective Equations: It is an expected feature that the equations of motion for the "Classical" particle are:

$$M_x \ddot{X} + M_x \omega_x^2 X + \lambda^2 \int_0^t dt' G(t,t') X(t')$$
$$= \lambda y_{\Sigma_0} \cos \omega_y t + \lambda \frac{k_y}{\omega_y} \sin \omega_y t$$

  The "Classical" Equations of Motion of the Quantum System act as a Forcing Term with initial data distributed according to its initial Wigner Function. Remains to be checked...

- Examine Different Model: The quantum particle interacts with an environment as well. Will this lead to different results concerning Quantum Entanglement?
  - In this model the Classical Particle is to be interpreted as a measuring device just recording but not inducing Classiclality.

- Examine connection to traditional (Semiclassical) approach and if any meaning can be given to it.